A mathematical multiverse without postulates

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Abstract

I show that the notion of a physical world is problematic. The known laws of physics should be taken to give an effective description of a mathematical multiverse. In this setting there is no room for any postulates.

1 Introduction

Physics as it is conventionally practiced is inherently phenomenological in nature. One postulates a certain number of fundamental concepts and a set of rules that relate experimental outcomes to these concepts. A theory is then a useful theory of physics if it is falsifiable. If there exists a hypothetical set of experimental outcomes that would be logically inconsistent with a theory, then the correctness of that theory implies some constraints on the possible experimental outcomes. Proceeding in this way, we have been very successful in describing Nature. Considering the phenomenological nature of physics, one should question if the postulates that define the theories really define the physical world in some approximation.

The whole concept of a physical world is actually rather problematic. We take for granted that it exists and assume that the postulates refer to it, but it's not clear what it is supposed to be. One can take the view that this is just an irrelevant philosophical matter that can be ignored. But this isn't actually the case; the ill defined concept of a physical world does come with its own baggage and should therefore be questioned. In cosmology, assuming that there exists a physical universe leads to questions that may not have any answers. A universe that as the Big Bang theory suggest did not always exist, motivates the question: Where did the universe come from? Obviously, if the universe doesn't exist physically, this question doesn't make sense. The assumption of a physical universe can thus lead to the wrong questions being asked and lead to wrongly motivated theories that answer those questions. If there are no experimental or observational results, these theories may lead physics to go off the rails and stray into the religious domain.

One obvious objection to dropping the assumption of a physical world is that one can simply define the world we observe as being "physical". I don't object to doing that, but the word "physical" must then not carry any hidden baggage. Since the word "physical" is then meaningless, it would be better to not use it, just to be sure it indeed doesn't bring along with it any hidden baggage. A more serious objection is to ask if there is no physical world, how we can exist. As some authors have pointed out, mathematical existence may be the only form of existence [1]. It is then tempting to identify the known laws of physics as an element of a mathematical multiverse, as e.g. Tegmark has done [12]. However, this actually doesn't lead to anything new. One has to assume a measure over the set of all possible mathematical models, anthropic reasoning and observations can then be used to compute probabilities of being in some universe. But this isn't fundamentally different from how cosmologists actually attempt to explain the observable universe we find ourselves in.

What I will argue for in this essay is something radically differently. Like Tegmark, I will argue for a mathematical multiverse, but each element of this multiverse is an observer, not some universe. By doing that I make sure that no physical baggage gets smuggled in via the backdoor. Clearly observers exist and mathematics exists. Then if we don't want to assume anything else, all we can do is identify observers with the mathematical models that describe them. The world we observe, which appears to be described by laws of physics, is then assumed to be fictitious. While these laws, by virtue of defining a set of mathematical laws, are also members of the mathematical multiverse, this should not be taken as describing a universe within we exist. The laws of physics are nothing more than an effective description of our local neighborhood in the mathematical multiverse. So, unlike in Tegmark's picture where the mathematical multiverse is the set of all possible laws of physics, here the laws of physics are meta-laws that describe the mathematical multiverse. These laws should in principle follow from mathematics alone, so there isn't any room for postulates at the fundamental level.

My arguments in the following sections are heavily based on my article "Entangled states considered as physical representations of classical algorithms" [2] from which I draw freely.

2 Observers

The reason why we tend to think that we live in a physical world, is because we experience it this way. We can touch and feel things; intuitively this is proof of the existence of a physical world. Yet, precisely these experiences are usually regarded to be beyond what physics can explain. Even though the relevant physics that describes the fundamental brain processes is well understood, how the brain gives rise to a conscious person, is seen to be mysterious. The most reasonable position one can take i.m.o. is to accept the computationalist theory of the mind. Here conscious experiences are identified with computational states of algorithms [3, 4]. While some authors have argued that quantum computing plays an essential important role in some brain processes [5], this has been proven wrong by Tegmark [6] who showed that the time scale for elementary information processing events in the brain to unfold is many orders of magnitude larger than the time it takes for superpositions of such processes to decohere. This means that whatever human consciousness is, it has to be identified with the running of certain *classical* algorithms.

There are problems, however, when one contemplates an algorithm (running in a classical deterministic setting) representing a conscious human being, some of which are discussed in [4]. I'll focus here on what I think is the most serious objection, which is based on the existence of mappings of the states of a machine that runs an algorithm to the states of trivial systems [7].

Consider a simulation of a person in a virtual environment such that the whole evolution of the system is deterministic. A simulation over some finite time interval will simply cause the system to evolve through some finite number of predetermined states. This means that this system behaves just like any trivial system that also evolves through at least the same number of different states, say a clock with a number of independent dials that run at different rates. Why then would such a trivial system not generate the same subjective experiences? Clearly the time evolutions of the two systems can be mapped onto each other. Nevertheless, the two systems are not the same, but to see this one has to consider a range of different initial conditions. Only then can one see that the computer is actually performing non-trivial computations, while the clock is just running in the same trivial way [8]. No one to one mapping between the states of the systems is possible when one considers counterfactual initial conditions.

While it is a compelling argument that counterfactuals must be considered to see if a computation is really being implemented by a system, one still has to deal with the fact that counterfactuals are events that, by definition, did not happen. To see that this is not just a philosophical problem with no relevance to physics, we can reformulate it into one where the same paradox manifests itself as ambiguities in probabilities of experimental outcomes [9]. Suppose we run three simultaneous simulations of a person, one in a virtual room with white walls and two simulations in a virtual room with black walls. The latter two simulations are exactly identical; the processors will behave in exactly the same way. The probability that the person finds himself in the black room would seem to be 2/3. However, since the two processors rendering the person in the black room are performing identical operations, we could imagine replacing one processor by a "dummy processor" that simply copies whatever the other one is doing. Since no real computations are performed by the dummy processor, we should expect that the probabilities are now 1/2. In terms of counterfactuals, we can say that when both processors are working independently, one can choose independent input for both systems. But then a similar objection can be made as in the previous case: In a situation where both processors are going to perform identical computations, why does it matter if both processors can function correctly when fed different input?

The fundamental problem these paradoxes point to is that in classical deterministic models, the information about the evolution laws of the system is not present in the physical states of the system itself. While this poses no problems when doing computations in terms of the physical states, it becomes a problem when we imagine that an observer is present in a universe described by such deterministic laws and we attempt to compute the probabilities of certain observations. However, we've only seen that this is a problem for a hypothetical world that is exactly described by a classical deterministic model. While the real world at the macro-level is effectively classical in the sense that probabilities behave in a classical way, i.e. without exhibiting interference or violations of Bell's inequalities, the reason why the above paradoxes arise has nothing to do with classicality in this sense. It thus makes sense to re-examine these paradoxes in a real world setting.

3 Classical algorithms run on real machines

The world we live in is described by quantum mechanics. This means that even macroscopic machines running classical algorithms are described by quantum mechanics. The classical behavior of such a machine is explained by decoherence: On a time scale that is much smaller than the time needed to perform an elementary computational step, any superposition of machine states would have decohered; in fact such superpositions won't arise due to fast decoherence in the first place. As mentioned in the previous section, we note that Tegmark has shown that this picture is adequate for the human brain [6].

The computational state a classical machine is in, can be specified by a bit string $\mathbf{b}$. The quantum states corresponding to these bit strings, which we'll denote as $|\mathbf{b}\rangle$, form a subset of the pointer basis. Normalized states of the universe containing the machine can be denoted as

$$|\psi\rangle = \sum_{\mathbf{b}} |\mathbf{b}\rangle \otimes |E(\mathbf{b})\rangle \tag{1}$$

where $|E(\mathbf{b})\rangle$ denotes the unnormalized states of the environment which we define as all the degrees of freedom that are not described by the bit string. We assume that all the environmental states $|E(\mathbf{b})\rangle$ describe the machine running the same algorithm reliably; under the time evolution according to the Schrödinger equation, the bit string changes after some time step according to the algorithm that the machine is supposed to be running.

The states $|E(\mathbf{b})\rangle$ are orthogonal and thus contain perfect information about the computational state. In fact, the whole computational history of the machine will be contained in the states $|E(\mathbf{b})\rangle$. In the Copenhagen and Consistent Histories interpretations, one assumes that only one term of (1) refers to the real world. Here we won't make any such assumptions, and work within the Many Worlds Interpretation (MWI) [10].

The state (1) seems to have a straightforward interpretation: According to the Born rule, the probability for the observer generated by the machine to find herself in the computational state $\mathbf{b}$ should be $|\langle E(\mathbf{b})| E(\mathbf{b})\rangle|^2$. However, we would then assume that an observer with some definite experience is described by a single term of (1). The only assumption about consciousness we'll make in this article is that the opposite is true: A definite conscious experience is consistent with an astronomically large number of computational states. We can be confident that this must be the case for at least us. A huge amount of brain processes are going on at any given moment. Many of these processes implement pattern recognition algorithms which leads to awareness of the patterns at the expense of awareness of the many parts that make up the patterns [11]. Since the patterns will have a significant entropy in terms of the computational states, this leads to the conclusion that any particular state of consciousness should be consistent with a large number of computational states.

We can now see how this solves the paradox on counterfactuals discussed in the previous section. An observer with a definite experience is represented by an entangled state of the form (1), where the summation is over many computational states. Since the environmental states contain the complete information about the computational history, we can read such an entangled state as a table that specifies the output of the algorithm as a function of the input. The state of the observer thus (partially) specifies the algorithm that generates the observer. Note that interpreting the entangled state in this way is not an ad hoc choice. Compare this to writing down a table on paper specifying each output corresponding to each input. This does not single out the correct interpretation of whatever is written as the algorithm. We may write in the caption how to interpret the table, but whatever is written there is not a law of physics and can thus be ignored.

In contrast, the entangled state (1) automatically implies that the components of the state are correlated. This correlation does not define the algorithm fully, as the summation over the computational states in the entangled state is only over a restricted set of states that are consistent with whatever the observer is aware of. It is then natural to assume that "awareness" *is* such a correlation. Quantum mechanics allows for such correlations to exist in the form of entangled states at any particular moment, while within classical mechanics there is no room for this.

One obvious objection against this picture can be raised: Since the computational states are macroscopic, one can imagine a machine looking at itself and simply observing its own parts. So, why won't the machine be able to observe its own computational state? To answer this, we note that the information the machine has about the external world makes up part of the computational state, therefore the machine cannot have enough memory to store the exact computational state it is in. Moreover, as explained above, it is reasonable to assume that whatever we are aware of, are patterns in the raw information present in the brain, so the number of different states of conscious experiences should be far less than the number of states the memory can encode. We can still imagine a "super-observer" with a memory capacity that is vastly larger than that of the machine that can observe the precise computational state of the machine, but there is then no way that the observation can be communicated to the machine.

4 The mathematical multiverse

The main conclusion we've reached is that classical mechanics is does not correctly describe the state of a classical observer, observing the effectively classical world he is in, while quantum mechanics has no trouble doing so. This is due to classical mechanics throwing away too much information; just because certain effects can no longer be observed, does not mean that a model in which they are always exactly zero is adequate. Compare this to general relativity versus Newtonian gravity. If the global structure of space-time is described by a Robertson-Walker metric with a nonzero curvature, then it can be the case that at all points, locally, the deviation from Newtonian gravity are negligible. But obviously, a global model based on special relativity would be not even be approximately correct. Similarly, what I've shown is that while the typical quantum effects are vanishingly small when describing brain processes or the running of a classical computer, the way a classical computation is physically represented is as an entangled quantum state. Any attempts to describe this using only classical language leads to paradoxes.

The Hamiltonian that determines the dynamics plays a redundant role in this picture. Since we are always localized in time at some given moment, the information we have about the dynamics must be contained in the physical state of the universe at any given time. This suggests that the dynamics may not play the fundamental role in physics as one commonly assumes. The conventional picture may actually not work; it has been hard to find a solution to the so-called Boltzmann brain problem: Contrary to our observations, typical observers in the universe are predicted to arise randomly due to fluctuations and will thus have awareness of false information. But if we simply ignore this problem and focus on a sector containing a normal observer and apply the time evolution operator to predict the probabilities of outcomes of experiments, then no problems occur.

It thus makes sense to throw away the redundancy to prevent it from making trouble. We can do this by postulating the existence of a mathematical multiverse, similar to a proposal by Tegmark [12]. The difference is that what we postulate is simply a set of all possible mathematical statements defining functions or algorithms. Unlike Tegmark, we don't take each element of the ensemble to be a model of a universe in the conventional sense, i.e. one which evolves according to some laws. Such laws contain redundant information that we don't want. The elements of the ensemble can be formally represented by quantum states, as explained above. For a given observer in some conscious state, there will be many elements that can represent her. One can e.g. append extra qubits to the element, without affecting the interpretation of the algorithm defining the observer.

5 Conclusion

Observers play a central role in physics, yet physics has not been successful in describing observers adequately. Assuming that observers are algorithms implemented on some machine, I have argued that within a classical physics setting, one cannot get a satisfactory description of observers, while within quantum mechanics no problems occur. This then leads to a picture of quantum mechanics describing a mathematical multiverse. If indeed there only exists a mathematical multiverse, the known laws of physics should follow from pure mathematics without any assumptions. While it is not clear how to do this, it is also not clear that this isn't possible.

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