Why is the world as it is?

Jarmo Mäkelä*

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Abstract

A discussion with Nicole Oresme, a medieval philosopher and mathematician.

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What is the fundamental reason for why the Universe is as it is? Curiously, this problem is somehow related to the question of where all odd socks vanish. According to a still controversial theory of Stephen Hawking the odd socks are swallowed by microscopic black holes. [1] With a small modification a black hole may be turned into a worm hole, and hence it is possible that the Universe may be criss-crossed by worm holes carrying now and then macroscopic objects (socks, for example) from a one to another point of space and time.

All of this is just wild speculation, of course, but to my view it began to gain plausibility after I discovered one day that several of my cherished physics books had mysteriously vanished from my bookshelf. The missing books ranged from basic undergraduate physics textbooks to very advanced books on gravitation and cosmology. I searched for my books everywhere, but I found no traces of them. It really seemed as if my books had been swallowed by a worm hole.

Several months later I gave a lecture in a freshman mechanics course on the kinematics of uniformly accelerating bodies. The problem of the distance covered by a uniformly accelerating body during the given time interval was first solved explicitly by a medieval French philosopher and mathematician Nicole Oresme (1323-1382). His brilliant solution contained germs of both calculus and analytical geometry, which were really discovered only about 300 years later. I have always been an ardent admirer of Oresme and his work in mathematics and the sciences. Given that he lived during the darkest times of the Middle Ages, his achievements were amazing.[1] [2]

After the lecture I returned back to my bachelor quarters (I was still unmarried at the time). Determined to make the very last attempt to find my missing books I climbed to the top of my bookshelf to see, whether I had possibly left my books there. It was a dangerous enterprise, because the bookshelf was not properly fixed to the wall. I had just reached the top of the bookshelf, when I discovered, quite unexpectedly, that Nicole Oresme was standing in my study in his own person, carrying my books under his arm.

"Bonjour, Monsieur", he said in French. "Some months ago I found these books in the private library of His Majesty Charles V, the King of France.[2]

*Vaasa University of Applied Sciences, Wolffintie 30, 65200 Vaasa, Finland, email: jarmo.makela@vamk.fi

Books are very expensive, you know, and because your name and address was written in every book, I decided to return them back to you. Wonderful books, by the way. I have read every single word of them."

I have never been more surprised in my life. I shall always regret that in my agitation I even forgot to ask how he had managed to get from the 14th century Paris to my study. Nevertheless, I was overjoyed for receiving back my books, and I asked Monsieur Oresme, whether he would like to get some dinner. That was an invitation he accepted with pleasure, and after some minutes I had prepared for my illustrious guest cheese sandwiches and tea. He seemed to like the tea, even though I think that he would have preferred wine, instead. He also had an excellent appetite for cheese sandwiches.

I am far from strong in French, but fortunately I know a bit of Latin. Nicole Oresme had also learned some English from my physics books, even though he pronounced it in a very original way, and so we could discuss. My guest made a very good company. As a councellor of the King of France and the bishop of Lisieux he had courteus and confident manners of a man of world, and he carried a general air of benevolence, which is typical for the best ecclesiastics.

"Reading your books made me very impressed", said he, while we were still drinking tea. "The scholars and the scientists of the 19th and the 20th centuries got very far, indeed: You have found out why the stars shine, and how they move, revealed the inner structure of the atoms and the human cells, cured the diseases, sent men to the Moon, discovered the fundamental laws of nature, and so on. I must say that this is really something I have been dreaming of during the lonely hours spent in my study, attempting to make humble contributions in mathematics and the sciences. Yet I am delighted to observe that even you, the people of the 21st century, do not know everything, but there is at least one huge problem, which still puzzles you."

"Which problem do you mean?", asked I with keen interest.

"The accelerating expansion of the Universe", said he. "This problem is of fundamental importance by itself, but the reason why it really interests me is that the accelerating expansion of the Universe may hold the key to the problem of how the Universe got into being, and why is it such as we know it. I learned from your books that the accelerating expansion of the Universe is predicted by general relativity, a theory constructed by a person named Albert Einstein, whom, I gather, most people of your time consider as the greatest savant of all ages. Would you like to give a brief summary of the problem of the accelerating expansion of the Universe, just to make sure that I have understood everything correctly?"

"With pleasure", said I."General relativity is based on *Einstein's field equation* which, in its most general form, involves a parameter known as the *cosmological constant*.[3] In appropriate units the cosmological constant may be understood as the energy density of the vacuum, or empty space. This energy is known as *dark energy*, and it drives the accelerating expansion of the Universe. According to the present estimate about 70% of all energy in the Universe consists of the dark energy. For a long time the cosmological constant was thought to be zero. However, at the end of the 1990's conclusive evidence was found to the effect that the Universe is not only expanding, but its expansion is *accelerating* [3], and so we must really include the cosmological constant to the field equation. The problem with the cosmological constant, and hence with the accelerating expansion of the Universe is that according to the present observations the cosmological constant is about 10^{121} times smaller than its theoretically predicted value. We are thus faced with two questions: Why is there the cosmological constant at all, and why is it so small?"

"Exactly", said my new friend. "That is how I have understood the problem after reading your books. What I find particularly interesting in the accelerating expansion of the Universe is that it implies an existence of the so-called *cosmological horizon*[4] which, in the isotropic universe, is a closed two-sphere beyond which no information may be gained by the observer. To me it resembles an idea of celestial spheres embraced by the philosohers of my time. In those theories one assumes an existence of the outermost celestial sphere, beyond which we cannot gain any information. Of course, I know now that there are not any material spheres in the Universe, but nevertheless the idea made me easy to understand the concept of cosmological horizon."

Nicole Oresme took yet one more cheese sandwich, thought for a moment, and said: "Actually, I am a little bit surprised that you have not managed to solve the problem. To me its solution seems pretty obvious."

The remark made me curious, and I asked: "Would you like to explain further?"

"I am not a very great fan of the Englishmen", said he, "because we Frenchmen were engaged in a very cruel war with them just when I left from Paris.[5] However, I have learned from your books that the country has produced some very great men of learning. One of them was Sir Isaac Newton. Do you still remember the acceleration produced by a point-like mass M at a distance r to another point-like mass according to Newton's theory of gravitation?"

"Of course", said I. "In the natural units, where all natural constants are set to one, it is the mass M devided by the square of the distance r."[6]

"Suppose that you surround the point-like mass M with a sphere with radius r, multiply its area by the gravitational acceleration, and divide the result by 4π. What will you get?"

"Well", said I. " Since the area of the sphere is $4\pi r^2$, the result is of course the mass M of the source of the gravitational field."

"Exactly", said he. "So we may conclude that acceleration times the area divided by 4π equals with the mass inside of the surface. According to Einstein's famous formula $E = Mc^2$ we may interpret this mass M, in the natural units, as the energy of the gravitaional field created by the particle. Is this obvious for you?"

"Yes", said I.

"Very good", said he, "and now, do you have some pens and paper?"

I hurried to my study to fetch some paper and pencils. My friend was now becoming very enthusiastic, and it was surprising how much he, a medieval scholar coming from an entirely different world, had learned from my physics books.

"Einstein's general relativity brings along a slight modification to our result", he said. "If the region of space is surrounded by a *horizon*, the correct formula for the energy of the gravitational field may be obtained by dividing the Newtonian expression by two. So we get: [4]

$$E = \frac{a}{8\pi}A, \tag{1}$$

where a is the acceleration measured by an observer close to the horizon for falling bodies, and A is the area of the horizon".

"An interesting feature of this result", he continued, "is that it depends entirely on the properties of the horizon only. As I learned from your books, the energy of any system may be identified with what you call as the *Hamiltonian* of the system. From the Hamiltonian of the system, in turn, one may deduce all of its properties. Hence we arrive at an unavoidable conclusion that all information about the properties of the region of space inside of a horizon is included in the properties of its horizon. Do you follow me?"

"Perfectly", said I. "I think I understand now, where you are aiming at. Do you want to say that all information about the properties of the visible, accceleratingly expanding Universe is really contained in the properties of its cosmological horizon or, as you might say, in the properties of its outermost celestial sphere?"

"Right!" he said. "You have got the point!" He drank some tea, took up a pencil and a piece of paper, and went on: "The next question, of course, is what are the properties of the cosmological horizon. Among your books was one written by Rovelli, or something. [5] I must confess that reading the book was very hard to me. However, from the book I gathered an idea that space may be thought as a network of links and nodes, somehow like a fisherman's net, but three-dimensional. Whenever you have a two-dimensional surface, its area is of the form:

$$A = 8\pi\gamma \sum_p \sqrt{j_p(j_p+1)}, \tag{2}$$

where we have summed over those links p, which pass through the surface. In this formula γ is a pure number of the order of unity, and the possible values taken by the numbers j_p are $0, \frac{1}{2}, 1, \frac{3}{3}$, and so on."

"You are talking about loop quantum gravity", said I. "The network you mentioned is known as *spin network*, because the links have properties similar to the spins of the elementary particles."

"That is right", said he. "I found very interesting the idea that the area of the given surface is not allowed to take arbitrary values, but the possible areas are given by Eq. (2). I also learned from Rovelli's book that for every j_p you may associate a quantity m_p, which takes all $2j_p + 1$ values, with spacing 1, from $-j_p$ to j_p. So, for instance, the only possible value of m_p is zero, when $j_p = 0$, but when $j_p = \frac{1}{2}$, the possible values of m_p are $\pm\frac{1}{2}$."

"That is all very interesting", said I, "but what does that have to do with the accelerating expansion of the Universe?"

"I shall come to that in a moment", said he. "I have learned from your books that whenever you have a system with a very large number of constituents, its properties may be investigated by means of its *partition function*

$$Z(\beta) = \sum_n e^{-\beta E_n} \tag{3}$$

where we have summed over all states n of the system, and E_n denotes the energy of the state n. $\beta = \frac{1}{T}$, which is known as the *temperature parameter*, is the inverse of the absolute temperature T of the system. Identifying the area A in Eq. (1) as the area of the cosmological horizon we may understand the right hand side of Eq. (1) as the energy of the Universe from the point of view of an observer with acceleration a, very close to the horizon. Employing Eqs. (2) and

(3) we find that the partition function of the Universe from the point of view of such observer takes the form:

$$Z(\beta) = \sum_{j_1 \cdots j_N} (2j_1 + 1) \cdots (2j_N + 1) \exp\left[-\gamma a \sum_{p=1}^{N} \sqrt{j_p(j_p+1)}\right]. \tag{4}$$

When writing this expression we have assumed that the number N of those links, which pass through the horizon is fixed. We have also taken into account that for all j_p there are $2j_p + 1$ allowed values of m_p."

Nicole Oresme drank again a bit of tea and continued: "The next question is how to count the states. To keep the things simple, we shall allow only the values 0 and $\frac{1}{2}$ for the numbers j_p. We say that the link p is in *vacuum*, if $j_p = 0$; otherwise it is in an *excited state*. In what follows, we shall sum over the excited states only; *i. e.* we shall assume that the vacuum states do not contribute to the partition function."

"It seems to me that your system of links passing through the horizon is somewhat similar to a system of particles with spin $\frac{1}{2}$", I remarked.

"Indeed", said he. "Using the standard rules of powering we may write Eq. (4) as:

$$Z(\beta) = \frac{1}{z} + \frac{1}{z^2} + \cdots + \frac{1}{z^N}, \tag{5}$$

where we have denoted:

$$z = z(\beta) := \frac{1}{2} \exp\left(\beta \frac{\sqrt{3}}{2} \gamma a\right). \tag{6}$$

In the first term on the right hand side of Eq. (5) just one link is in an excited state, in the second two links are in excited states. Finally, in the last term all N links are in the excited states."

"As you may observe", he continued, "on the right hand side of Eq. (5) we have a simple geometric series. Can you calculate its sum by yourself?"

"Of course", said I. "The sum is

$$Z(\beta) = \frac{1}{z} \frac{1 - \frac{1}{z^N}}{1 - \frac{1}{z}} = \frac{1}{z-1}\left(1 - \frac{1}{z_N}\right), \tag{7}$$

where I have used the formula first discovered, according to my understanding, by yourself."[7]

Nicole Oresme waved away the compliment, but it was obvious that it had pleased him. "The average energy of any system as a function of the temperature parameter β", he continued, "is

$$E(\beta) = -\frac{\partial}{\partial \beta} \ln Z(\beta). \tag{8}$$

Reading your books I learned to differentiate. The result is:

$$E(\beta) = \frac{\sqrt{3}}{2} a\gamma \left(\frac{z}{z-1} - \frac{N}{z-1}\right). \tag{9}$$

At this point the the cosmological constant, which we shall denote by Λ, will finally enter the stage. Since about 70% of the energy of the Universe is dark energy, the cosmological constant will ultimately dominate the time evolution of the Universe. In a universe with no matter, but just the cosmological constant, the radius of the cosmological horizon is, in the natural units:

$$r_C = \sqrt{\frac{3}{\Lambda}}. \tag{10}$$

Using Eq. (1) we find that the energy of the Universe from the point of view of an observer just inside of the cosmological horizon is

$$E = \frac{a}{8\pi} 4\pi r_C^2 = \frac{3a}{2\Lambda}. \tag{11}$$

Identifying the right hand sides of Eqs. (9) and (11) we obtain an expression for the cosmological constant Λ in terms of the temperature parameter β:

$$\Lambda(\beta) = \frac{\sqrt{3}}{\gamma} \left(\frac{z}{z-1} - \frac{N}{z^N - 1} \right)^{-1}, \tag{12}$$

and it is a function of z only."

It was astonishing, how quickly Nicole Oresme was able to perform all these calculations. That was the more remarkable, since the standard notations of mathematics were not established before the 17th century. Obviously, he had learned the notations from my books, and practiced a lot.

"Now, can you see anything special in Eq. (12)?" he asked.

"I must confess that I am in dark", said I.

"Well", he said, "can you see what happens, when $z = 1$, assuming that N, the number of the links passing through the horizon, is very large?"

"Oh, yes", said I. "If $z > 1$, then z^N is enormous for large N, the first term inside of the brackets in Eq. (12) will dominate, and we may write the cosmological constant, in effect, as:

$$\Lambda = \frac{\sqrt{3}}{\gamma} \frac{z-1}{z}. \tag{13}$$

However, if $z < 1$, then z^N becomes very small for large N, and the second term inside of the brackets will dominate. As a consequence, we have:

$$\Lambda = \frac{\sqrt{3}}{N\gamma}, \tag{14}$$

which is very small for large N."

"Exactly", said Nicole Oresme. "Our calculations indicate that at the temperature, where $z = 1$ from the point of view of our observer, the Universe went through a *phase transition*, where the cosmological constant dropped enormously. When the Universe was created, the entropy and the temperature of the Universe were presumably very low. In the low-temperature limit the temperature parameter β, and therefore z, are very large, and we may write Eq. (13) as:

$$\Lambda = \frac{\sqrt{3}}{\gamma}. \tag{15}$$

So we find that during the phase transition the cosmological constant drops by the factor $\frac{1}{N}$."

"I see", said I. "That is very interesting. In the metric system of units the cosmological constant takes before the phase transition, assuming that γ is of the order of unity, the form:

$$\Lambda = \frac{\sqrt{3}}{\gamma}\frac{c^2}{\ell_{Pl}^2} \sim 10^{86}s^{-2}, \tag{16}$$

which is very large, indeed. In this equation

$$\ell_{Pl} := \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-35}s^{-2} \tag{17}$$

is the so-called *Planck length*, which is generally viewed as the smallest possible length. However, if we take N to be around 10^{121}, then the cosmological constant after the phase transition is

$$\Lambda \sim 10^{-35}s^{-2}, \tag{18}$$

which agrees with the present-day observations."

"You are right", said Nicole Oresme. "The reason why the presently observed value of the cosmological constant is 10^{121} times smaller than its theoretically predicted value is, quite simply, that the links of the spin network pass through the horizon 10^{121} times. However, the cosmological constant is still non-zero, and hence we have managed to find an explanation both to the presence and the smallness of the cosmological constant."

"Your explanation seems very plausible to me", said I. "Do you have any idea of what happened during the phase transition?"

"During the phase transition", said he, "the cosmological constant went down very rapidly, and the energy of the vacuum was converted to the energy of the matter fields. In other words, matter was created. Since the cosmological constant, which drives the accelerating expansion of the Universe, was very large at the moment of its creation, the Universe went through a period of an extremely rapid expansion. Actually, the expansion was so rapid that if the cosmological constant had decreased slowly enough, the whole Universe as such as we observe it could have been created in a tiny fraction of a second."[8]

"That is most interesting", said I. "You told that matter was created, when the cosmological constant decreased. Was the creation of matter a completely random process, or was it governed by certain rules?"

"You may still remember", said Nicole Oresme, "that all information about the properties of the Universe is contained in the properties of its cosmological horizon, and that with each link of the spin network passing through the horizon one may associate the number j_p, which takes the values 0 and $\frac{1}{2}$, and the number m_p, which is 0, when $\jmath_p = 0$, and either $\frac{1}{2}$ or $-\frac{1}{2}$, when $j_p = \frac{1}{2}$. I am of opinion that the properties of matter depend on the values taken by these two numbers at each link during the phase transition. In other words, all information about the properties of matter, you, I and everything in the Universe is contained in the combination of those numbers. Assuming that after the phase transition $j_p = \frac{1}{2}$ at each of the 10^{121} links passing the cosmological horizon, the number m_p is either $\frac{1}{2}$ or $-\frac{1}{2}$ at each link, and there are

$$2^{10^{121}}$$

possible combinations of the numbers m_p. This is an enormous number of different possibilities, and the properties of the Universe are, I believe, uniquely determined by the combination taken by the numbers m_p. In short, the world is as it is, because the numbers m_p in the links passing through the cosmological horizon have a certain ordering."[9]

"I think I understand what you mean", said I. "Actually, there is an interesting analogue between the properties of the Universe and those of the human cells: Each cell of a living organism contains the so-called DNA molecule, where a large number of four different base molecules are arranged in a very long chain. It is the ordering of those base molecules in the chain, which determines the properties of the cells of the organism, and thereby the properties of the organism as a whole. With some ordering of the base molecules we get a human being, with another a donkey, a cat, or whatever. So it seems to me that with a different ordering of the numbers m_p at the links the Universe would have become completely different, in the same way as the properties of a living organism are dramatically changed, if the ordering of the base molecules in the DNA molecule in its cells is changed. In other words, the ordering taken by the numbers m_p almost immediately after the creation determined the evolution of the Universe in the same way as does the ordering of the base molecules in the DNA molecule in the first cell of an embryo determine the developement of the embryo to a human being. This, of course, leaves us with a perennial question of what determined the ordering of the numbers m_p in the links?"

"Ah", said Nicole Oresme. "It is here, where you take me from the realm of physics to that of metaphysics. The probability of a random creation of a universe exactly like ours is extremely low. It is just 1 out of $2^{10^{121}}$.[10] Given this incredibly small probability I find very hard to believe that the ordering of the numbers m_p, and hence the properties of the Universe, would be mere chance. Yet I cannot see any physical, underlying principle, which would determine those numbers. As a faithful servant of the Church I am tempted to believe that the ordering of those numbers was chosen by God for reasons, which are beyond the human comprehension."

Our discussion had reached its conclusion. "I have very much enjoyed this conversation", said my guest. "My current responsibilities allow me very little time for deep philosophical discussions. And now", he continued, "I should like to get yet one more cup of tea."

We had already drunk all tea and eaten the sandwiches. I went to the kitchen to prepare more tea. "Would you like to get some more cheese sandwiches?" I asked.

There was no answer. I walked back to the sitting room. There was no-one. I could not find my guest anywhere. Where had he gone? Had he gone out, perhaps? I was afraid that a man, coming from the 14th century France in his celestial robes would probably not survive in our society, no matter how intelligent he was.

I ran to the street in front of my house. I saw no traces of him. A huge lorry was rolling towards me with a high speed. I knew I could not avoid it...

I was lying on the floor of my study suffering from a terrible headache. My books weere all scattered around the floor. I decided to fix my bookshelf properly to the wall. I do not want to be knocked out by a falling bookshelf again.

Notes:

1) Nicole Oresme may be considered as the first notable mathematician in Europe since the Ancient. Among other things, he investigated the properties and the convergence of infinite series, introduced the concept of fractional power, and applied to physical problems methods, which were closely related to differential and integral calculus. He also considered seriously a heliocentric theory of the Solar System. His derivation of the expression for the distance covered by a uniformly accelerated body during the given time interval by means of geometrical methods inspired the mathematicians of the 17th century. In modern terms, he interpreted the definite integral as area. He was educated as a theologian, and he worked, during the last five years of his life, as a bishop of Lisieux.

2) Charles V, the King of France during the years 1364-1380, was born in the year 1338. He appreciated learning and the sciences, and he had a large private library with more than 1200 books, which was really something, because at those times all books were still hand-written. Nicole Oresme was his close friend and councellor.

3) Einstein's general theory of relativity explains the properties of gravitation by means of the properties of space and time. In this theory space and time in together constitute a four-dimensional spacetime. Einstein's field equation reads, in the natural units, as:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}.$$

The quantity $G_{\mu\nu}$ describes the geometry of spacetime, and $T_{\mu\nu}$ the properties of matter. The parameter Λ is known as the cosmological constant, and $g_{\mu\nu}$, which is known as the metric tensor, tells how to measure distances between points of spacetime.

4) In spherically symmetric, empty spacetime with positive cosmological constant Λ the so-called line element of spacetime may be written as:

$$ds^2 = -(1 - \frac{\Lambda}{3}r^2)\,dt^2 + \frac{dr^2}{1 - \frac{\Lambda}{3}r^2} + r^2\,d\theta^2 + r^2\sin^2(\theta)\,d\phi^2.$$

At the cosmological horizon the roles played by the time coordinate t and the radial coordinate r are interchanged. The radius of the cosmological horizon is:

$$r_C = \sqrt{\frac{3}{\Lambda}}.$$

5) The so-called Hundred Years War was fought between England and France during the years 1337-1453.

6) According to Newton's theory of gravitation point-like masses m_1 and m_2 attract each other with the gravitational force

$$F = G\frac{m_1 m_2}{r^2},$$

where $G \approx 6.67 \times 10^{-11} Nm^2kg^{-2}$ is Newton's gravitational constant, and r is the distance between the masses.

7) In the modern notation this formula reads as:

$$a + aq + aq^2 + \cdots + aq^{n-1} = a\frac{1-q^n}{1-q}.$$

Among other things, Nicole Oresme performed a detailed study of the properties and the convergence of the geometric series.

8) In empty, spherically symmetric spacetime with positive cosmological constant Λ the so-called *scale factor* R, which describes the radius of the Universe, evolves as a function of the time t according to the formula:

$$R(t) = c\sqrt{\frac{3}{\Lambda}}\cosh(\sqrt{\frac{\Lambda}{3}}t).$$

According to the present estimate R is around 10 billion light years, or $10^{26}m$. Hence it follows that if we take $\Lambda \sim 10^{86}s^{-2}$, the time needed for the expansion of the Universe to its present size is

$$t = \sqrt{\frac{3}{\Lambda}}\cosh^{-1}(\frac{R}{c}\sqrt{\frac{\Lambda}{3}}) \sim 10^{-41}s.$$

9) The *entropy* of the system may be defined, under certain conditions, as the natural logarithm of the number of the microscopic states associated with the given macroscopic state. Identifying the number Ω of these microscopic states as the number of the combinations of the numbers m_p in the N links passing through the cosmological horizon we find $\Omega = 2^N$, and therefore the entropy is:

$$S = \ln(2^N) = N\ln(2).$$

However, according to Eq. (2) each link contributes to the horizon an area $4\pi\sqrt{3}\gamma$. Hence the area of the cosmological horizon is

$$A = 4\pi\sqrt{3}N\gamma,$$

and if we put $\gamma = \frac{\ln(2)}{\sqrt{3}\pi}$, the entropy is

$$S = \frac{1}{4}A,$$

which is exactly the expression obtained by Gibbons and Hawking in Ref. [6].

10) To illustrate how small this probability really is, suppose that we have written the digits of the number $2^{10^{121}}$ on paper sheets, which are 0.1 mm thick, one digit per each square millimeter. It turns out that the total volume of the paper sheets needed would be around 10^{31} times the volume of the observable Universe. It is interesting that a somewhat similar estimate for the probability of a random creation of a universe exactly like ours was arrived at on entirely different grounds by Roger Penrose in Ref. [7].

References

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