Fluctuation and Superposition of Digital and Analog Aspects of Spacetime

Willard Mittelman

Abstract

Motivated by the problem of obtaining a plausible account of dark energy, we describe the microstructure of spacetime as continually fluctuating between discreteness and a generally continuous state, with dark energy taking the form of a quantum potential of spacetime that emerges from this process of fluctuation. In addition, we consider the problem of explaining the initial conditions of our universe, which leads us to entertain the idea that spacetime itself is a superposition of discrete and continuous metrics.

In confronting the question of whether reality is analog or digital, the strategy we adopt here is to consider certain approaches and ideas that are useful in connection with other fundamental cosmological questions, and to see if these ideas enable us to get a handle on the analog vs. digital question. Certainly, this is not the only strategy that one could adopt; and even assuming the adoption of this strategy, one could still choose different approaches and ideas to work with than the particular ones chosen here. Nonetheless, we feel that the above strategy is at least worth exploring, and that the ideas examined here are at least worthy of consideration; and as will be seen, we do obtain some interesting perspectives on the question of whether reality is analog or digital. It should be added that we do not accept uncritically the ideas that are explored here; rather, we undertake modifications of these ideas as needed, both in order to deal with certain problems that these ideas face and in order to connect the ideas together in a way that helps answer some fundamental questions, such as whether reality is analog or digital.

We begin with the perspective of causal set theory, according to which the four-dimensional spacetime of our universe is describable as a set of discrete, indivisible Planck-sized elements, or "causet elements." Each of these elements is subject to random, Planck-scale fluctuations of its volume, and hence also of its density $\rho = 1/v$, where v is the volume of the relevant causet element. There are two main reasons for considering this theory here. First, it offers a promising account of black hole entropy, in particular the Bekenstein entropy bound [1, 2]. And second, the theory naturally yields an account of dark energy, and hence of the universe's accelerated expansion, that is attractive in various respects [3]. There is, however, a significant problem with this account of dark energy: namely, the theory treats dark energy in terms of stochastic vacuum fluctuations, and these fluctuations entail an inhomogeneity of dark energy that does not correspond to actual observations of the CMB [4]. This raises the question of whether causal set theory's account of dark energy can be modified so as to resolve this problem, or whether the theory – and perhaps the idea of spacetime as discrete – should simply be abandoned altogether.

Our response to this problem is to modify causal set theory by positing that nonlocal, mutual cancellations of the volume-fluctuations of causet elements occur frequently and regularly, and on a cosmological scale. Such cancellation may be viewed, in the spirit of Mach's Principle [5], as reflecting an interplay among all parts of the universe, no matter how widely separated they may be; it is also consistent with the strong nonlocality of causal set theory. A crucial point to note is that the cancellation of volume-fluctuations at a given time t is not complete, or total. For, given the existence of N causet elements that constitute the four-volume V of the universe at t, there will be an uncanceled remnant at t consisting of approximately $\sqrt{N}$

causet elements; this is a consequence of both the random character of these fluctuations and the Law of Large Numbers, together with the fact that N is indeed a very large number. As a result of N's largeness, the value of $\sqrt{N}$ is, *in relation to N*, very small; hence, uncanceled fluctuations are *relatively* rare at any given time. The vast majority of causet elements, due to the mutual cancellation of their fluctuations, become virtually indistinguishable from each other, so that even at the Planck scale spacetime takes on a uniform character that approximates a continuum, or is analog-like. Thus, the cancellation of volume-fluctuations here represents a movement or "flow" of spacetime from the digital – i.e., a set of distinct elements, each with its own individual volume-fluctuations – toward the analog. As indicated, however, spacetime never becomes *completely* analog, but contains roughly $\sqrt{N}$ causet elements exhibiting volume-fluctuations that make them stand out like isolated digital "islands" in a vast analog "sea." These volume-fluctuations affect the size, and hence also the density ρ, of each of these $\sqrt{N}$ elements, so that the relation between each element and the surrounding "sea" is characterized by a nonzero density-gradient. This nonzero gradient, combined with the fact that the density-gradient *within* the surrounding sea is zero, gives rise to a nonzero Laplacian $\Delta\rho$ for the density ρ; and this Laplacian in turn entails the existence of a nonlocal "quantum potential of spacetime" that acts as dark energy, as explained in [6]. (Note, by the way, that the continuum-like character of the surrounding sea allows the employment of "continuum" concepts such as the gradient.)

The concept of a quantum potential is familiar both from the Bohmian interpretation of quantum mechanics [7] and from the Madelung or hydrodynamic representation of wave functions, a representation that is useful in the treatment of Bose-Einstein condensates [8]. The quantum potential concept can be made applicable to the 4D spacetime of causet elements if spacetime is itself treated like a Bose-Einstein condensate, as proposed, e.g., by B.L. Hu [9]. Such a treatment of spacetime finds motivation in causal set theory itself. For this theory, as the above discussion has made clear, employs the number density $\rho = n/v$ of causet elements, where n is the number of causet elements and v is the total volume of these elements; and this number density is formally similar or analogous to the particle density $\rho = n/v$ used in connection with Bose-Einstein condensates. In the Madelung representation, the condensate wave function ψ is given by the following equation: $\psi = \sqrt{\rho}(\exp[iS/\hbar])$. Here ρ is the above-mentioned particle density of the condensate; in addition, we have $|\psi|^2 = \rho$, and S has the dimensions of an action [8]. The expression for the quantum potential is obtained by substituting the Madelung representation of ψ into the Schrodinger equation, a substitution that yields two equations, one of which is the Quantum Hamilton-Jacobi Equation (QHJE); writing "R" for $\sqrt{\rho}$, the quantum potential is given by the term "$[(-\hbar^2/2m)(\Delta R/R)]$" occurring in the QHJE, which has the following form [7, 8]:

$$\partial S/\partial t + (\nabla S)^2/2m + V - [(\hbar^2/2m)(\Delta R/R)] = 0.$$

For present purposes, we may disregard the first three terms in the above equation. Our focus is solely on the quantum-potential term, since it can be demonstrated by explicit calculation that the magnitude of the quantum potential Q of spacetime is comparable to the observed magnitude of dark energy [6]. Furthermore, the nonlocality of Q entails a homogeneous distribution of dark energy, thereby avoiding the problem of inhomogeneity mentioned earlier.

A crucial point is that the dark energy Q produces *new* causet elements that perturb or disrupt the calm analog "sea" described above. The result of this perturbation is an outbreak of pervasive volume-fluctuations, until a new round of fluctuation-cancellations restores a generally (but not totally) analog-like character to spacetime. The quantum potential is therefore associated with a "flow" of spacetime from analog to digital, which is followed immediately by an opposite flow from digital toward analog that reflects the mutual cancellation of volume-fluctuations described earlier. Thus spacetime reality, at a fundamental level, is characterized by a continual back-and-forth movement between the analog and the digital. There is an asymmetry here, however: the movement from digital to analog is an effect of nonlocal relations between causet elements, and hence is instantaneous; the opposite movement, however, is a (non-instantaneous) *process* in which dark energy produces new causet elements, and as such it represents a flow of time itself – at least if we parametrize time by the number N of cauet elements, as suggested in [3]. Now this latter movement, involving as it does the breakup of a generally homogeneous and continuous arrangement of causet elements – with this arrangement being replaced by a set of discrete, randomly fluctuating causet elements – yields Local Lorentz Invariance (LLI), in accordance with causal set theory, while the opposite movement breaks LLI (see [3, 10] on LLI as a feature of causal set theory, and on the connection between LLI and random – specifically, Poisson – fluctuations of causet elements). But because LLI is broken only *instantaneously*, LLI is "temporally predominant," so to speak; and so, to a good approximation, LLI can be taken as holding generally, thereby alleviating the tension between causal set theory and the idea of nonlocal cancellations of volume-fluctuations. (It is worth noting, in addition, that even in general relativity LLI does not hold exactly [11].)

We note in passing that random fluctuations of causet elements produce a dimensional reduction of spacetime at the Planck scale, from four dimensions (3,1) to two dimensions (1,1) [12]. This reduction effectively "collapses" causet elements into each other, or blurs the distinction between them, so that the number of distinct causet elements – and hence, the number of degrees of freedom of the spacetime – is the square root of what it would be in the absence of such reduction (since two-dimensional area is the square root of four-dimensional volume). Specifically, the number of degrees of freedom is on the order of 10^{123}, which is in accordance with the holographic entropy bound for our universe (i.e., the maximum entropy allowed by the holographic principle). The nonlocal cancellation of volume-fluctuations, however, leads to a general cessation or "suspension" of dimensional reduction. When this happens, the distinction between causet elements still "collapses," or is "reduced," *but in a different way*: specifically, the general cancellation of volume-fluctuations effectively "merges" the vast majority of causet elements into a single, continuous whole – the "analog sea" mentioned earlier – that is *not* characterized by multiple degrees of freedom. The spacetime degrees of freedom here are represented instead by the approximately $\sqrt{N}$-many, volume-fluctuating causet element "islands" which, by virtue of their fluctuations, stand apart from the analog sea. Since there are thus only $\sqrt{N}$ degrees of freedom, the holographic entropy bound is still respected. Hence, the very *way* in which spacetime degrees of freedom, or "atoms of spacetime," are determined for a causal set is something that continually fluctuates back and forth; i.e., the very *nature* of spacetime's microstructure is constantly shifting. But the holographic entropy bound remains in effect, despite these microstructural shifts that represent changes or fluctuations in the particular way in which this bound is respected.

This is not all that can be said, however, concerning the question of whether spacetime is analog or digital. To see how the need for additional ideas regarding this question arises, we begin by noting that an important unresolved issue of modern cosmology is the problem of explaining the unusual initial conditions of the universe, in particular the existence of an initial singularity (or near-singularity) characterized by extremely high temperature and extremely low entropy. This problem is present in causal set theory, e.g., according to which the universe initially is represented by a causal set consisting of a single "origin element" [13]. It is simply unclear how the bare idea of such an element can yield an explanation of the initial co-existence of very high temperature with very low entropy. In order to deal with this problem here, we consider the intriguing idea, developed in [14] and connected with M-theory, that spacetime itself – and hence, causal sets and their elements – emerges from the dynamical state space of a generalized quantum mechanics, where the (continuous) metric on this state space is a generalization of the Fubini-Study (FS) metric on a non-linear Grassmannian that generalizes the complex projective phase space $\mathbf{CP}^n$ of quantum mechanics. The key result utilized in [14] is that the generalized FS metric induces a vanishing geodesic distance on the state space, i.e. the geodesic distance between any two points on this space is zero, due to unbounded positive curvature in some directions that causes the space to curl up on itself arbitrarily tightly [15]. (In what follows, we refer simply to the "FS metric," with its generalized character being understood.) This vanishing-geodesic property is given a physical interpretation in [14]: namely, it is the key defining characteristic of the cosmological singularity at the origin of our universe. The physical consequence of the vanishing geodesic distance here is that vacuum fluctuations of the state space are effectively "jammed," leading to an unstable "crystalline" state of low entropy and high temperature, with the instability of this state giving rise to a phase transition consisting of an explosive inflationary/Big Bang event in which the vast fabric of spacetime is produced, together with an arrow of time that runs from low entropy/high temperature conditions to high entropy/low temperature conditions [14]. We regard this as an interesting and attractive way of addressing the problem of the universe's unusual initial conditions. Nonetheless, as explained below, we wish to modify the ideas of [14] in one important respect, a modification that is relevant to the account of dark energy given above.

From the standpoint of causal set theory, the production of the fabric of spacetime is tantamount to the creation of a large number of causet elements. Due to the discreteness of these elements, the spacetime metric at the Planck scale exhibits discreteness, though this is combined, as indicated above, with fluctuations toward a continuous metric. Now on the account given in [14], the emergence of large-scale spacetime from the initial singularity represents an evolution of the state space itself, with the FS metric being replaced by, or evolving into, a different metric. The nature of this process of "metric evolution," however, is not described or explained. This motivates us to consider an alternative possibility here, according to which the FS metric does not completely disappear: specifically, we propose that, at a scale close to the Planck scale, the state space metric evolves into a superposition that includes among its eigenstates the FS metric itself, as well as discrete metrics with non-zero geodesics. (The wave function describing this superposition may plausibly be viewed as undergoing factorization, so that causally disconnected sub-regions of the larger universe that emerges from the initial singularity are each characterized effectively by their own separate wave function and superposition.) The question then becomes, what are the physical effects or manifestations of this superposition – or more particularly, of the FS metric that is one of the eigenstates of this superposition? We propose here, in answer to this question, that the FS metric, with its vanishing geodesic, is "felt" in the form of nonlocal relations between causet elements. The effect of this nonlocality is just as if the geodesic distance between any two points actually were zero; hence we get, at a given time t, a near-total, nonlocal cancellation of the volume-fluctuations of the N causet elements that constitute the four-volume V of the universe at t. Such a cancellation, as explained earlier, represents a fluctuation of the very microstructure of spacetime toward an analog state; hence, at a fundamental level (i.e., at the Planck scale), the superposition of discrete and continuous metrics is reflected or manifested in a continual process of microstructual fluctuation between the digital and the analog (or near-analog). In addition, the occurrence of nonlocal action – specifically, the nonlocal cancellation of volume-fluctuations – on a cosmological scale implies the existence of a cosmological preferred frame; hence, the eigenstate consisting of the FS metric also makes its presence felt, or manifests, in the form of such a preferred frame. The existence of a preferred frame, admittedly, goes against the tenets of causal set theory itself [3, 10]. Nonetheless, there is no *inherent* contradiction between the idea that a preferred frame exists and the idea that volume-fluctuating, Planck-sized (causet) elements exist. These two ideas can be brought together by simply supplementing causal set theory's account of the volume-fluctuations of causet elements with the claim that the process of fluctuation here is continually "interrupted" or "suspended" by instantaneous, nonlocal cancellations of these fluctuations, cancellations that represent fluctuations of the very microstructure of spacetime and that presuppose the existence of a preferred frame. Although the existence of a preferred frame does entail some tension with causal set theory due to the fact that a preferred frame breaks Local Lorentz Invariance, this tension is substantially alleviated, as indicated earlier, by virtue of the fact that cancellation of volume-fluctuations here is *instantaneous*.

The key consequence of this nonlocal, near-total cancellation of volume-fluctuations, as already noted, is the formation of a highly uniform, nearly-continuous arrangement of causet elements, an arrangement that recalls the (unstable) "crystalline" state described in [14] as immediately preceding the Big Bang. In the post-Big Bang era, however, the fact that cancellation here is not total, i.e. the fact that uncanceled fluctuations exist, entails the existence, as we have seen, of dark energy – in the form of a quantum potential of spacetime – that quickly "dissolves" the crystalline state through the production of new causet elements that disturb the generally orderly arrangement of existing elements.

Thus dark energy, conceived here as the quantum potential of spacetime, results in part from the existence and effects of a preferred frame. The conception of a cosmological preferred frame is, of course, not new. What is new here is the idea of *explaining* the existence of a preferred frame, and its nonlocal effects, in terms of an underlying "analog" aspect of reality, namely a continuous (FS) metric that is one of the eigenstates of a superposition of metrics, a superposition in which discrete metrics are also among the eigenstates. The presence of discrete metrics here reflects the existence of causet elements as fundamental constituents of the four-dimensional spacetime of our universe. And as explained earlier, the dark energy represented by the quantum potential of spacetime is an effect of *both* the digital *and* the analog aspects of reality just mentioned.

The analog character of the FS metric can be regarded as reflected in the very character of the cosmological preferred frame that manifests this metric. Specifically, we can view this preferred frame as a "gravitational aether" [16] that is represented by a cosmological stress tensor, the latter of course being an analog or continuum concept. One motivation for positing the existence of such a tensor is its ability to accommodate the many-fingered-time formalism of quantum field theory [17]. Another motivation is the contribution that such a tensor makes to a degravitation scenario that resolves (part of) the cosmological constant problem [16]. In addition, the idea of a continuum viscous stress tensor appears in a Weylian approach that offers promising new perspectives on such things as the Pioneer anomaly and Dirac's Large Numbers Hypothesis [18]. The feature of viscosity is important in connection with the gravitational aether of [16] as well, since it can help ensure the satisfaction of observational constraints on the time-variation of the gravitational constant G [19].

Since the FS metric, in contrast to the discrete metric(s) associated with causet elements, arises naturally in connection with the phase space $\mathbf{CP}^n$ (a naturalness that carries over to the *generalized* forms of both the FS metric and $\mathbf{CP}^n$), we make the assumption here that the FS metric is a preferred eigenstate of the superposition of metrics. Furthermore, we assume that, as the universe continues to expand, there comes a point in the distant future at which the energy density of the aether or preferred frame gets so close to zero that quantum fluctuations of this density can take the value of this density all the way to zero. As a result, the aether effectively fluctuates in and out of existence, thereby destabilizing the superposition of metrics and leading it to collapse to the preferred eigenstate, namely the FS metric. The upshot is that we get a Big Crunch in which the geodesic distance between any two points shrinks to zero, thereby setting the stage for a new Big Bang of the sort already described; and this pattern is capable of being repeated indefinitely.

Summarizing the overall argument here, we began with the idea of four-dimensional spacetime as fundamentally discrete, motivated both by this idea's promise in connection with the explanation of dark energy, and by its role in understanding of black hole entropy. Nonetheless, in order to deal with the objection that the resulting account of dark energy is too inhomogeneous to satisfy observational constraints, we were led to the idea that the microstructure of spacetime fluctuates in such a way that spacetime continually gets close to an analog structure; these fluctuations of the microstructure entail the existence of dark energy, in the form of a nonlocal quantum potential of spacetime, that is suitably homogeneous. Furthermore, in considering the problem of the unusual initial conditions of the universe, we were led to the idea of spacetime, in its initial state, being represented by a manifold having a continuous FS metric, a metric such that the geodesic distance between any two points is zero. By taking this metric to be present not only at the universe's beginning, but also at later times (where it is an eigenstate of a superposition of metrics), we can motivate the idea of a cosmological preferred frame; such a frame makes possible the nonlocal interactions that are crucial to the microstructural fluctuations of spacetime mentioned above, and hence it is essential to the present account of dark energy. In conclusion, then, spacetime – and, *ipso facto*, reality itself – has a fundamentally dual nature marked by (i) a superposition of discrete and continuous metrics, and hence of the digital and the analog, and (ii) microstructural fluctuations between digital states and near-analog states. In addition, the "analog component" of the above superposition manifests as a continuum viscous stress tensor, which adds a further analog aspect to reality.

REFERENCES

[1] R.D. Sorkin, “The statistical mechanics of black hole thermodynamics,” in R.M. Wald, ed., *Black holes and relativistic stars* (University of Chicago Press, 1998) [arXiv:gr-qc/9705006]
[2] S. Zohren, “A causal perspective on random geometry” Ph.D. thesis (Imperial College London, 2008) [arXiv:0905.0213]
[3] M. Ahmed, S. Dodelson, P.B. Greene, and R. Sorkin, “Everpresent Lambda” Phys. Rev. D69:103523 (2004) [arXiv:astro-ph/0209274];
R. Katti, J. Samuel, and S. Sinha, “The universe in a soap film” Class. Quant. Grav. 26:135018 (2009) [arXiv:0901.0978]
[4] J.D. Barrow, “A strong constraint on ever-present Lambda” Phys. Rev. D75:067301 (2007) [arXiv:gr-qc/0612128];
J. Zuntz, “The CMB in a causal set universe” Phys. Rev. D77:043002 (2008) [arXiv:0711.2904]
[5] F. Darabi, “Quantum gravity: a solution for current problems in cosmology and particle physics” [arXiv:0809.1282]
[6] W. Mittelman, “Ever-present Lambda and the quantum potential of spacetime” Prespacetime Journal v.1 no.7 (2010) [viXra:1010.0028]
[7] D. Bohm and B.J. Hiley, *The undivided universe: an ontological interpretation of quantum theory* (Routledge, New York, 1993)
[8] C. Barcelo, S. Liberati, and M. Visser, “Analog gravity from Bose-Einstein condensates” Class. Quant. Grav. 18:1137 (2001) [arXiv:gr-qc/0011026]
[9] B.L. Hu, “Can spacetime be a condensate?” Int. J. Theor. Phys. 44:1785-1806 (2005) [arXiv:gr-qc/0503067];
B.L. Hu, “Emergent/quantum gravity: macro/micro structures of spacetime” J. Phys. Conf. Ser. 174:012015 (2009) [arXiv:0903.0878]
[10] J. Henson, “The causal set approach to quantum gravity” [arXiv:gr-qc/0601121]
[11] W.A. Rodrigues, Jr. and M. Sharif, “Equivalence Principle and the Principle of Local Lorentz Invariance” Found. Phys. 31:1785-1806 (2001) & 32:811-812 (2002) [arXiv:math-ph/0302009]
[12] L. Modesto and P. Nicolini, “The microstructure of a quantum universe” [arXiv:0912.0220]
[13] M. Ahmed and D. Rideout, “Indications of de Sitter spacetime from classical sequential growth dynamics of causal sets” Phys. Rev. D81:083528 (2010) [arXiv:0909.4771]
[14] V. Jejjala, M. Kavic, D. Minic, and C.-H. Tze, “On the origin of time and the universe” Int. J. Mod. Phys. A25:2515-2523 (2010) [arXiv:0804.3598]
[15] P.W. Michor and D. Mumford, “Vanishing geodesic distance on spaces of submanifolds and diffeomorphisms” Documenta Math. 10:217-245 (2005) [arXiv:math/0409303]
[16] N. Afshordi, “Reviving gravity’s aether in Einstein’s universe” [arXiv:1004.2901]
[17] A. Anderson, “Generalized Einstein theory with fundamental cosmological stress tensor” [arXiv:gr-qc/9902027]
[18] M.V. Gorbatenko, “On the possibility to explain ‘The Pioneer Anomaly’ within the framework of conformal geometrodynamics” [arXiv:gr-qc/0509017];
M.V. Gorbatenko, “On physical meaning of Weyl vector” [arXiv:1002.2748]
[19] X.-M. Kuang and Y. Ling, “Viscous gravitational aether and the cosmological constant problem” JCAP 0910:024 (2009) [arXiv:0907.3180]