THE UNCERTAINTY PRINCIPLE: THE END AND THE BEGINNING OF DREAMS

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Abstract. Is it possible to have a unique interpretation of quantum mechanics and quantum gravity? This article proves that the Heisenberg uncertainty principle implies that it is not possible. Moreover, it raises the question to know if there is a finite number of interpretations of these theories.

1. Introduction

Werner Heisenberg has written an incredible article in 1927 containing a result which now bears his name: the Heisenberg uncertainty principle [1]. If the Heisenberg uncertainty principle is the end of the dream of an absolute knowledge of physics, it is also the beginning of new scientific developments beyond our dreams. The door of an absolute knowledge of our universe has been closed but the one of the multiple interpretations is now open. Quantum mechanics has several coherent interpretations and we can expect the same for quantum gravity. This article points out the fact that a unique interpretation of quantum mechanics contradicts the uncertainty principle which is a limit to our knowledge. Similarly, a unique interpretation of quantum gravity contradicts the existence of a minimal length which is also a limit to our knowledge. Then, we can wonder whether there is a finite number of interpretations of quantum mechanics and quantum gravity.

The article is organized as follows. The Heisenberg uncertainty principle is recalled in section 2. The interpretations of quantum mechanics, quantum field theory and quantum gravity are discussed respectively in sections 3, 4 and 5. Section 6 gives a look beyond the uncertainty principle and below the minimal length. Finally, a conclusion is addressed in section 7.

2. The Heisenberg uncertainty principle

Let us recall how the uncertainty principle is originally obtained from a thought experiment called the *Heisenberg microscope* which is based on a particle approach (see for instance [2, page 342] or [3, page 21]). Suppose that we use an optical microscope with photons of wavelength λ to observe a body B moving on the $x-$axis below the microscope. Due to the laws of optics, the position of the body on the $x-$axis can be measured with an accuracy Δx satisfying

$$\Delta x \gtrsim \frac{\lambda}{\sin \epsilon} \tag{2.1}$$

where ϵ is the angle of the cone of light rays focusing on the particle also called the aperture angle. The momentum of photons which strike the particle is given by $\frac{h}{\lambda}$. The recoil of the observed particle is uncertain because the angle of impact is uncertain. By using the definition of the sine function, we have that the momentum on the $x-$axis of the observed particle can be measured with an accuracy Δp_x satisfying

$$\Delta p_x \gtrsim \frac{h}{\lambda}\sin\epsilon. \tag{2.2}$$

So, it leads to the original Heisenberg uncertainty principle

$$\Delta x \cdot \Delta p_x \gtrsim h \tag{2.3}$$

where Δx and Δp_x are respectively the uncertainties about $x-$position and projection of momentum

$$\vec{p}_B := m_B \cdot \vec{v}_B \tag{2.4}$$

on the $x-$axis of the observed body of mass m_B with velocity $\vec{v}_B$ and h the Planck's constant.

Another version of the Heisenberg uncertainty principle developed by Earle Kennard [4], using the probability theory and compatible with the Copenhagen interpretation of quantum mechanics, states that

$$\sigma_x \cdot \sigma_{p_x} \geq \frac{\hbar}{2} \tag{2.5}$$

where σ_x and σ_{p_x} are respectively the standard deviations of $x-$position and projection of momentum $\vec{p}_B$ on the $x-$axis of the observed body and $\hbar = \frac{h}{2\pi}$ the reduced Planck's constant.

Both inequalities (2.3) and (2.5) state that it is not possible to measure simultaneously the position and the velocity with an arbitrary accuracy. Nevertheless, there exists a difference between these two inequalities. The original result of Werner Heisenberg (2.3) states that every measurement of the position and the velocity of a body implies an uncertainty whereas the second result (2.5) states that among a large number of measurements there is an uncertainty on average values. Strictly speaking, it seems possible to violate the original uncertainty principle (2.3) on a measurement by using the second uncertainty principle (2.5). However, this is not allowed even on a virtual measurement involving a single photon. So, if the second version of the Heisenberg uncertainty principle (2.5) is best suited with the Copenhagen interpretation of quantum mechanics where only quantum probabilities exist, it can lose some of its physical meaning with other interpretations of quantum mechanics, as the de Broglie-Bohm interpretation, where position and velocity always exist.

The Heisenberg uncertainty principle was a revolution for the scientific community in 1927. A limit to our knowledge was erected as a fundamental principle concerning the foundations of physics. The limit imposed by the Heisenberg uncertainty principle did not prevent the development of physics but it required an interpretation. Several years later, the same occurs in the foundations of mathematics with the Gödel's incompleteness theorems developed by Kurt Gödel in 1931 [5].

3. The interpretations of quantum mechanics

Quantum mechanics is a physical theory for predicting the behaviors of particles whose mathematical formalism was developed by Paul Dirac [6]. The standard interpretation of quantum mechanics is the Copenhagen interpretation which was developed by the founding fathers of quantum mechanics Niels Bohr, Max Born, Werner Heisenberg, Pascual Jordan and Paul Dirac and is indeterministic [7, 8]. It postulates that particles have no intrinsic position or velocity, only quantum probabilities of measuring these quantities. Supporters of this interpretation postulate that there is nothing of physical nature beyond the Heisenberg uncertainty principle. As it is not possible to know the position and the velocity of a particle with an arbitrary accuracy, the Copenhagen interpretation does not assign an objective reality to particles, apart from measurements when the wave function is supposed to collapse. Suppose that this interpretation can be considered as the only interpretation of quantum mechanics. So, we know what is beyond the Heisenberg uncertainty principle: nothing of physical nature. But this is a knowledge and also a problem because the Heisenberg uncertainty principle (2.3)-(2.5) is supposed to be a limit to the knowledge. So, we can state the first main result of this article.

Proposition 1. *The uniqueness of interpretation of quantum mechanics leads to a paradox.*

In a logical point of view, several coherent interpretations of quantum mechanics should exist in order to avoid this paradox. Actually, this is the case and we are particularly interested in three of them:

- the Copenhagen interpretation;
- the de Broglie-Bohm interpretation;
- the many-worlds interpretation.

The de Broglie-Bohm interpretation was developed by Louis de Broglie [9], a founding father of quantum mechanics, and David Bohm [10]. It uses a guiding equation and is a "quasi-deterministic" non local theory. The term "quasi-deterministic" means that this theory is deterministic, with position and momentum well-defined at any time, when the initial conditions are given. A residual indeterminism remains on the initial conditions given at the Big-Bang [11] and this is due to the existence of a minimum length developed in section 5. Supporters of this interpretation argue that uncertainty does not imply indeterminism. The many-worlds interpretation was developed by Hugh Everett in [12]. It denies the actuality of wavefunction collapse and then leads to many-worlds where all quantum probabilities are realized. Contrary to the Copenhagen interpretation, the de Broglie-Bohm interpretation and the many-worlds interpretation suppose that there is some physics beyond the Heisenberg uncertainty principle.

The objective of this article is not to discuss about these interpretations and the reader may refer to [13] for more details. There exist other interpretations of quantum mechanics: relational quantum mechanics, the transactional interpretation, the stochastic interpretation, the consistent histories etc. A question which arises is the following one:

Question 1. *Is there a finite number of interpretations of quantum mechanics?*

This is an interesting open question. It is difficult to see why there should be a limit to the number of interpretations of quantum mechanics. However, an interpretation must be mathematically coherent and must lead to the same experimental results as the other interpretations. This is the case for the three previous interpretations [11, 14]. However, the interpretation of Albert Einstein of quantum mechanics as a local deterministic theory is false because quantum mechanics violates the Bell's inequality, as proved by the Bell test experiments of Alain Aspect [15].

4. The interpretations of quantum field theory

We make a few remarks about quantum field theory where things are much more complicated. Quantum field theory is a theory used to describe the physics of elementary particles and its building block is the quantum field which can be a scalar, vector or spinor field [16]. It is a kind of "dynamical" quantum mechanics adapted to the fluctuations of fields which give birth to pair production and pair annihilation of particles, taking into account the special relativity. This is the reason why the standard mathematical framework of quantum field theory is called the second quantization, quantum mechanics being the "first" quantization. The second quantization is an Hamiltonian approach first developed by Paul Dirac, Eugene Wigner and Pascual Jordan. There exists an equivalent formalism of the second quantization called the path integral quantization which is a Lagrangian approach developed by Richard Feynman [17].

The uncertainty principle (2.5) was generalized by Howard Percy Robertson in [18] and then refined by Erwin Schrödinger, a founding father of quantum mechanics, in [19] to any observables A and B in the following way

$$\sigma_A^2 \cdot \sigma_B^2 \geq \left(\frac{1}{2}\langle\{\hat{A},\hat{B}\}\rangle - \langle\hat{A}\rangle\langle\hat{B}\rangle\right)^2 + \left(\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle\right)^2 \tag{4.1}$$

where $\hat{A}$ and $\hat{B}$ are corresponding operators of observables A and B, $[\hat{A},\hat{B}] := \hat{A}\hat{B} - \hat{B}\hat{A}$ the commutator of $\hat{A}$ and $\hat{B}$, $\{\hat{A},\hat{B}\} := \hat{A}\hat{B} + \hat{B}\hat{A}$ the anticommutator of $\hat{A}$ and $\hat{B}$, $\sigma_A := \sqrt{\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2}$ the standard deviation of A where $\langle\hat{A}\rangle$ is the expectation value of $\hat{A}$. So, the formulation (4.1) of the uncertainty principle can be applied to the classical field theory, for instance the electric and magnetic field strengths. As it is not possible to know the observables with an arbitrary accuracy, the standard interpretation does not assign an objective reality to classical fields. Only quantum fields are supposed to have a meaning. Only quantum probabilities exist and they are given by the cross sections corresponding to the fluctuations of quantum fields.

As particles in quantum mechanics, quantum fields require an interpretation in quantum field theory, but this is a challenge because of their abstract feature. A quantum field $\hat{\phi}(t,x)$ is a system containing an infinite number of degrees of freedom which come from the possible values of the field $\phi(t,x)$, contrary to particles in quantum mechanics with momentum and position. The standard interpretation of the second quantization is a "field interpretation" where the field is supposed to be more fundamental than the particle because the mathematical formalism of quantum field theory is based on field operators that create or annihilate particles in space [20]. However, quantum fields are not directly related with physical quantities. There is also the question of the meaning of measurements in quantum field theory where particles are created and destroyed all the time.

The Bohmian quantum field theory is developed in [21, 22] by keeping a particle theory and leads to a "particle interpretation". This is the counterpart of the de Broglie-Bohm interpretation for quantum field theory. The path integral quantization can be seen as another interpretation of quantum mechanics and quantum field theory, also known as sum-over-histories [23]. Finally, the uncertainty principle (4.1) can also be applied to quantum field theory and it is quite possible that we find new interpretations in the future.

5. The interpretations of quantum gravity

Quantum gravity attempts to unify quantum mechanics with the general relativity. The success of quantum fields theories for the electroweak interaction and the strong interaction [23, 24] has led physicists to use the same strategy for gravity. However, perturbative quantum gravity is nonrenormalizable [25], and this is a problem in the use of the classical quantum field theory.

It is possible to combine the uncertainty principle and the general relativity and it leads to a minimal length [26, 27]. A thought experiment based on a particle approach and known as the *general relativistic Heisenberg microscope* gives the result as follows. Let us observe a body B at rest of mass m_B with a photon. We have the following conditions:

– the Compton wavelength defined by

$$\lambda_C := \frac{h}{m_B c} \tag{5.1}$$

is the maximal wavelength of a photon that can be used to observe a body of rest mass m_B at quantum scale. Indeed, quantum field theory implies that below this Compton wavelength, a new body of rest mass m_B can be produced during the observation and this renders questionable the notion of position of the observed body B by a photon.

– the Schwarzschild radius defined by

$$r_{\rm s} := \frac{2Gm_B}{c^2} \tag{5.2}$$

is the radius at which a body of mass m_B would become a black hole. If a black hole is created during the observation of a body B then the observation itself becomes impossible because light cannot escape from a black hole.

These two conditions imply the following inequality

$$\lambda_C \gtrsim \alpha\, r_{\rm s} \tag{5.3}$$

which is a limit to the observation of a body B of mass m_B with a photon having a wavelength equals to the Compton wavelength of the body. The positive constant $\alpha > 0$ depends on the fact that it is possible to take into account:

– the half of the Compton wavelength $\frac{h}{2m_B c}$ because a pair production creates a particle and an antiparticle;

– the Schwarzschild diameter $2r_{\rm s}$ rather than the Schwarzschild radius $r_{\rm s}$.

Equation (5.3) leads to the following inequality

$$m_B \lesssim \sqrt{\frac{hc}{2\alpha G}}. \tag{5.4}$$

The mass $\sqrt{\frac{hc}{2\alpha G}}$ is the maximum mass of a body that can be observed with a photon having a wavelength equals to the Compton wavelength of the body. Above this mass, it is possible to generate a black hole during the observation with such a photon. The inequality (5.4) does not mean that a black hole is always created during the observation of a body whose mass is higher than $\sqrt{\frac{hc}{2\alpha G}}$ by a photon having a wavelength equals to the Compton wavelength of the body, because it depends on the density of the body B obtained during the observation.

Then, the momentum of B satisfies

$$p_x := m_B v_x \lesssim \sqrt{\frac{hc}{2\alpha G}} v_x \tag{5.5}$$

and then

$$\Delta p_x \lesssim \sqrt{\frac{hc}{2\alpha G}} \Delta v_x \lesssim \sqrt{\frac{hc^3}{2\alpha G}} \tag{5.6}$$

because the speed of light c is also a limit for Δv_x. By using the Heisenberg uncertainty principle (2.3), we have

$$\Delta x \gtrsim \frac{h}{\Delta p_x} \gtrsim \sqrt{\frac{2\alpha h G}{c^3}} \tag{5.7}$$

where $\sqrt{\frac{2\alpha h G}{c^3}}$ is the *minimal length*.

The Hoop conjecture [28] proposed by Kip Thorne states that black holes with horizons form when, and only when, a mass m_B gets compacted into a region whose circumference in every direction is

$$C \leq 2\pi r_{\mathrm{s}}. \tag{5.8}$$

By using the Hoop conjecture and the Heisenberg uncertainty principle (2.5), it is possible to infer the inequality

$$\Delta x \geq \beta\, \ell_p = \beta \sqrt{\frac{\hbar G}{c^3}} \tag{5.9}$$

where ℓ_p is the Planck's length and β a positive constant which has the same purpose than α [29]. However, the Hoop conjecture is not proved and the interpretation of the circumference of a black hole is already unclear. The inequality (5.9) should become clearer with a future probabilistic theory of quantum gravity adapted to the use of the Copenhagen interpretation of the Heisenberg uncertainty principle (2.5).

The Heisenberg uncertainty principle, with the general relativity, is a limit to the knowledge of the space itself. Once again, it is possible to consider that there is nothing of physical nature below the minimal length. This is the standard interpretation which does not prevent the development of quantum gravity. The term "standard" reflects the fact that it is probably the most common opinion within the physics community. As for quantum mechanics, we are facing a similar paradox. Indeed, if we know that there is nothing of physical nature below the minimal length then it contradicts the fact that there is a limit to our knowledge. We can expect several interpretations for this problem which is known as the interpretation of quantum gravity. So, we can state the second main result of this article.

Proposition 2. *The uniqueness of interpretation of quantum gravity leads to a paradox.*

In a logical point of view, several coherent interpretations of quantum gravity should exist in order to avoid this paradox. Among the different theories of quantum gravity:

- string theory;
- loop quantum gravity;
- noncommutative geometry;
- twistor models;

etc.

the boundary of the minimal length is different. String theory, which is the first consistent theory of quantum gravity, attempts to unify gravity with the other fundamental interactions by adding supplementary dimensions and by postulating that there exist strings whose size is of the minimal length order [30]. Loop quantum gravity tends to quantize the gravitational field by quantizing the space and by postulating that there exist loops whose size is related to the minimal length [31]. At the moment, a general mathematical formalism for quantum gravity is still missing and we are waiting a "new" Paul Dirac to unify the different theories which may be different sides of a more general mathematical theory. When the mathematical framework will be established, the same open question remains:

Question 2. *Is there a finite number of interpretations of quantum gravity?*

As for quantum mechanics, it is difficult to see why there should be a limit to the number of interpretations of quantum gravity. It must be mathematically coherent and must lead to the same experimental results as the other interpretations. We can also expect several interpretations of quantum gravity using for instance the Copenhagen, de Broglie-Bohm and many-worlds interpretations of quantum mechanics. In particular, this implies that a de Broglie-Bohm type interpretation of quantum gravity could remove the residual indeterminism on the initial conditions at the Big Bang, and could lead to a deterministic non local interpretation of physics. But this would be one interpretation among many.

6. Below the uncertainty principle and the minimal length

If the standard interpretation of quantum mechanics and quantum gravity is: there is nothing of physical nature beyond the Heisenberg uncertainty principle and below the minimal length, should we abandon the search for other interpretations of quantum mechanics and quantum gravity? Why should we give up our dreams to know the nature of particles and the early universe?

First of all and as it is explained in this article, the standard interpretation cannot be the only interpretation because it leads to paradoxes. Then, we must try to answer the question: is there a finite number of interpretations of quantum mechanics and quantum gravity? It is useless to want a single interpretation and clan rivalries have no meaning. The important thing is to know the different interpretations and let our imagination do the rest. Since several decades, mathematicians have accepted the Gödel's incompleteness theorems as the limit of decidability. Physicists should accept the Heisenberg uncertainty principle as a limit to the uniqueness of the interpretation of quantum mechanics and quantum gravity.

7. Conclusion

Which of our basic physical assumptions are wrong? The uniqueness of interpretation of quantum mechanics and quantum gravity. Indeed, almost all physics textbooks present the Copenhagen interpretation as the only interpretation of quantum mechanics. It will probably be the same for quantum gravity in the future. This article has shown that a unique interpretation is not possible because it contradicts the limit of our knowledge imposed by the Heisenberg uncertainty principle.

The Gödel's incompleteness theorems is a limit to our mathematical knowledge and we know that there exist undecidable conjectures, as the continuum hypothesis. We cannot do anything else in mathematics and the David Hilbert's dream of an absolute knowledge is gone. Even if the Albert Einstein's dream of an absolute knowledge of physics is also gone with the Heisenberg uncertainty principle, things are more complicated in physics. We know that there is a limit to our knowledge in physics but it is possible to give several interpretations of what is beyond the Heisenberg uncertainty principle in quantum mechanics and below the minimal length in quantum gravity. This research is the essence of science: answer to the question *why?*, knowing very well that the answer is not definitive.

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