Electromagnetic Waves for the 21^{st} Century

Mark A. Newstead and Stephen C. Newstead
email: mark.newstead@alumni.lboro.ac.uk

In this paper, we take the ideas surrounding electromagnetic waves having a wave-particle duality, matter, antimatter collisions and mass-energy equivalence to bring electromagnetic waves into the 21^{st} century.

1 A History of Electromagnetism

In 1861, James Maxwell published a paper entitled "On Physical Lines of Force" [14], where he expanded on the work of Gauss, Faraday and Ampere. In this paper he showed that electricity and magnetism were not independent from each other, as was thought at the time, but were the two components of electromagnetism. For example a magnet can be created by simply passing electricity through a coil of wire [16]. Maxwell's equations that describe electromagnetism (written in modern form) are [8]:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{4}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, t is time, ρ is the total charge density, ϵ_0 is the permittivity of free space, μ_0 is the permeability of free space and $\mathbf{J}$ is the total current density. In simple terms these equations tell us the following:

- Equation 1 (Gauss's law) describes the relationship between the electric charge and the electric field it creates.
- Equation 2 (Gauss's law for magnetism) states that there are no magnetic monopoles (i.e. a magnetic north and south can never be separated).
- Equation 3 (Faraday's law of induction) describes how a changing magnetic field will create an electric field perpendicular to the magnetic movement.
- Equation 4 (Ampere's law with Maxwell's correction) describes how a magnetic field can be generated by either a changing electric field or by the constant flow of electric charge. It also states (similar to equation 3) that the generated magnetic field will be perpendicular to the electric field and charge.

A few years later, in 1864, Maxwell published another paper entitled "A Dynamical Theory of the Electromagnetic Field" [15]. In it he showed how one solution of his equations described an oscillating electric and magnetic field, which were perpendicular to each other, that could and would be able to travel through space at a constant speed. We now refer to these oscillating fields as electromagnetic waves (or photons, which we define to be a single electromagnetic wave) that travel at the speed of light in a vacuum. In fact light (i.e. the thing we use to see with) is only a very small part of the electromagnetic spectrum [7]. Electromagnetic waves go from ultra long waves, radio and TV waves, microwaves, infrared waves, visible light, ultraviolet rays, X-rays and gamma rays, in order of lowest to highest energy.

Moreover, although electromagnetic waves travel at the speed of light in a vacuum (i.e. almost 300 million metres a second, or equal to $1/\sqrt{\mu_0\epsilon_0}$), this is their maximum speed [8]. Note that we have said “in a vacuum”, as this defines what medium the wave is travelling through and is often not mentioned. However, this is important since the waves travel at different constant speeds depending upon the medium they are travelling through. For example in water, electromagnetic waves travel at about 225 million metres a second [1]. This change in speed occurs as the electromagnetic wave enters or leaves a medium, which also causes the length of the wave (i.e. its wavelength) to increase or decrease, depending upon whether the wave speeds up or slows down, respectively [10]. It is this change in speed and wavelength that causes a straight pencil to appear bent in a glass of water and why a prism splits white light into a rainbow.

Sometimes though, electromagnetic waves do not behave like waves, but like particles [10]. By particles we are meaning something that is very small, but has mass. An example of an electromagnetic wave behaving like a particle, is when one with sufficient energy hits an electron ejecting it from its atom. This is known as the photoelectric effect [1, 10] and is the principle behind solar panels. Thus electromagnetic waves are said to have a wave-particle duality to them.

2 Mass-Energy Equivalence

In 1905, Albert Einstein proposed that mass and energy were equivalent to each other, in his paper entitled “Does the inertia of a body depend upon its energy-content?” [5]. This equivalence is summed up in the simply equation

$$E = m_r c^2, \tag{5}$$

where E is the energy, m_r is the relativistic mass and c is the speed of light in a vacuum. Note that relativistic mass means that people (observers) travelling at different speeds will obtain different values for the object’s mass [6]. The important thing about this equation though is the speed of light squared on the right hand side, as this is an incredibly large number. What this implies is that mass is a very dense form of energy, since only a tiny piece of mass will produce a very large amount of energy. Moreover, it could be argued that if energy is sufficiently compressed then some of it would portray itself as mass. Furthermore, this equation implies that mass can be created and destroyed. An example of this is when a particle and antiparticle pair collide, all their mass energy is converted into electromagnetic waves [4, 9].

3 Electromagnetic Waves for the 21^{st} Century

We have seen that Maxwell's equations describe what electromagnetic waves are and how they move through space, but there are some questions that we do not know the answer to. One of these questions is whether or not electromagnetic waves have mass. It is theorised by both relativity and quantum mechanics that they do not [2, 6]. Experiments though regarding their mass can only currently say that their mass is less than 10^{-49}g [17]. Although this is an extremely small value, we have to remember that the energy of a single electromagnetic wave is also extremely small [10, 18] and thus we would not expect them to have a large mass. Furthermore, if electromagnetic waves were found to have mass then this would have wide implications in physics [17]. Examples of these implications are that it would allow magnetic monopoles to exist, define whether charge is conserved and quantized, as well as whether there could be charged black holes in the universe.

We have already mentioned that when an electromagnetic wave slows down, its wavelength decreases, and this is governed by [10]

$$v = f\lambda, \tag{6}$$

where v is the speed of the electromagnetic wave in the medium, f is its frequency and λ is its wavelength. Thus if the wave was travelling really slowly, it would have an incredibly short wavelength, implying that its energy was contained in a very small volume. Now from the mass-energy equivalence this would potentially imply that some of the energy may portray itself as mass, instead of electromagnetism. Additionally, we know that when a particle, antiparticle pair collide all their mass is converted into electromagnetic waves [9, 4]. This would further imply that there is indeed some link between mass and electromagnetism. Moreover, the amount of mass portrayed can be calculated by equating $E = m_r c^2$ and $E = hf$ (i.e. the energy of an electromagnetic wave of frequency f, where h is Planck's constant) [18], giving

$$m_r = \frac{hf}{c^2}. \tag{7}$$

However this is the relativistic mass of the wave not its rest mass. The conversion between the two is given by [6]

$$m_r = \frac{m_v}{\sqrt{1 - v^2/c^2}}, \tag{8}$$

where m_v is the rest mass. Thus substituting equation (7) into equation (8) we obtain

$$m_v = \frac{hf}{c^2}\sqrt{1 - \frac{v^2}{c^2}}. \tag{9}$$

Interestingly, this equation implies that the mass of an electromagnetic wave is inversely proportional to its velocity. Thus, if the wave was travelling at the speed of light then its mass would be zero correlating with what has been theorised by relativity and quantum mechanics [2, 6]. Furthermore, it would agree with the relativistic statement that only massless particles can travel at the speed of light [6], since as stated, at this speed the wave would have zero mass. Conversely, if the wave were to impact something, say an electron, then at that moment, before its energy was transferred, it would obtain its maximum mass.

Moreover, the fact that an electromagnetic wave had mass when not travelling at the speed of light would significantly help to explain its wave-particle duality. This would follow since at all times, apart from the moment it became stationary, it would be a wave. However, we would now have the concept that the wave can have mass, especially at the point when its velocity becomes zero. Hence in this situation it would act exactly the same as if it were two particles colliding together.

Furthermore, we have that at all times the total energy of the electromagnetic wave is conserved; its energy is just transferred between electromagnetism and mass. Hence the equation

$$E = hf, \tag{10}$$

would always calculate the total energy of the wave, since its frequency always remains fixed, i.e. it is the total energy of the wave that defines the frequency not just the electromagnetism energy. Additionally, we note that the change in mass with speed is different for electromagnetic waves than for particles, as defined by relativity [6]. The reason for this is due to the total energy of the electromagnetic wave remaining constant, whilst energy is continuously added to the particle, to accelerate it. This extra energy gained by the particle then appears in the form of mass, which is why its mass increases with velocity. Again the added energy often comes in the form of electromagnetism, which ends up manifesting itself as mass.

Also, if we substitute equation (9) into the energy, mass and momentum equivalence [6], i.e.

$$E^2 = m_v^2 c^4 + p^2 c^2, \tag{11}$$

and the rearrange for p, the momentum, we obtain

$$\begin{aligned} p &= \frac{hfv}{c^2}. \qquad (12) \\ &= m_r v. \qquad (13) \end{aligned}$$

Thus, if we let m_r be the relativistic mass of a wave or particle, then we end up with a consistent momentum equation, the latter of which is the same as defined by relativity [6]. Additionally, if we assume that an electromagnetic wave is travelling at the speed of light (i.e. $v = c$) then equation (12) becomes the standard equation for the wave's momentum [6]. Overall, therefore this consistency in how we calculate the momentum of a particle or wave would seem to add further credence to the idea that electromagnetic waves can have mass.

Lastly, although we stated at the start of this section that experiments had narrowed down the rest mass of an electromagnetic wave to less than 10^{-49}g, this may no longer hold based upon this work. The reason for this is that scientists have appeared to assume that the rest mass for an electromagnetic wave is a constant, independent of the wave's frequency or velocity [17]. Thus some of the experiments have tried to measure the rest mass of a wave whilst it is travelling through space (i.e. a vacuum) and in that situation we would agree with relativity and quantum mechanics that all waves would have no mass. Other experiments though have looked at radio waves travelling through the air. In this situation, the mass of any one particular wave would have been extremely tiny, even compared with 10^{-49}g. There are two reasons for this; firstly radio waves have very low frequencies (from 1KHz to 1GHz [10]) and therefore even when "stationary" their mass is still extremely small. Secondly, electromagnetic waves travelling through the air are still moving at 99.97% of the speed of light in a vacuum [1]. Hence again our work states that most of the wave's energy would be in the form of electromagnetism and not mass; and thus a very low rest mass would be seen, if any.

Overall therefore, we propose that electromagnetic waves do have mass, when they are travelling slower than the speed of light in a vacuum. Furthermore, this mass is proportional to the frequency of the wave and thus a gamma ray would portray more mass at a given speed than a radio wave.

4 An experiment to test this idea

In the previous section we stated that the mass of an electromagnetic wave is inversely proportional to its velocity, as described by equation (9). Therefore in this section we will describe an experiment that may be able to test whether indeed this is the case. Now in order to best detect whether electromagnetic waves have mass, we should slow the waves down as much as possible, as this "generates" more mass. In fact over the last couple of decades scientists have been able to purposely slow electromagnetic waves to several miles an hour in various experiments [12, 13]. For example the Rowland Institute for Science slowed these waves down to 38 mph in 1999. Even though scientists can significantly slow down the wave speed, the mass of a bunch of electromagnetic waves is still minute. Therefore we propose that in order to measure this minute mass, a very sensitive balance scale is used, as shown in Figure 1. Moreover, since the mass of the wave is proportional to frequency then gamma rays should be used. These will then cause the greatest unbalancing of the scales for a given wave speed.

Figure 1 shows a schematic diagram of an experiment that should be able to test whether electromagnetic waves have mass. On the left hand side of the balance are two closed tubes (represented by the white and grey circles) that the gamma rays will pass through. The ball on the right hand side is the counterbalance weight, which is sufficiently heavy to balance the scales when no gamma rays (or any other electromagnetic waves) are passing through either of the left hand tubes. The reason for having two tubes on the left hand side is that they contain different mediums. The top white tube contains a vacuum, and thus when the waves pass through it, the balance scale should remain level. We note that the balance will only remain level if a very thin material, which has a minimal affect on the speed of electromagnetic waves, is used on the two ends of the

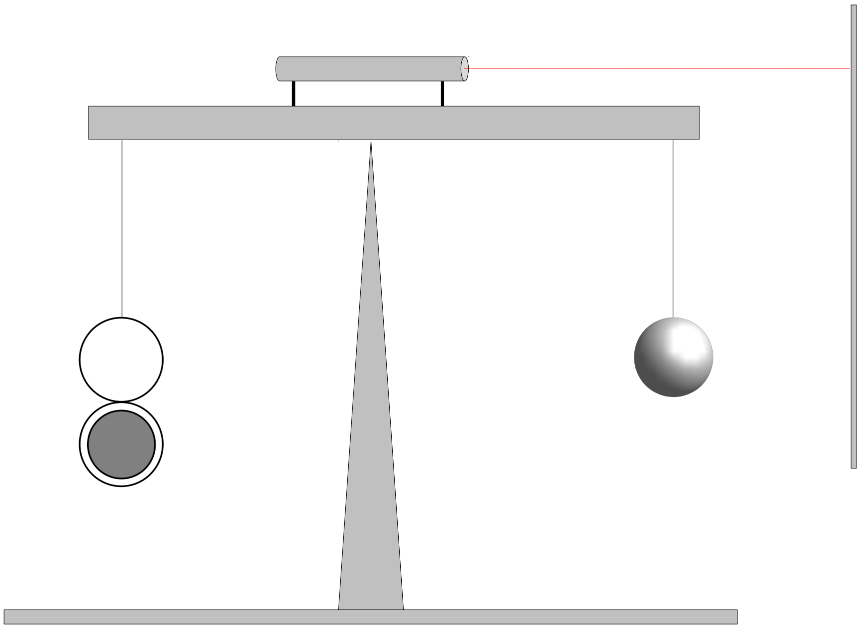

Figure 1: *A schematic diagram for an experiment that would be able to test whether electromagnetic waves have mass as we have proposed, or not. Moreover, it should be able to determine whether the mass of an electromagnetic wave is proportional to its frequency and inversely proportional to its velocity.*

tube. Otherwise the mass "generated" by the gamma rays as they pass through the ends of the tub will be sufficient to unbalance the scales. The bottom grey circle (tube) on the left hand side contains the medium that significantly slows electromagnetic waves down. Hence by only changing which tube the electromagnetic waves pass through, determines whether the balance is balanced (i.e. in the case of the vacuum tube) or not (i.e. in the case of the grey tube). Also some of the waves maybe absorbed by the medium in the grey tube, as they pass through it, producing heat which may affect the results. Therefore we propose that as much as possible of the tube is covered in a reflective surface, such that the minimum amount of energy is lost externally, along the tube. This reflective surface may require multiple layers to retain the maximum amount of energy. These reflective surface(s) are represented by the black circle, which surrounds the grey one. Additionally, as this is such a fine experiment, we have added a laser to the top of the balance beam. Therefore any movement in the beam is amplified by the movement of the laser dot on the screen (shown on the far right hand side). Moreover, the further the screen is from the balance the greater the amplification of any movement, allowing the minute mass "generated" by the electromagnetic waves to be detected.

Furthermore we note that gamma rays can also destroy atomic nuclei and may therefore cause the apparatus to lose mass, since the nuclei will be heavier than the resulting electromagnetic waves [4, 9]. However as the mass of the electromagnetic waves should "instantly" become non-zero as they enter the medium slowing the wave down, then the experiment should be able to detect the change in weight almost immediately. Thus the experiment should not need to run for a long time, which has the side effect of limiting the number of atoms destroyed by the gamma rays. Also it would be important to make the whole balance as light as possible. This would minimise the inertia effect of the balance as well as the bearing friction at the balance point. Overall though, if the experiment is set up carefully enough, then it should be able to detect the mass of an electromagnetic wave, as well as having the possibility of measuring it.

5 How Gravitational Fields Affect Electromagnetic Waves

We have already mentioned that the speed of an electromagnetic wave is dependent upon the medium it is passing through [1]. Therefore unlike particles [11], waves are unable to speed up or slow down as they are entering or leaving a gravitational field. However this does not mean that they are unable to gain or lose energy, it is just that the energy is in a different form, frequency. This can be easily seen if we initially take a Newtonian approach [11, 3], which will then be extended into relativity. Let us take two points, the first one being far away from any gravitational fields, whilst the second is within a weak gravitational field. Thus by conservation of energy we have that

$$E_2 = E_1 - U, \tag{14}$$

where E_1 and E_2 are the energy of the wave at points 1 and 2 respectively and U is the potential energy. In the "Electromagnetic Waves for the 21st Century" section, we stated that the total energy of a wave is given by $E = hf$, thus we have

$$\begin{aligned} E_1 &= hf \qquad (15) \\ E_2 &= hf_g \qquad (16) \\ U &= -\frac{GMm}{r}, \qquad (17) \end{aligned}$$

where f is the frequency outside the gravitational field, f_g is the frequency inside the gravitational field, G is the Newtonian gravitational constant, M is the active mass creating the gravitational field and m is the relativistic mass of the wave (which follows exactly the approach as we saw with the momentum equation above, i.e. we replace mass with the relativistic mass of the wave). Therefore substituting these equations into equation (14) we obtain

$$\begin{aligned} hf_g &= hf + \frac{GMhf}{c^2 r}, \qquad (18) \\ \implies \qquad f_g &= f\left(1 + \frac{GM}{c^2 r}\right). \qquad (19) \end{aligned}$$

This equation tells us that the stronger the gravitational field (i.e. the larger the mass M), the higher the frequency change. Moreover, this equation represents the first two terms of the series (in a weak gravitational field) for the frequency change as given by the Schwarzschild metric in general relativity [6]. It states that

$$\begin{aligned} f_g &= \frac{f}{\sqrt{1-\frac{2GM}{c^2r}}}, &(20)\\ &= f\left(1+\frac{GM}{c^2r}+\frac{3G^2M^2}{2r^2c^4}+\frac{5G^3M^3}{r^3c^6}+\frac{35G^4M^4}{8r^4c^8}+\dots\right), &(21)\end{aligned}$$

Thus, just as a particles speed changes as it enters or leaves a gravitational field, so does an electromagnetic wave's frequency.

Moreover, we know that gravitational fields exist over vast distances and are non-linear in strength with distance [6, 11]. Therefore, if a wave was passing through the gravitational field of a massive object then the frequency change across the width of the electromagnetic wave would be different. Thus the troughs of the waves (being closer to the massive object) would increase in frequency more than the peaks of the waves. Although this would only be a very slight difference, it would cause the waves to be bent towards the object creating the gravitational field, especially as gravity acts over large distances. Furthermore, this bending process is very similar, although not as abrupt as what occurs in refraction. In the case of refraction it is the speed and wavelength that change, whilst the frequency remains a constant, whereas gravitational bending causes the frequency and wavelength to change. We should note though that gravity does not cause the dispersion of different frequency of electromagnetic waves, as refraction does [1]. The reason for this difference is that in the case of gravity the wave's momentum and bending force cancel each other out, causing all waves to be bent equally (just as two particles of different mass fall at the same rate within a gravitational field). Although this result, that gravity bends electromagnetic waves has been observed and explained by relativity [6], its explanation only points out that the waves follow the geodesic of space-time (i.e. a straight path on a curved surface). However the arguments made here would explain what an external observer, observing an electromagnetic wave oscillating past a massive object would "see".

6 Conclusions

In this paper, we have tried to take electromagnetic waves into the 21^{st} century by investigating whether they could have mass, since this would have wide implications from astronomy to nuclear physics [17]. To start with we mentioned that mass is just a very dense form of energy, which can be fully released when a particle, antiparticle pair collide [4, 9]. We then took this idea and applied it to electromagnetic waves, since the slower they travel the more compact their energy is [18] while noting that particle, antiparticle collisions produce these waves [4, 9]. Thus we proposed and calculated that electromagnetic waves do have mass when they are travelling slower than the speed of light (i.e. when they are not in a vacuum). In particular we found that the amount of mass they portray monotonically increases as their speed decreases, and that their mass is directly proportional to their frequency.

From this description of an electromagnetic wave's mass being dependent upon its speed and frequency, we stated that some of the experiments to find its mass would be invalid. This is due to them searching either for a fixed mass for all electromagnetic waves, measuring their mass whilst they were travelling through space or using low frequency waves within the air [17]. Due to this and the novel idea that a wave's mass could be speed and frequency dependent, we proposed a new experiment that may be able to test our proposals. The essence of the experiment was to pass gamma rays through a medium that slowed them down to a few miles a hour, that was attached to a balance scale. Thus, if the balance moved whilst the gamma rays were passing through the medium then this would imply that the waves indeed had mass, although as we pointed out this experiment would be very difficult to accomplish.

Lastly we explained how gravity affects electromagnetic waves, since unlike particles [6, 11], their speed is fixed for a given medium [1]. What we showed is that as a wave enters a gravitational field it frequency increases, directly proportional to the strength of the field. Moreover, this additionally frequency energy came directly from the gravitational potential energy the wave was losing. Furthermore, we proposed that it was actually this frequency change happening, at a different rate across the wave that causes it to be bent as it passes a massive body, like the Sun. Therefore, if an external observer was able to measure the distance between the peaks and troughs of a wave, they would see that the troughs, being closer to the massive body, were closer together.

Overall therefore this paper gives a new approach to the mass of an electromagnetic wave, as well as outlining an experiment that may well help prove these propositions.

References

[1] M. Born and E. Wolf. *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*. Cambridge University Press, 7 edition, 1999.

[2] C. E. Burkhardt and J. J. Leventhal. *Foundations of Quantum Physics*. Springer, November 2008.

[3] S. Chandrasekhar. *Newton's Principia for the common reader*. Oxford University Press, April 2003.

[4] F. Close. *Antimatter*. OUP Oxford, January 2009.

[5] A. Einstein. Does the inertia of a body depend upon its energy-content? *Annalen der Physik*, 1905.

[6] A. Einstein and R. W. Lawson. *Relativity: The Special and General Theory*. Dover Publications, March 2001.

[7] D. S. Falk, D. R. Brill, and D. G. Stork. *Seeing the light: optics in nature, photography, color, vision, and holography*. Longman Higher Education, January 1986.

[8] D. A. Fleisch. *A student's guide to Maxwell's equations*. Cambridge University Press, January 2008.

[9] G. Fraser. *Antimatter: The Ultimate Mirror*. Cambridge University Press, 1st edition, July 2002.

[10] I. Kenyon. *The Light Fantastic A Modern Introduction to Classical and Quantum Optics*. Oxford University Press, 2008.

[11] J. M. Knudsen and P. G. Hjorth. *Elements of Newtonian mechanics: including nonlinear dynamics*, volume 3. Springer, June 2000.

[12] P.-C. Ku, F. Sedgwick, C. Chang-Hasnain, P. Palinginis, T. Li, H. Wang, S.-W. Chang, and S.-L. Chuang. Slow light in semiconductor quantum wells. *Optics Letters*, 29(19):2291–2293, 2004.

[13] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau. Observation of coherent optical information storage in an atomic medium using halted light pulses. *Letter to Nature*, 409:490–493, 2000.

[14] J. C. Maxwell. *On physical lines of force*. s. n., 1861.

[15] J. C. Maxwell and T. F. Torrance. *A Dynamical Theory of the Electromagnetic Field*. Wipf and Stock Publishers, 1996.

[16] J. C. Slater and N. H. Frank. *Electromagnetism*. Dover Publications Inc., June 1969.

[17] L.-C. Tu, J. Luo, and G. T. Gilles. The mass of the photon. *Reports on Progress in Physics*, 68:77–130, 2005.

[18] F. Wilczek. The universe is a strange place. *International Journal of Modern Physics A*, November 2005.

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