What is Ultimately Possible in Physics?
A Geometrical Approach Towards a Theory of Everything (TOE)

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Abstract – In 2007, A. Garrett Lisi published "An Exceptionally Simple Theory of Everything" [1] (TOE) in which he presented a geometrical approach towards TOE based on *E*8 and the Gosset lattice. Although Lisi's approach has been very well received by FQXi members and pop culture, it has received some serious physics critique – most notably from Prof. Jacques Distler of the University of Texas. Distler's [2] fundamental complaint is that *E*8 is not large enough to properly contain three chiral generations. Still, it seems appropriate to consider Lisi's geometrical approach a reasonable way to model an approach towards a TOE – a "toy model" TOE as such.

Introduction – The author recently posted "A Case Study of the Geometrical Nature of Exceptional Theories of Everything" [3] and published a book on "New Approaches Towards A Grand Unified Theory" [4]. These two papers present the possibility of a geometrical approach towards a TOE. Geometry enters into this approach to TOE in two different ways: 1) Yang-Mills Boson GUT's are derived by recognizing similarities between certain crystal symmetries and certain *SU*(N) Lie Algebra symmetries, and 2) Particle multiplets are constructed from Simplices, and the product of these Simplices builds representative multi-dimensional lattices. It is anticipated that this geometrical approach may be an axiomic breakthrough that allows us to bypass the apparent complications of Gödel's Incompleteness Theorem and ask the question "What is Ultimately Possible in Physics?" – A Geometrical Approach Towards a TOE.

The Geometry of Yang-Mills Theories – A simple example of the geometrical nature of Yang-Mills Boson GUT's is provided in the comparison between the Tetrahedral Conjugacy [5] classes and the Georgi-Glashow *SU*(5) Boson GUT [6]:

$$\begin{pmatrix} \mathrm{x} & C_3 & C_3 & S_4 & \sigma_d \\ C_3 & C_3 & C_3 & S_4 & \sigma_d \\ C_3 & C_3 & C_3 & S_4 & \sigma_d \\ S_4 & S_4 & S_4 & 1 & C_2 \\ \sigma_d & \sigma_d & \sigma_d & C_2 & C_2 \end{pmatrix} \Rightarrow \begin{pmatrix} \mathrm{x} & g & g & X & Y \\ g & g & g & X & Y \\ g & g & g & X & Y \\ X & X & X & B & W \\ Y & Y & Y & W & W \end{pmatrix} \Rightarrow \begin{pmatrix} \mathrm{x} & g & g & X & Y \\ g & g^3 & g & X & Y \\ g & g & g^8 & X & Y \\ X & X & X & \gamma^0 & W^- \\ Y & Y & Y & W^+ & Z^0 \end{pmatrix} \quad (1)$$

Equation (1) Legend – C_2 = Rotation by 180º (degree 2), C_3 = Rotation by 120º (degree 3), *1* = Identity, S_4 = Rotoreflection by 90º (degree 4), σ_d = reflection in a plane through two rotation axes (degree 4), *B* = Weak Hypercharge of $U(1)_Y$ (degree 1), *g* = Gluons of $SU(3)_C$ (degree 3), γ = Photon, *W* = neutral and charged *W*'s of $SU(2)_L$ (degree 2), *X* & *Y* = hypothetical Georgi- Glashow leptoquark bosons (degree 4), and *Z* = neutral Weak IVB.

Here, the Strong/ Color force is placed in the top left position, followed along the diagonal by the next stronger Electromagnetic force (related to Weak Hypercharge), followed by the weaker Weak force in the bottom right position. The Georgi-Glashow *SU*(5) GUT has an order of 24 as does the total number of tetrahedral conjugacy classes. Color Theory has degree and order of three and eight (8 gluons) as does C_3, the class of tetrahedral rotations by 120º. All other sub-symmetries follow similar comparisons. These reflection symmetries S_4 and σ_d have a higher degree of symmetry than the rotation symmetries C_2 and C_3, and are, therefore, intentionally placed in off-diagonal positions, thus representing higher rank terms. The *B* and *W* names reflect the unbroken Electroweak symmetries. After Spontaneous Symmetry Breaking of the Electroweak symmetry, these $\left(B^0, W^0, W^{\pm}\right)$ mix quantum states to become $\left(\gamma^0, Z^0, W^{\pm}\right)$.

These Yang-Mills Boson GUT's extended to analogies between the Octahedral Conjugacy classes and a proposed *SU*(7) Boson Gut with an order of 48 (Equation 2),

$$\begin{pmatrix} \mathrm{x} & C_3 & C_3 & S_4 & \sigma_4 & S_6 & S_6 \\ C_3 & C_3 & C_3 & S_4 & \sigma_4 & S_6 & S_6 \\ C_3 & C_3 & C_3 & S_4 & \sigma_4 & C_2 & C_4 \\ S_4 & S_4 & S_4 & 1 & C_4^2 & C_2 & C_4 \\ \sigma_4 & \sigma_4 & \sigma_4 & C_4^2 & C_4^2 & C_2 & C_4 \\ S_6 & S_6 & C_2 & C_2 & C_2 & i & \sigma_h \\ S_6 & S_6 & C_4 & C_4 & C_4 & \sigma_h & \sigma_h \end{pmatrix} \Rightarrow \begin{pmatrix} \mathrm{x} & g & g & X & Y & V & V \\ g & g & g & X & Y & V & V \\ g & g & g & X & Y & D & E \\ X & X & X & B & W & D & E \\ Y & Y & Y & W & W & D & E \\ V & V & D & D & D & \phi & C \\ V & V & E & E & E & C & C \end{pmatrix} \Rightarrow \begin{pmatrix} \mathrm{x} & g & g & X & Y & V & V \\ g & g & g & X & Y & V & V \\ g & g & g & X & Y & w & w \\ X & X & X & \gamma & W & w & w \\ Y & Y & Y & W & Z & w^0 & w^0 \\ V & V & w & w & w^0 & z & H \\ V & V & w & w & w^0 & H & z \end{pmatrix} \quad (2)$$

Equation 2 Legend – Same as Equation 1 plus: C_4^2 = Rotation by 180° about a 4-fold axis, C_4 = Rotation by 90° (degree 4), i = Inversion, S_6 = Rotoreflection by 60º (degree 6), σ_h = reflection in a plane perpendicular to a 2-fold axis (degree 2), σ_4 = reflection in a plane perpendicular to a 4-fold axis (degree 4), *C*, *D*, *E*, *w* and *z* = Hyperflavor $SO(2,4)$ bosons (degree 4 – see Ref. [3]), H, ϕ = Higgs, and V = *SU*(7) Grand bosons (degree 6).

…and the Icosahedral Conjugacy [7] classes and a proposed *SU*(11) Boson GUT [3,8] with an order of 120 (Equation 3).

$$\begin{pmatrix} \mathrm{x} & C_3 & C_3 & C_3 & C_3 & C_5 & C_5 & S_{10}^3 & S_{10}^3 & S_{10} & S_{10} \\ C_3 & C_3 & C_3 & C_3 & C_3 & C_5 & C_5 & S_{10}^3 & S_{10}^3 & S_{10} & S_{10} \\ C_3 & C_3 & C_3 & C_3 & C_3 & C_2 & C_2 & S_{10}^3 & S_{10}^3 & S_{10} & S_{10} \\ C_3 & C_3 & C_3 & 1 & C_5 & C_2 & C_2 & S_6 & S_6 & C_5^2 & C_5^2 \\ C_3 & C_3 & C_3 & C_5 & C_5 & C_2 & C_2 & S_6 & S_6 & C_5^2 & C_5^2 \\ C_5 & C_5 & C_2 & C_2 & C_2 & C_5 & C_2 & S_6 & S_6 & C_5^2 & C_5^2 \\ C_5 & C_5 & C_2 & C_2 & C_2 & C_2 & C_2 & S_6 & S_6 & S_6 & S_6 \\ S_{10}^3 & S_{10}^3 & S_{10}^3 & S_6 & S_6 & S_6 & S_6 & i & \sigma & \sigma & \sigma \\ S_{10}^3 & S_{10}^3 & S_{10}^3 & S_6 & S_6 & S_6 & S_6 & \sigma & \sigma & \sigma & \sigma \\ S_{10} & S_{10} & S_{10} & C_5^2 & C_5^2 & C_5^2 & S_6 & \sigma & \sigma & \sigma & \sigma \\ S_{10} & S_{10} & S_{10} & C_5^2 & C_5^2 & C_5^2 & S_6 & \sigma & \sigma & \sigma & \sigma \end{pmatrix} \Rightarrow \begin{pmatrix} \mathrm{x} & a & a & a & a & c & c & T & T & R & R \\ a & a & a & a & a & c & c & T & T & R & R \\ a & a & a & a & a & d & d & T & T & R & R \\ a & a & a & b & c & d & d & U & U & S & S \\ a & a & a & c & c & d & d & U & U & S & S \\ c & c & d & d & d & c & d & U & U & S & S \\ c & c & d & d & d & d & d & U & U & U & U \\ T & T & T & U & U & U & U & G & F & F & F \\ T & T & T & U & U & U & U & F & F & F & F \\ R & R & R & S & S & S & U & F & F & F & F \\ R & R & R & S & S & S & U & F & F & F & F \end{pmatrix} \quad (3)$$

Equation 3 Legend – Same as Equations 1 and 2 plus: C_5 = Rotation by 72° about a 5-fold axis (degree 5), C_5^2 = Rotation by 144° about a 5-fold axis (degree 5), S_{10} = Rotoreflection by 36º (degree 10), S_{10}^3 = Rotoreflection by 108º (degree 10), σ = reflection (degree 2), *a* = "Color" 20-plet, *b* = $U(1)_Y$ "Photon", *c* = "Higgs-Weak" 12-plet, *d* = Hyperflavor $SO(2,4)$, *F* = "Fifthons" = WIMP-Gravitons, *G* = Graviton, *R*, *S*, *T* and *U* = $SU(11)$ Grand bosons.

Simplices as Particle Multiplets – Consider the example of a 3-simplex (tetrahedron) as a particle multiplet. This is the simplest example that demonstrates all of the basic properties of these simplices. We will assume an $SU(4)$ Lie algebra with diagonal operators (C_3, C_8, C_{15}).

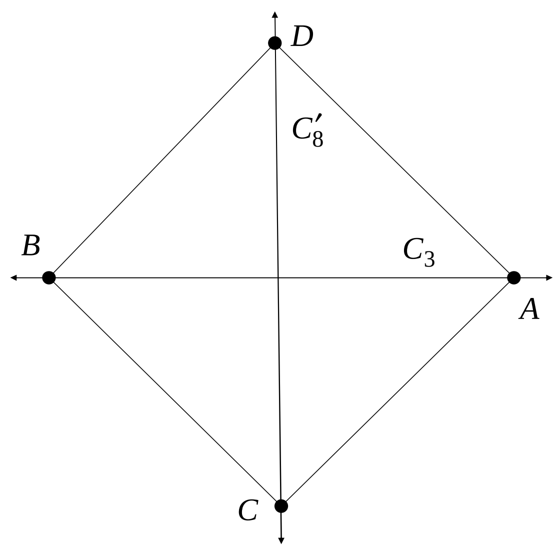
We want to construct a simplex with the following properties: 1) the sum of all charges within a particle multiplet equals zero, and 2) all particles have the same distance from each other. As a consequence of these two requirements, we realize that all particles must also have the same radius about the origin.

Table 1 is deduced by process of trial and error. Note that the strengths of the charges (C_3, C_8, C_{15}) are introduced in a ratio of $(1, \sqrt{3}, \sqrt{6})$. In the general case, this approaches a ratio of the square root of the progression of Special Orthogonal orders $(1, \sqrt{3}, \sqrt{6}, \sqrt{10}, \sqrt{15}, \ldots, \sqrt{n(n+1)/2})$. These four particle vectors (A, B, C, D) exist in a three-dimensional space (C_3, C_8, C_{15}), are each one unit from each other, and are each $\sqrt{3/8}$'s of a unit from the origin. In the general case, our *n*-simplex will exist in *n*-dimensions; have $(n+1)$ particle vectors that are one unit from each other, and $\sqrt{n/(2n+2)}$ of a unit from the origin. Note, that by construction, we have $\sum_{A,B,C,D} C_3 = \frac{1}{2} + \frac{-1}{2} + 0 = 0$, $\sum_{A,B,C,D} \sqrt{3}\, C_8 = \frac{1}{2} + \frac{1}{2} - 1 + 0 = 0$, and $\sum_{A,B,C,D} \sqrt{6}\, C_{15} = \frac{-1}{2} + \frac{-1}{2} + \frac{-1}{2} + \frac{3}{2} = 0$.

Table 1 – A 3-Simplex Multiplet

Charges $\rightarrow$ $\downarrow$ Fermions	C_3	$\sqrt{3}\times C_8$	$\sqrt{6}\times C_{15}$	C_8'
A	$1/2$	$1/2$	$-1/2$	0
B	$-1/2$	$1/2$	$-1/2$	0
C	0	-1	$-1/2$	$-1/2$
D	0	0	$3/2$	$1/2$
Sum	0	0	0	0

Figure 1 – Petrie Diagram of a 3-Simplex



Grand Unified Theories (GUT's) generally require this feature within a particle multiplet. Interestingly enough, a secondary conserved quantum number emerges from the mathematics: $C_8' = \left(\sqrt{3}\, C_8 + \sqrt{6}\, C_{15}\right)/3 = 0, \pm 1/2, \pm 1,$ etc. This is due to the fact that both charges have a common factor of $\sqrt{3}$, and has a net effect of collapsing the algebra down into one fewer dimensions and introducing a broken symmetry. In the general case, we will have more secondary conserved quantum numbers, such as $C_{24}' = \left(\sqrt{10}\, C_{24} + \sqrt{15}\, C_{35}\right)/5 = 0, \pm 1/2, \pm 1,$ etc., and so on. These geometrical constraints may be related to Clifford bivectors and the first-class constraints of BRST formalism (Becchi, Rouet, Stora and Tyutin) [9]. Note that antiparticles could simply be the inversion operator applied to these particle states, thus yielding a nested dual tetrahedron.

A direct application of the 2-Simplex (equilateral triangle) is Color Theory. With the substitution of A = Red (*r*), B = Green (*g*), C = Blue (*b*), $C_3 = g^3$ and $C_8 = g^8$, we are immediately led to the same definition of Color Theory that Lisi used in his paper.

A relevant application of the 3-Simplex (tetrahedron) is Electro-Color. With the additional substitution of D = White (*w*) (the "color" of leptons) and $\sqrt{6}\,C_{15} = {}^{-3}\!/_{2}\,Y'$ (where *Y'* is a "universal" hypercharge that accounts for proposed "Weak" interactions of Left or Right-handed helicity, see Ref's [3, 4]), then we have a 3-Simplex of Electro-Color. The four "colors" (*r,g,b,w*) are the four corners of this 3-Simplex. This tetrahedron and its dual $\left(\bar{r}, \bar{g}, \bar{b}, \bar{w}\right)$ collectively comprise a cube. One of the triangular sides of this tetrahedron contains color theory, and we obtain the important GUT result:

$$\sin^2\theta_W = \left(\frac{C_{15}}{Y'}\right)^2 = \left(\frac{-3}{2\sqrt{6}}\right)^2 = \frac{3}{8}\,. \tag{4}$$

A Proposed TOE Lattice – The author is proposing a TOE based on a "new" lattice [i] called *K*12' because of its similarities to the 12-dimensional Coxeter-Todd *K*12 lattice [10]. The *K*12 lattice has $756 = 12\times(7\times 9)$ minimal roots, and the similar *K*12' lattice has $672 = 12\times(7\times 8)$ roots (plus 12 basis vectors for an order of 684). This *K*12' shares an isomorphism with the semi-simple $E8\times H4$ product of Lie Algebras, and contains two of Klein's $\chi(7)$ hyperbolic curves [11] (or 10-dimensional laminated lattices Λ_{10} [12]) with an order of 336 each. Coincidentally, *K*12' and Λ_{10} are both shallow holes of the 24-dimensional Leech lattice [12].

An important decay route for *K*12' is $K12' \to SU(13)\times SO(24)\times E8$, where the interpretation is that the $SU(13)\times SO(24)$ of rank 12 and order 444 is a Super Yang-Mills Boson GUT with tensor, vector and scalar boson content – many of which are hypothetical and as yet undiscovered (Ref. [3] has an expansion of the prior Icosahedral example – Equation 3), and the *E*8 of rank 8 and order 240 is a Fermion particle multiplet. From its Dynkin diagram, *E*8 has symmetries of $240 = 8\times(2\times 3\times 5)$, and thus exhibits two-fold "duality", three-fold "triality" and five-fold "pentality" symmetries in an eight-dimensional "octality" space. To the author's knowledge, Lisi never identified the pentality symmetry. Curiously, *H*4 has the same symmetries of $120 = 4\times(2\times 3\times 5)$ in a four-dimensional "tetrality" space. Table 2 represents these *E*8 component symmetries as products of Simplices within a 12-dimensional *K*12' lattice. We already reviewed the 3-Simplex of Electro-Color, and learned that this sub-theory implies that leptons possess the neutral Strong Color charges of white and anti-white. The next new physics is revealed in a study of the 4-Simplex of "Gravi-Weak".

[i] Ref's [3, 4] called this lattice *E*12 because of its isomorphism with *E*8 x *H*4.

Table 2 – Component Simplex Symmetries of an *E*8 within a *K*12'

*E*8 Roots 240 =	4 - plet ×	5 - plet ×	2 - plet ×	2 - plet ×	3 - plet
Component Simplices	3-Simplex of Electro-Color	4-Simplex of Gravi-Weak	Collapsed "C_3" and "C_8'" of a 3-Simplex of Helicity and Matter/ Anti-Matter		2-Simplex of Generations (Lisi's Triality)
12 Dimensions = (acts like 11-D =)	3-D +	4-D +	3-D + (acts like 1-D + 1-D +)		2-D

A 4-Simplex of Gravi-Weak – Lisi's *E*8 TOE used a Gravi-Weak unification of a Pati-Salam Weak force and a Gravitational force based on Clifford algebra. At first glance, this seems to be as odd of a pairing of forces as Electro-Color, but these strange relationships may yield clues to the structure of Spacetime and Hyperspace.

The 4-simplex of Gravi-Weak in Table 3 is partially inspired by Hyperflavor-Weak Theory (a hypothetical extension of the Standard Weak Force that is defined in Ref's [3, 4]), and the fact that this *K*12' lattice requires the five-fold symmetry of a 4-Simplex (pentachoron) particle multiplet. This secondary conserved quantum number is important, and has the interpretation of a right-handed weak isospin projection operator, $T_{3R} = \left(\sqrt{3}\,T_{3HF} + \sqrt{6}\,T_{8HF}\right)/3$. The fifth vertex of this 4-simplex has unusual characteristics.

Table 3 – A 4-Simplex of Gravi-Weak

Charges → ↓ Fermions	T_{3L}	$\sqrt{3}\times T_{3HF}$	$\sqrt{6}\times T_{8HF}$	$\sqrt{10}\times T_G$	T_{3R}	T_G'
$f_{1L}^{\wedge} = (u_L, \bar{e}_L)$	½	−½	½	½	0	0
$f_{1L}^{\vee} = (d_L, \bar{\nu}_{eL})$	−½	−½	½	½	0	0
$f_{1R}^{\wedge} = (u_R, \bar{e}_R)$	0	1	½	½	½	0
$f_{1R}^{\vee} = (d_R, \bar{\nu}_{eR})$	0	0	−3⁄2	½	−½	0
$sf_1 = (sq, s\bar{l})$	0	0	0	− 2	0	−½

This new "particle" appears to have properties equivalent to a "scalar fermion" [13]. It is a "fermion" because it belongs to this fundamental particle multiplet. But it has quantum numbers that imply a zero spin: $T_{3L} = 0$ and $T_{3R} = 0$, thus it has neither a left-handed nor a right-handed isospin projection. Does $s_F = T_{3L} + T_{3R} + T_G' = \pm 1/2, \pm 3/2, \ldots$ define a generalized Gravi-Weak Fermion? Are the spin projections for these new quanta hidden in a Hyperspace dimension? Do these "scalar fermions" manifest themselves as tachyons, BRST or Faddeev-Popov ghosts, or physical particles? The designation for these new quanta in Table 3 is preceded by an "s" to indicate generation-dependent scalar fields $\left(sf_1, s\bar{f}_1, sf_2, s\bar{f}_2, sf_3, s\bar{f}_3\right)$ with all four electro-color quantum numbers (r, g, b, w). A 4-simplex plus its dual contains 10 particle states $\left(f_L^{\wedge}, f_L^{\vee}, f_R^{\wedge}, f_R^{\vee}, sf, \bar{f}_L^{\wedge}, \bar{f}_L^{\vee}, \bar{f}_R^{\wedge}, \bar{f}_R^{\vee}, s\bar{f}\right)$. These new sf scalar fermions have electric charges of $\left(\tfrac{1}{6}, -\tfrac{1}{2}, -\tfrac{1}{6}, \tfrac{1}{2}\right)$ for color charges of $\left((r, g, b), w, (\bar{r}, \bar{g}, \bar{b}), \bar{w}\right)$, and are thus not the supersymmetric partners to the known fermions. Table 4 enumerates the Electro-Color-Gravi-Weak quantum numbers for the first generation of fermions. Note the new conservation laws: $g_8' = \left(\sqrt{3}\, g^8 - \tfrac{3}{2} Y'\right)/3 = 0, \pm\tfrac{1}{2}, \ldots$, $T_{3R} = \left(\sqrt{3}\, T_{3HF} + \sqrt{6}\, T_{8HF}\right)/3 = 0, \pm\tfrac{1}{2}, \ldots$ and $T_G' = \left(\sqrt{10}\, T_G + F_3\right)/5 = 0, \pm\tfrac{1}{2}, \ldots$, where the secondary gravity quantum number T_G' is defined with the expectation that Gravity and a new WIMP-Gravity (denoted by F) will collectively comprise a Clifford bivector and mix charges such that $T_G' = \left(\sqrt{10}\, T_G \mp \tfrac{1}{2}\right)/5 = 0, \pm\tfrac{1}{2}$, etc.

Table 4 – Electro-Color-Gravi-Weak Quantum Numbers for Select Fundamental Fermions

Charges $\rightarrow$ $\downarrow$ Fermions	g^3	$\sqrt{3}\times$ g^8	$-\tfrac{3}{2}\times$ Y'	T_{3L}	$\sqrt{3}\times$ T_{3HF}	$\sqrt{6}\times$ T_{8HF}	$\sqrt{10}\times$ T_G	F_3	g_8'	T_{3R}	T_G'
u_L^r	½	½	−½	½	−½	½	½	−½	0	0	0
u_L^g	−½	½	−½	½	−½	½	½	−½	0	0	0
u_L^b	0	−1	−½	½	−½	½	½	−½	−½	0	0
e_L^w	0	0	3⁄2	−½	½	−½	−½	½	½	0	0
d_L^r	½	½	−½	−½	−½	½	½	−½	0	0	0
d_L^g	−½	½	−½	−½	−½	½	½	−½	0	0	0
d_L^b	0	−1	−½	−½	−½	½	½	−½	−½	0	0
ν_{eL}^w	0	0	3⁄2	½	½	−½	−½	½	½	0	0
u_R^r	½	½	−½	0	1	½	½	−½	0	½	0
u_R^g	−½	½	−½	0	1	½	½	−½	0	½	0
u_R^b	0	−1	−½	0	1	½	½	−½	−½	½	0
e_R^w	0	0	3⁄2	0	−1	−½	−½	½	½	−½	0
d_R^r	½	½	−½	0	0	−3⁄2	½	−½	0	−½	0
d_R^g	−½	½	−½	0	0	−3⁄2	½	−½	0	−½	0
d_R^b	0	−1	−½	0	0	−3⁄2	½	−½	−½	−½	0
ν_{eR}^w	0	0	3⁄2	0	0	3⁄2	−½	½	½	½	0
sq_1^r	½	½	−½	0	0	0	−2	−½	0	0	−½
sq_1^r	−½	½	−½	0	0	0	−2	−½	0	0	−½
sq_1^r	0	−1	−½	0	0	0	−2	−½	−½	0	−½
sl_1^w	0	0	3⁄2	0	0	0	2	½	½	0	½

A 3-Simplex of 4-Spinors – The next relevant Simplex represents Dirac γ 4-Spinors. Table 5 demonstrates how a 3-Simplex of 4-Spinors might decompose into the product of two 1-Simplices of Spinors and Matter/ Anti-Matter with charges given by F_3 and the secondary effective charge $F_8' = \left(\sqrt{3}\,F_8 + \sqrt{6}\,F_{15}\right)/3$ (note the similarities to Figure 1 with the substitutions $C_3 = F_3$ and $C_8' = F_8'$). This application yields either: 1) a properly defined 3-Simplex of Dirac γ 4-Spinors or 2) pairs of two-component (1-Simplex) Pauli σ spinors that collectively comprise a twistor [14]. This dimensional collapse reflects a broken symmetry. Prior to collapse, the 3-Simplex is three dimensional. However, this apparent collapse into Spinors and Matter seems to decrease the effective dimensionality of this component of the theory into 2 = 1 + 1. Thus the complete theory with all of the Simplices listed in Table 2 might collapse from twelve to eleven effective dimensions – possibly consistent with M-Theory.

Table 5 – A 3-Simplex of 4-Spinors Collapsing Into Two 1-Simplices of Helicity & Matter

Charges $\rightarrow$ $\downarrow$ Spinors	F_3	$\sqrt{3}\,F_8$	$\sqrt{6}\,F_{15}$	F_8'
σ_A	$1/2$	$1/2$	$-1/2$	0
π_A	$-1/2$	$1/2$	$-1/2$	0
$\sigma_{A'}$	0	-1	$-1/2$	$-1/2$
$\pi_{A'}$	0	0	$3/2$	$1/2$

Charges $\rightarrow$ $\downarrow$ Spinors	F_3	F_8'
matter σ_1	$1/2$	0
matter σ_2	$-1/2$	0
anti-matter σ_1	0	$-1/2$
anti-matter σ_2	0	$1/2$

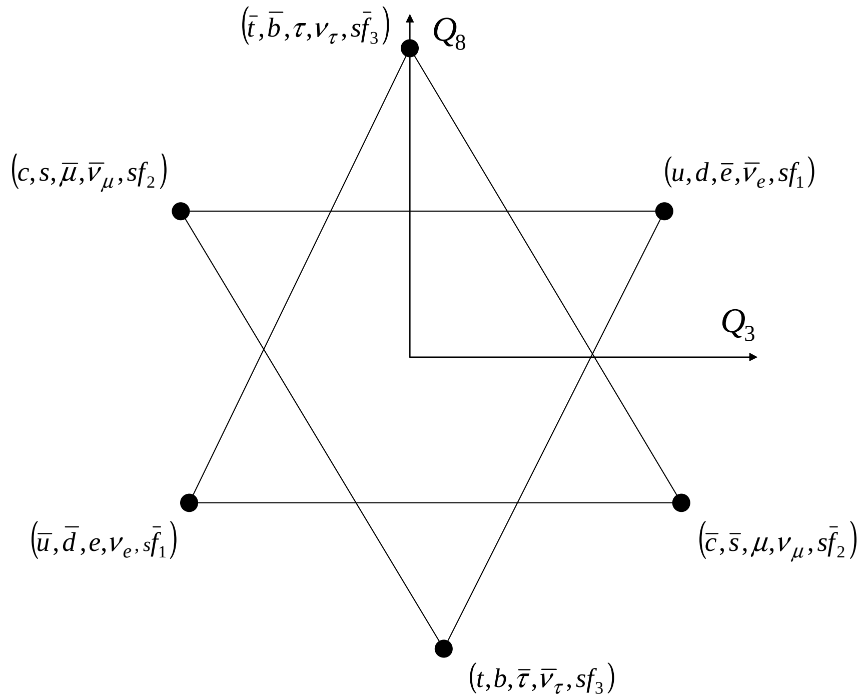
A 2-Simplex of Generations – The final component of this theory is to include Lisi's triality concept as two nested 2-Simplices (equilateral triangles) of Generations shown in Table 6 and Figure 2. Note that if we multiply the twenty Electro-Color-Gravi-Weak Fermion root states of Table 4 with the four spinor states of Table 5 and the three generation states of Table 6 (anti-particles were already counted in Table 5), then we have a Fermion multiplet with $240 = 20 \times 4 \times 3$ distinct particle states as proposed in Table 2. This *E*8 240-plet bears many similarities with Lisi's, but with important differences: 1) the eight-dimensional *E*8 does not include all of Spacetime and Hyperspace (of 11 or 12 dimensions) but is an important subset, 2) this *E*8 does not include any bosons, they are a Super Yang-Mills 444-plet as described in Ref. [3], and 3) this *E*8 predicts new fermions (*sf*) with scalar or anyonic [15] behavior that may be tachyons.

Table 6 – Dual 2-Simplices of Generations

Charges $\rightarrow$ $\downarrow$ Gen's	Q_3	$\sqrt{3}\,Q_8$
1st Gen	$1/2$	$1/2$
2nd Gen	$-1/2$	$1/2$
3rd Gen	0	-1

Charges $\rightarrow$ $\downarrow$ Gen's	Q_3	$\sqrt{3}\,Q_8$
Anti-1st Gen	$-1/2$	$-1/2$
Anti-2nd Gen	$1/2$	$-1/2$
Anti-3rd Gen	0	1

Figure 2 – Dual 2-Simplices of Generations



What about Bosons? In this Geometrical *K*12' Theory, Fermions are eight-dimensional lattice points embedded in a twelve dimensional theoretical framing. Reference [3] predicts a large number of hypothetical Super Yang-Mills Bosons of various dimensionalities including dimensions one through seven plus nine and eleven. These multi-dimensional Bosons allow transitions from one Fermion/ lattice point to another Fermion/ lattice point. As such, all of the basic interactions between Bosons and Fermions are three-legged Fermion-Boson-Fermion Feynman diagrams. Higher-legged Feynman diagrams have their origins in non-Abelian Lie Algebras, which are contained in this Super Yang-Mills Theory. Because these Bosons are differences between Fermions/ lattice points, we may represent Bosons as a reciprocal lattice. Table 7 is a listing of the six non-basis gluons as two-dimensional basis difference vectors $(\Delta g^3, \Delta g^8)$. A simple example of how these operators perform is: $q^{c_2} = g^{c_2\bar{c}_1} q^{c_1}$, where q^{c_1} is the initial quark "lattice point" state (such as a "red" left-handed up, u_L^r), $g^{c_2\bar{c}_1}$ is the gluon "difference vector" operator (such as a "green-anti-red" gluon, $g^{g\bar{r}}$), and q^{c_2} is the final quark "lattice point" state (such as a "green" left-handed up, u_L^g).

Table 7 – The Off-Diagonal Gluons as Translation Vectors

Charges → ↓ Bosons	Δg^3	$\sqrt{3}\times \Delta g^8$	$-\frac{3}{2}\times \Delta Y'$	$\Delta g_8'$
$g^{r\bar{g}}$	1	0	0	0
$g^{r\bar{b}}$	½	3⁄2	0	½
$g^{g\bar{b}}$	−½	3⁄2	0	½

Charges → ↓ Bosons	Δg^3	$\sqrt{3}\times \Delta g^8$	$-\frac{3}{2}\times \Delta Y'$	$\Delta g_8'$
$g^{g\bar{r}}$	−1	0	0	0
$g^{b\bar{r}}$	−½	−3⁄2	0	−½
$g^{b\bar{g}}$	½	−3⁄2	0	−½

Brane-Structure – In the author's opinion, these lattices are "stringy lattices" or nets of Simplices on branes, and share similarities with M-Theory [16] and with Causal Dynamical Triangulation (CDT) [17]. Analysis of the Super Yang-Mills Boson GUT's (Ref. [3]) implies that Space is a 3-brane, Time is a 1-brane, and Hyperspace is an M2-brane, a 1-brane, and a D5-brane (that further decomposes into a 3-brane and a 2-brane).

The origin of Entanglement may be hidden in the twelve primary dimensional basis quantum numbers $\left(g^3, g^8, Y', T_{3L}, T_{3HF}, T_{8HF}, T_G, F_3, F_8, F_{15}, Q_3, Q_8\right)$ and in the four secondary geometrically-conserved quantum numbers $\left(g_8', T_{3R}, T_G', F_8'\right)$. These conserved quanta may remain as fossils of the now-collapsed multi-dimensional lattice.

What is Ultimately Possible in Physics? This paper implies that a Geometrical TOE may ultimately be possible, and laid out some of the ground work for such an approach. This geometrical approach has similarities to Lisi's approach, and may be an axiomic breakthrough that allows us to bypass the apparent complications of Gödel's Incompleteness Theorem.

One version of Occam's razor says "Plurality ought never be posited without necessity." And although this theory introduces a large amount of Plurality/ Complexity/ non-Simplicity by virtue of a 444-plet of mostly hypothesized Super Yang-Mills bosons, it does so in an interest to respect important or naturally-occurring symmetries (Beauty?) such as Tetrahedral (Equation 1), Octahedral (Equation 2), Icosahedral (Equation 3), multi-dimensional Simplices (Table 1), Gosset (Table 2 and Ref. [1]) and Leech (*K*12' is one of its shallow holes, see Ref. [12]) lattice symmetries. Is this a matter of coincidence, or should we naturally expect Nature to repeat and reapply useful structures? Are Beauty and Symmetry necessary reasons to trump Simplicity? If Simplicity always trumps Necessity, then we should be satisfied with the "ugly but practical" Standard Model of $SU(3)_C \times U(1)_Y \times SU(2)_L$ and a separate General Theory of Relativity, and we need to stop talking about such "foolishness" as Theories of Everything or Not Everything.

This TOE is not complete. Although many geometrical details were presented in this paper and in References [3, 4], it will take time to enumerate all of the Feynman diagrams, derive all of the Lagrangian components, and formally tie this *K*12' Theory into General Relativity, M-Theory, CDT or whatever other Theories may be related.

What is ultimately possible? Certainly, the founders of Quantum Mechanics had no idea of the profound effects that QM would have on our modern lives. Likewise, it is impossible to know if or how a 21st Century TOE will affect the people of the 22nd Century. But the ultimate possibility of a Geometrical TOE that contains all particles and all interactions is worth the effort!

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