The holographic principle and the digital universe

Joakim Munkhammar *

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Abstract

Many modern approaches to quantum gravity such as various string theories and loop quantum gravity implicitly contain a holographic principle which states that the universe is based on information encoded on two dimensional surfaces. If this is true the universe is undoubtedly digital and currently contains roughly 10^{123} bits.

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*MRC, Studentstaden 23:230, 752 33 Uppsala, Sweden, joakim.munkhammar@gmail.com

1 Introduction

Physics is historically both analog and digital. But over time conceptual explanations for physical phenomena have changed dramatically. The discussion whether matter is infinitely divisible or atomic and the discussion if light is corpuscular or wave-like are perhaps canonical examples of these historically transient concepts. If one seeks to find an all-underlying set of principles for physics this prompts the question if physics inherently is either analog, digital or somehow a combination of both at a fundamental level. In this essay we shall discuss how modern attempts to establish a theory of quantum gravity largely suggests that physics rests on a historically growing trend in physics which postulates that information is as fundamental as matter or energy. If physics is fundamentally based on information we shall assume by definition that it is digital.

2 Entropy and information

"Information is the resolution of uncertainty."

- Claude Shannon

Thermodynamic entropy was coined by Rudolf Clausius in 1865 as a measure of the "useless" energy in a thermodynamic process [15]. Entropy prevented the perfect utilization of energy in a thermodynamic process. Derived from his theorem the Clausius equality between entropy, heat and temperature appeared as [15]:

$$dS = \frac{\delta Q}{T}. \tag{1}$$

This equality, while prominent and useful in a practical sense since the mid 1850's, has been revitalized in thermodynamic theories of gravitation more recently [12]. We will return to this recent research in section 3. Together the laws of thermodynamics, in particular the second - the maximization of entropy - formed an axiomatic basis for thermodynamics from which the dynamics of a thermodynamic process could be obtained [15]. The notion of entropy was further extended by Ludwig Boltzmann when he developed statistical mechanics in the 1870's; establishing the entropy in statistical mechanics as a measure of disorder[15]:

$$S = -k_B \sum_i p_i \log p_i, \tag{2}$$

where p_i is the probability of state i and k_B is Boltzmanns constant. In 1949 Claude Shannon established a canonical form of information theory which now bears his name; Shannon information theory [16]. Upon his discovery he stated the following regarding entropy after a discussion with John von Neumann [19]:

"My greatest concern was what to call it. I thought of calling it 'information', but the word was overly used, so I decided to call it 'uncertainty'. When I discussed it with John von Neumann, he had a better idea. Von Neumann told me,'You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already has a name. In the second place, and more important, nobody knows what entropy really is, so in a debate you will always have the advantage.'"

Thus in Shannon's information theory entropy was in practice synonymous with information [16]. Entropy had historically in some sense risen from a measure of the "useless" in Clausius scenario to a measure of the "useful" in Shannon's scenario. Shannon's entropy was defined as [13]:

$$S = -\sum_{i} p_i \log p_i \tag{3}$$

which is equivalent to Boltzmann's expression for entropy (2) from statistical mechanics (stripped of the Boltzmann constant). Shannon's information theory is the most widely used framework in information theory and it is still used for the construction of every communication device for example [2]. Shannon's information theory had no obvious connection to physics, and it was derived on pure mathematical grounds alone, how could it actually be a physical phenomenon?

3 The holographic principle and quantum gravity

It is not unreasonable to imagine that information sits at the core of physics, just as it sits at the core of a computer.

- John Wheeler [21].

The deep connection between physics and information was perhaps first noticed by Jaynes in the 1950's [13]. But it was not until the early 1970's when the connection started to become prominent; the eminent physicist John Wheeler prompted a question to his student Jacob Bekenstein on what would happen if one took a cup of hot tea, stirred it and poured it through the event horizon of a black hole [2]. Since information could not be destroyed the information had to be stored somehow, but without an obvious resolution to the problem there was a paradox: the black hole information paradox. A possible solution to the problem was to consider the possibility that the information was never lost in the black hole but was instead stored on the event horizon [2]. This initiated a massive scientific discussion on what exact function information had in physics. This discussion led Jakob Bekenstein to develop his famous Bekenstein bound, which is a theorem about the maximum amount of information possible within an enclosed sphere. It states the following inequality [4]:

$$S \leq \frac{2\pi k_B R E}{\hbar c}, \tag{4}$$

where R is the radius of the sphere and E is the energy contained within the sphere. Further developments were made by Stephen Hawking as he had started working on black hole thermodynamics. He arrived the following startling formula for the entropy of a Schwarzschild black hole [7]:

$$S = \frac{k_B c^3 A}{G\hbar} \tag{5}$$

which contained the Boltzmann constant k_B, the gravitational constant G, the speed of light c, the reduces Planck's constant $\hbar$ and the event horizon area $A = 4\pi R_S^2$ (where R_S is the Schwarzschild radius). This entropy (5) is also the maximum entropy possible in the Bekenstein bound (4). Among other things the development of black hole thermodynamics paved the way for the concept of Bekenstein-Hawking radiation [7], which might recently have been discovered experimentally for the first time [1]. In a wider sense black hole thermodynamics had opened up a whole new way of looking at fundamental physics where information was not just useful but instead perhaps paramount to any theory. Physicist Charles Thorn came up with a concept that perhaps physics was based on information encoded on two dimensional surfaces [17]. This was then rigorously developed by Gerard t'Hooft and further adopted within string theory by Leonard Susskind [17]. This "holographic principle" which stated that all information in a volume of space was stored on two dimensional surfaces - preferably a light-like boundary like a gravitational horizon - had turned out to be more important than suspected as it was an integral part of several approaches to quantum gravity [17]. Thus perhaps information was even at the very foundation of quantum gravity. More recently a very interesting "conceptual reversal" of physics was made by Erik Verlinde who proposed that even the laws of Newton were subordinate to the laws of holography in his recent preprint "On the origin of the laws of Newton" [11, 20]. This work was based on the results of Thanu Padmanabhan, Ted Jacobson and strongly related to the AdS/CFT principle proposed by Juan Maldacena [9]. A key concept in Jacobson's, Padmanabhan's and subsequently Verlinde's studies were to utilize the Clausius equality in thermodynamics and apply it to derive the Einstein field equations as an entropic force [12, 14]. Generally these theories of gravity - by being based on entropy and holographic arguments - indicated the revolutionary possibility that gravity might not be a fundamental force but an emergent phenomenon [20]. This suggested that physics could be reformulated in such a way that information constituted the foundation for physics and that the laws which govern it could - to some extent - be derived from that foundation. Thus in the holographic perspective the concept of information - as it turned out - is at least as fundamental as matter or energy in formulating theories in physics [2].

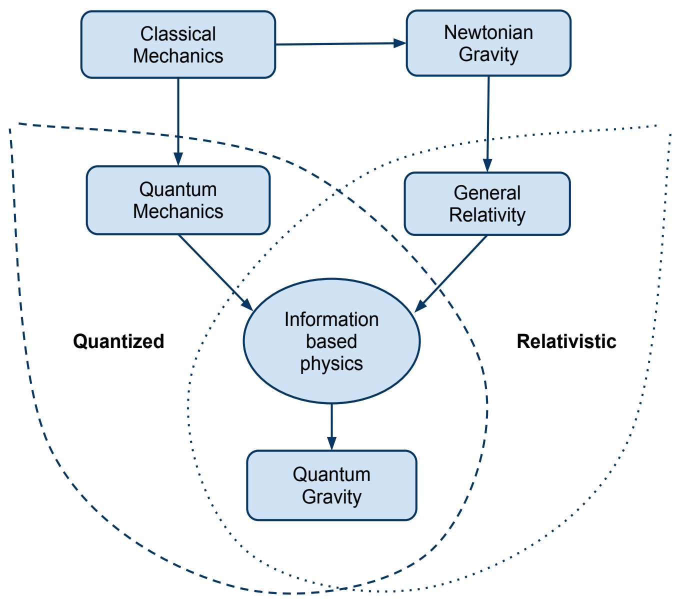

Figure 1. A schematic illustration of how quantum gravity emerges in an information-based theory of physics.

4 Quantum cosmology and the information of the universe

If a reasonable theory of quantum gravity is based on the holographic principle then this has implications for cosmology. Since information is stored on two dimensional surfaces enclosing a volume it might be interesting to ask the question about the amount of information in the observable universe. An explicit formula for the number of bits on a surface such as the horizon of the observable universe is given by [20]:

$$N = \frac{Ac^3}{G\hbar} = \frac{4\pi r^2 c^3}{G\hbar} \tag{6}$$

Given a cosmological horizon distance in the observable universe of about $R = 46$ ly then we get the following approximate quantity:

$$N \sim 10^{123}\ bits \tag{7}$$

which is about a *thousand vigintillion* bits in the Brittish system [10] (this was also evaluated by Johannes Koelman [8] and it is an expression about 10^{23} times larger than a similar estimate made earlier by Jakob Bekenstein [2]). Since the universe is expanding this expression might be mathematically invalid, however there are recent speculative studies relating the entropy of the holographic surface at the horizon distance which might provide an explanation for dark energy [6, 8]. Naturally since these are speculative theories there are opposing studies, see for example Ulf Danielsson's recent paper [5]. Quantum gravity, and subsequently quantum cosmology, based on holography remains an unsettled field of research where the proponents are optimistic on finally obtaining a viable such theory.

5 Discussion

Is the universe digital or analog? Many of our best efforts so far indicate that the universe might be based on information which then by definition makes it digital. Perhaps it is like the proposition by Erik Verlinde which suggests that information constitutes the foundation for even the laws of Newton - and not the other way around. Any final theory, based on the holographic principle or not, should eventually provide testable predictions with the hope of resolving some unsettled fundamental questions in physics. Perhaps quantum gravity in relation to the holographic principle could finally bring light on the origin of cosmology's greatest problems: dark energy and dark matter.

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