Can Mathematics Reasonably Represent Nature?

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Abstract

Starting with Eugene Wigner's 1960 article, an expansion of natural science is made to study both inanimate and animate nature. Further expanding the goal of science to the full description of nature, the uniqueness of all objects and events is introduced shifting this goal of science to the description of unique objects and events. As our current scientific methodology and tools attempt to eliminate uniqueness, we cannot use current scientific methodology or current mathematical tools to describe such a unique world. Therefore we must retreat to science being the study of general laws. Given the importance of our mathematical tools to science, the question of whether our current tools are sufficient is asked. Bringing in historical analogies, a need for inventing new numeric tools is presented.

There is a well-known 1960's article by Eugene Wigner on "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [1]. In this article, Mr. Wigner states, "The physicist is interested in discovering the laws of inanimate nature." He moves quickly from the study of 'natural science' to the discipline of physics and the study of 'inanimate nature'. While he does imply this is a limited view of nature[i], his entire discussion is about the use of mathematics in physics. R. G. Hamming, in his 1980 response to Wigner [2], limits the discussion similarly.

A particular aspect of physics (and science in general), which Wigner and Hamming explicitly single out[ii], is the identification of repeatable results and results that do not change under a variety of situations, also known as invariance. Invariance is constructed as a generality – results that are essentially the same when an experiment is performed under generally different circumstances. The law of gravity, for example, is the same for two falling rocks or one falling rock and one falling person. The speed of light is the same (invariant) no matter the inertial reference frame of the observer. This aspect of invariance, of generalizing results, is key in the identification of scientific rules or laws that apply in many apparently different situations.

This aspect can also be seen in how mathematics developed – through the generalization of concepts. The concept of a (whole) number is the generalization of what one or two or some group of objects hold invariant regardless of what those objects are. A similar situation exists for geometry, where the abstract line or figure is what is general to any such line or figure. Hamming explicitly adheres to this generalization concept while Wigner implies it.[iii] The abstraction is from the individual objects we observe to what is general about all such objects – something we do not directly observe and only observe in our mind (an abstraction).

It is this method of generalizing that connects physics and mathematics so closely. Rules that apply to the concept of 'one' in mathematics should also apply to any quantity of 'one' in nature. Or, as is more likely the historical case, the generalization from physical objects of 'one', 'two', etc. became the concept of 'number' that we now use in mathematics. Similarly, a line in the sand, or rectangular piece of metal or square tile, which we observe around us, can be generalized into an abstraction of any line or figure. Rules that apply to the generalization (eg. 1 + 1 = 2, 5 + 4 = 9) should be expected to apply back to specific or physical instances of the generalization (eg. 1 apple + 1 apple = 2 apples, 5 tires + 4 tires = 9 tires).

Given this method of generalization for both disciplines, it would seem expected that mathematics would be so reasonably effective for science and the search for general laws of nature. This generalization method applies to numbers as quantities, to geometry as space, and to logic as reasoning. This author finds it strange that we might consider this situation somehow an unreasonable expectation. The unreasonable expectation is that somehow nature operates according to our generalizations and theories[iv] – rather than our generalizations and theories approximating nature.

The growth of mathematics and of physics has been a mutual and inter-dependent growth. We might find areas of 'pure' mathematics that advance for significant periods of time without direct application, such as topology or complex numbers. However we also find the development of mathematics directly tied to attempts to expand and grow science, in particular physics; such as Newtonian physics and calculus, differential equations, and vector analysis. Archimedes, Newton, and Gauss, are particular examples of practitioners indicating the generalization principle working in both the scientific and mathematical arenas.

If we use Wigner's limitation of the study of nature to that of physics, then we immediately remove a huge swath of nature and knowledge – that of animate (or living) nature. In response to Mr. Wigner's original statement (the title of his article) we must expand our view of natural science to more than physics and include animate nature. We must also consider how reasonable (or unreasonable) mathematics is in representing all of nature – not simply physics.

Let us first expand our view of 'natural science' beyond Wigner's admittedly limited view. We should note that the scientific study of animate nature in 1960 was still developing and could not rival that of physics. However, today there have been many advances in the study of animate nature, including identifying the human genome, facial recognition, and modeling of animals through computer graphics. The study of animate nature uses many of the same tools as the study of inanimate nature, including increasingly heavy use of mathematics and computers. The distinction between animate and inanimate nature still remains a mystery, as the question of what is lost when a living being stops living is still unknown.

The goal for both divisions of natural science apparently remains the same, that of developing invariant laws that apply to as many different situations and types of objects (or beings) as possible. This may be less the case for animate science, since categories and divisions (eg. a species or cell) are not as neat as inanimate objects. Also, animate objects need to be studied in a 'timely' manner, where the growth and movement of an animate object is crucial to understanding it. Humans, cells, and viruses cannot be understood as frozen objects in time, but rather as related objects that interact and evolve across a direction-sensitive timeline. This is different than the current study of inanimate science, which tends to exclude time or freeze the objects (and movement) in time – consider the concept of 'spacetime' or the idea of time-reversal in physics.[v]

Given the above, we should note that different branches of science (currently) appear to emphasize different aspects of nature, especially down the split of animate vs inanimate nature. Further, not all areas are entirely in agreement about how to represent nature generally, in particular the evolutionary aspect of animate nature vs the time 'invariant' aspect of physics and inanimate nature.

There is another aspect of the animate world we should consider – the importance of uniqueness to animate objects. We have computer graphics modeling individual hairs on an animal, genetics that look at the uniqueness of an individual's genetic makeup, facial recognition that looks at what uniquely defines an individual's facial features. While the study of inanimate objects tends to focus on what is invariant across objects, the study of animate objects includes what is invariant in describing the uniqueness of those objects. Uniqueness appears to be a more fundamental concern for animate nature than inanimate nature.

To consider the topic of this essay, "Can mathematics reasonably represent nature?" we must first make a quick digression concerning the 'final' goal of natural science. Should the goal of science be simply about developing 'invariant' laws or are we attempting to describe nature? Can invariant laws, even theoretically, completely describe nature? This is an assumption, even presumption, of the scientific method and certainly of some 'Theory of Everything'. However, consider a hypothesis (more properly a 'conjecture') that a universal aspect of nature both animate and inanimate, is the uniqueness of objects, beings, and events. It would seem an obvious conjecture, since it is a common experience for us. However, this conjecture would mean that no two objects have precisely the same characteristics and the more accurate (precise) we care to be the more we will see these differences.

Note that the whole scientific method appears built to reduce the impact of this (unproven) conjecture, since we require many 're-runs' of an experiment to rule out the unique character of any single result. This allows us to provide evidence of patterns across many different situations and to develop 'laws' that apply to many different objects in many different situations. If science is about finding these 'universal laws' that apply to most objects and situations, we are applying a methodology that rules out describing nature in all its unique aspects. The 'conjecture of uniqueness' appears to be an assumption of the scientific method, which the method is explicitly devised to overcome or excise. Applying this method removes from consideration this potentially most 'universal law' of uniqueness, all the while presuming its existence.

If science is about finding a set of universal laws, which are invariant across all objects and could (theoretically) predict all motion and change, then this seems uniquely at odds with a universal property that all objects are unique. More to the point, to achieve a full description of nature means to be able to describe individual situations, in all their individuality and uniqueness. A methodology, like the scientific method, that removes individual characteristics would seem at odds with this goal. As well, the unique character of animate objects might undermine a scientific methodology that removes the uniqueness of the objects it studies.

The above discussion should bring into question whether the methodology and tools being applied to inanimate nature are adequate for completely describing nature. Further, do the (and how do the) methodology and underlying tools impact the study of all of nature? If the goal of science is to describe nature, including the unique aspects of nature, then are (mathematical) tools, which remove the unique aspects of nature, short-circuiting this goal? If this is the case, then mathematics (and science using this methodology) cannot represent nature in its entirety.

This short-circuiting can be seen in the application of statistics to physical phenomena, as in the statistical theory of gases. In this situation we consider objects at one level of nature (eg. gas molecules) and agree to limit our knowledge of actions at this level (through the process of 'averaging') in order to focus on objects and/or measurements at a 'higher' level (eg. temperature and pressure). Note that in this case we explicitly leave out information on the uniqueness of actions of objects at one level in order to gain a generalization or rule that applies to other objects. Also note that this occurs because of the mathematical tools employed (statistics and also probability), rather than some goal of physics. This technique is quite useful in developing invariant laws for certain objects, but at the expense of unique aspects of other objects.

Let us return from this digression and step back from attempting to describe the uniqueness of nature to the objective of science as discovering invariant laws. With this more limited goal, we can now feel comfortable about using 'averaging' tools, such as statistics and probability. Along with Thomas Brody [3], most scientists would admit our theories are approximations to nature anyway[vi]. Our theories do not pretend to be entirely precise models of nature. Lee Smolin, in his book 'Time ReBorn' [4], uses the term "effective theory"[vii] explicitly stating that the theories of physics are always approximations to nature. In what situations they are 'effective' is key to understanding their use and application.

It would be hard to find historical evidence that our current theories will continue to hold, no matter how precise our measurements become. If we are fine with approximations, then the (un-)reasonableness of mathematical models for revealing invariant laws can be directly correlated with approximating nature to whatever degree we desire – or our tools can measure. If, as occurred about 120 years ago, our degree of approximation lies in the 10^5 level, then we could feel (as Lord Kelvin allegedly stated[viii]) that we were near understanding the world and only needed more precise measurements. Today, after the 'quantum revolution' drastically changed the theories of Lord Kelvin's day and with a degree of approximation around 10^12, we might again feel we are near understanding the world (now universe). However our 'understanding' is strictly tied to our degree of approximation of that universe (and may not be entirely correct even at that agreed approximation).

Theories can be said to apply in specific situations and with specific limitations as to their application. Thomas Brody provides a good epistemology for the application of physical theories to specific areas of nature[ix]. Associating levels of approximation to our theories would go a long way to explaining why, a hundred years after it's discovery, the gravity of general relativity is not generally taught in high school physics while Newtonian gravity is. We continue to teach Newtonian gravity in our textbooks - even though it has been supposedly 'replaced'. The reason is that the equations of Newtonian gravity work quite well - for normal 'human' situations (human-level approximations). Since physical theories are always approximations, we can be justified in using one that works (is 'effective') for our approximate needs.

This brings up another point regarding the difference between mathematical equations used in a theory and the theory itself. Mathematical equations are theory-agnostic, since the same equation can be used in different theories or to different approximations. For gravity, Newtonian equations work up to a certain degree of accuracy, and then diverge from general relativity under different and more precise circumstances. So, the equations (not the theory) can be said to be 'effective' for both theories – within a given realm of application. Simply identifying the mathematics of a situation does not provide an understanding of it. The equations of Newton work, reasonably accurately, for many situations, even if Newton's theory has been replaced by Einstein's general relativity. The epicycles of Ptolemy worked at least as accurately as Copernicus' calculations, when Copernicus proposed them. The mathematical equations do not make a physical theory; they only model specific conditions and results – to a certain approximation.

This separation of mathematical models into approximate categories can be seen in a number of situations, including the shift from our 'classical' world to the 'quantum' world. Our older classical equations still hold (and we still teach them) - to a certain degree of accuracy – even though the underlying theory, to a higher level of accuracy, has been replaced with the quantum world. The history of knowledge about the natural world can be defined by the levels of approximation of the mathematical models devised to describe that world. These equations provide the predictive power we leverage for technology and further experiments. The equations do not, however, provide the understanding that a theory provides as to 'what is going on' and what else might we be able to grasp using a theory.

Ivor Grattan-Guinness, in his article "Solving Wigner's Mystery" [5], provides an evolutionary perspective about how theories are created (he uses 'theory-building') and the relationships between theories and 'notions', as he calls them[x]. He rightly observes that "When forming a problem and attempting to solve it, a scientist does not work in isolation: he is operating in various contexts, philosophical, cultural and technical, in some cases consciously recognised but in others intuitively or implicitly adopted. "

There is a lot that can be said about how we build new theories, which both Grattan-Guinness and Thomas Brody discuss. However, the assumption regarding the tools used is that we always model nature using mathematical models. As we observed earlier, our mathematical tools can impinge upon our goals (eg. the conjecture of uniqueness) and are also theory-agnostic. The question should be asked: Do the mathematical tools we use to build our models bias, impinge, or otherwise impact this process of 'theory-building'?

To gain an understanding of how implicit our use of models is and the potential impact of that use, let us consider a topic at the vertex of pure mathematics, applied mathematics, and the use of mathematics in science. This is the topic of numeric representations, of symbolic representations of quantity and measure – representations of numbers. We have the concept of a number and we have the representation of that concept – the model of that concept. Even at the most basic level of a number and of representing a quantity, we use representations (ie. numeric symbols), not the actual concepts.

Consider that we use fractions to represent Rational numbers and we use decimals (or positional base numerics) to represent Real numbers. Fractions are models for Rationals, however they do not fully represent a Rational number, which is quite unique. The Rational number 'one half' is a unique Rational number, however it can be represented by an infinite number of fractions: ½, 2/4, 3/6, 4/8, … The representation has characteristics which are not part of the underlying concept being represented. If we look at decimals, Real numbers, and fractions, we can find some easy examples where the representations do not match the concept being represented. Using fractions, we get exact finite results, while decimals, for the same numbers, sometimes provide only approximate results, such as: 1/3 + 2/3 = 1 while 0.333… + 0.666… = 0.999… Note that we are forced to make 0.999… equal to 1.000… in order for the two representational systems, representing the same numbers, to agree on the results. There are many situations where we have a decimal ending in …999… (eg. 0.1999…, 0.024999…) that needs to be forced to be the same as …000… (eg. 0.2000…, 0.02500…). Again, the representation has characteristics that are not part of the underlying concept of a Real number.

Let us be clear about the situation when science is involved (at the vertex): Not only are we using mathematical models to represent nature, but we are also using numeric representations to model numbers and number systems within mathematics. So the models used in science have a two-fold modeling going on – the mathematical models to represent nature and the numeric representations of numbers for quantities used in those mathematical models. We might be clear about the approximate nature of mathematical models in science, but have we understood the impact of numeric representations of numbers on mathematics (from a theoretic perspective) and on the application of mathematics to science?

Most critical would be the question of whether our current representational systems for numbers are actually adequate for modeling the natural world. We might think this question obvious ("it works, doesn't it?"), however consider the Pythagoreans, who believed in the ultimate character of nature through numbers and ratios. In their studies of mathematics and geometry, they discovered a relatively simple situation (the length of the diagonal of a 1 unit per side square) that they proved could not be represented by any ratio (our fractions or Rational numbers). It took close to two millennium to resolve this situation. The resolution required the expansion of what a number is, through the addition of 'irrational' numbers, all of which are encompassed by the Real numbers. This expansion only came about after a means of representing these 'new' numbers was invented – the decimal numeric system. And it should be noted that all of 'modern' science comes after this expansion of the concept of number and could not exist using only (as we now call them) Rational numbers and their representation as fractions. So there is an historic precedent we might need to consider: We are using a concept of number and representational system for numbers that is limiting our models - in both mathematics and science.

What evidence is there for this situation, especially given how amazingly (unreasonably?) accurate our current theories appear to be? The place to start might be with our assumption that Real numbers, especially as represented today (via decimals, logarithms, and various bases) are the end-all of linear numbers (comprising a line or continuum). What if we have limited our conception of number, and of a geometric continuum, to our conception of a 'Real' number – because representations like decimals for Real numbers are all we know? This would be similar to the situation of the Pythagoreans, where ratios (Rational numbers) were all they knew and comprised a continuum that 'irrational' numbers did not have a seat at. What if 'more' numbers than Real numbers can exist on a continuum? This would produce a 'denser' continuum than is possible with Real numbers. We know of 'more' numbers - 'imaginary' numbers. When combined with Real numbers, we obtain the Complex number system. This system is heavily used in models of nature today. What if, like the Pythagoreans, we have not properly understood the limits of this system – could such a misunderstanding have an impact on our mathematical models of nature?

We currently consider Complex numbers to be 2-dimensional numbers, since we represent them using two different Real numbers. Is this 2-dimensionality truly a property of Complex numbers, or might this be a consequence of how we represent them? Maybe we are mistaking characteristics of our representation (our model) of these numbers for characteristics of that which is being represented (Complex numbers). Consider the extension of Rational numbers into the range of Real numbers: This can be accomplished by taking two rational numbers 'a' and 'b' and combining them with an irrational number as a mathematical expression: (example: $a + b * \sqrt{2}$). Note how similar this is to how we represent Complex numbers, with ‘x’ and ‘y’ being Real numbers: (x + y * √-1). Note that we already consider Real numbers to exist on a linear continuum, even though we can devise a 2-dimensional representation of them.

Using this technique of extending the Rationals, we can create many Real numbers, although not all Real numbers. That is why this technique is an extension of the Rationals into the Real numbers and not an equivalent method of representing the Real numbers. Note that all this can be done strictly with fractions and does not require the decimal numeric system for this extension of Rationals into Reals. If we only use fractions, the primary issue with this expression is the undefined ‘irrational’ term involved (in this example √2). It is undefined precisely because our means of representing a number (via fractions) does not include this value as a representable value. Once we invent a means of representing √2 as a value (ie. using decimals), then the entire mathematical expression can be collapsed into a single value and the 2-dimensional character disappears.

In a similar vein, our current technique of representing Complex numbers is limited to representations of Real values (eg. decimals), along with an undefined ‘imaginary’ term. We are forced to represent Complex numbers as an expression (an ‘extension expression’) precisely because we do not have a means of representing an imaginary nusmber as a ‘value’ – like we can represent a Real number as a decimal ‘value’. The term ‘value’ means a representation capable of calculation – of adding, subtracting, multiplying, dividing, ‘exponentiation’, and ‘logarithms’. In particular, what we are completely unable to accomplish, without such a value, is the addition and subtraction operations. Without the ability to add or subtract a term, we are stuck with the expression of (x + y * √-1). This is the same situation we found for the extension of fractions into Reals.

What if we could invent a means of representing imaginary numbers as an ‘additable value’? This would involve inventing an entirely new means of representing numbers and could include inventing a representation of numbers that are defined using a negative base (since √-2 = (-2)^ ½). Note also that, for the extension of fractions into Reals, the expression (a + b * √2) does not give us all Real numbers. We might find that what we call Complex numbers are really just an extension into another system of numbers. We might find that Quaternions (employing ‘i’, ‘j’, ‘k’ imaginary values) are closer to this new system than Complex numbers are (Quaternions are used in a number of physical models today.)

What might be the impact to mathematical models of such a new representational system? First, a number of calculations could be tremendously simplified – since we could collapse any complex expression into a single value. Second, we would need to understand what a complex value (as a single value not separated into ‘real’ and ‘imaginary’ parts) maps to in nature. Is a single complex value something different than the two parts we are forced to work with today? Might such a value map to something we cannot map to with our current system?

Finally, what if these single values can be ordered, since if we can add and subtract them, they would seem to be ‘order-able’? Now we would have a different ‘Complex’ continuum and this continuum would extend to all cases where points are mapped to numbers – lines, surfaces, spaces, etc. This would also apply to other mathematical objects, such as vectors and matrices. The density of all these continuums would increase, in particular those of manifolds and spaces. Such an invention would appear to significantly impact that vertex of theoretic and applied mathematics to develop models of nature.

We need to question the adequacy of the underlying mathematics we apply to nature. We generally presume that our current representation of complex numbers (and quaternions) is adequate to the task of modeling nature. Do we have evidence of this? We are aware that we do not have fully formed values for complex or quaternion numbers. Our current representation always involves unknowns (i.e.. 'i', 'k', 'j'). So how can a theory built on these unknowns be 'real'? This situation is like pre-decimal times, when a Real number (at least an 'irrational' one) was not considered a real number. Only rational numbers (represented as integer ratios) were considered 'numbers'. Some people saw that solutions to certain algebraic problems could not be solved using ratios, but they could not properly represent these irrational (and 'transcendental') values – so the numbers were somehow phantom numbers. Only after decimals came into common use was representing them as values possible. We still do not have an adequate representation of complex or quaternion values. Since such numbers pervade many physical models, how can we expect these models to be adequate representations of nature?

References

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Notes:

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[i] Near the end of the article in [1], by asking "if we could, some day, establish a theory of the phenomena of consciousness, or of biology, which would be as coherent and convincing as our present theories of the inanimate world."; Wigner implies there is more than the study of inanimate nature.

[ii] In [2], Hamming states, "The fundamental role of invariance is stressed by Wigner. It is basic to much of mathematics as well as to science."

[iii] In [2], Hamming states, "we see that one of the main strands of mathematics is the extension, the generalization, the abstraction - they are all more or less the same thing-of well-known concepts to new situations." In [1], Wigner states, "it is unquestionably true that the concepts of elementary mathematics and particularly elementary geometry were formulated to describe entities which are directly suggested by the actual world"

[iv] From Boyer [6] "A new difficulty, however, then entered into Greek thought, for the Pythagorean mathematical concepts, abstracted from sense impressions of nature, were now in turn projected into nature and considered to be the structural elements of the universe…. Geometry was regardedbt them as immanent in nature, and the idealized concepts of geometry appeared to them to be realized in the material world."

[v] Smolin's book [4] is mostly about this time-less aspect of current physics.

[vi] Brody states [3] "The need for only a finite (and commonly even small) depth to adjust an epistemic model has its roots in another very fundamental aspect: the model need not be more than approximate."

[vii] Smolin heads a sub-section of his book [4]. "Effective but approximate theories." He starts the section with: "All the major theories of physics are models of the truncations of nature produced by experimenters. They may have been imagined as fundamental theories when they were invented, but over time theorists have come to understand them as effective descriptions of a limited number of degrees of freedom."

[viii] There is much dispute about this quote: "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement" http://en.wikiquote.org/wiki/William_Thomson

[ix]Brody states [3] "Relatively low-level models are built with fairly specific aims; a higher-level model is built so that with its help many different lower-level models can easily be built to suit quite a wide range of different aims. Its purpose can therefore not be very exactly circumscribed. As the level and hence generality of the model increases, its purpose becomes more wide-ranging."

[x] From [5], "In the discussion that follows, 'notion' is an umbrella term covering not only objects such as function and matrix but also concepts such as convexity, systems of symbols, and proof methods, that occur in mathematical theories; these latter are often called 'topics' when they include individual theorems or algorithms as well as larger- scale bodies of results."