The Top-Down Complexity

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The rising complexity of our terrestrial surrounding is an empirical fact. Details of this process evaded description in terms of physics for long time attracting attention and creating myriad of ideas including non-scientific ones. In this essay we explain the phenomenon of the growth of complexity by combining our up to date understanding of cosmology, non-equilibrium physics and thermodynamics. We argue that the observed increase of complexity is causal in nature, stands in agreement with the second law of thermodynamics and has it's origin in the cosmological expansion. Moreover, we highlight the connection between the leader of complexity growth in localized areas of space with free energy rate density, starting from the largest scales towards the smaller ones. Finally, in the light of recent advances in non-equilibrium statistical mechanics, our belief in the causal structure of modern scientific theories is transferred to biological systems. On relevant scales, adaptation and complexity growth follows a similar pattern, in which free energy rate density is provided by an external electromagnetic radiation. The presented, holistic approach, arms us with predicting power about variety of attributes of complex systems and leads to a chain of successful explanations on all scales of the Universe.

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The complexity of life on Earth can appear perplexing to our scientific apparatus. Simple microscopic laws that performed amazingly well in cases of small inanimate systems were in many cases less successful in the description of biological forms, let alone the ones with information processing capabilities (agency). This, however, does not mean that we need to discard this approach. Rather, we realize that emergence involves *history* and that we cannot understand microcosmos without comprehending the cosmos. Therefore, let us begin our discussion towards the origin of complexity rise on Earth from the largest scales perceived by the human mind.

The Universe (See Fig. 1), which we understand as basically everything that exist; is forming the largest, perfectly isolated system with no environment.

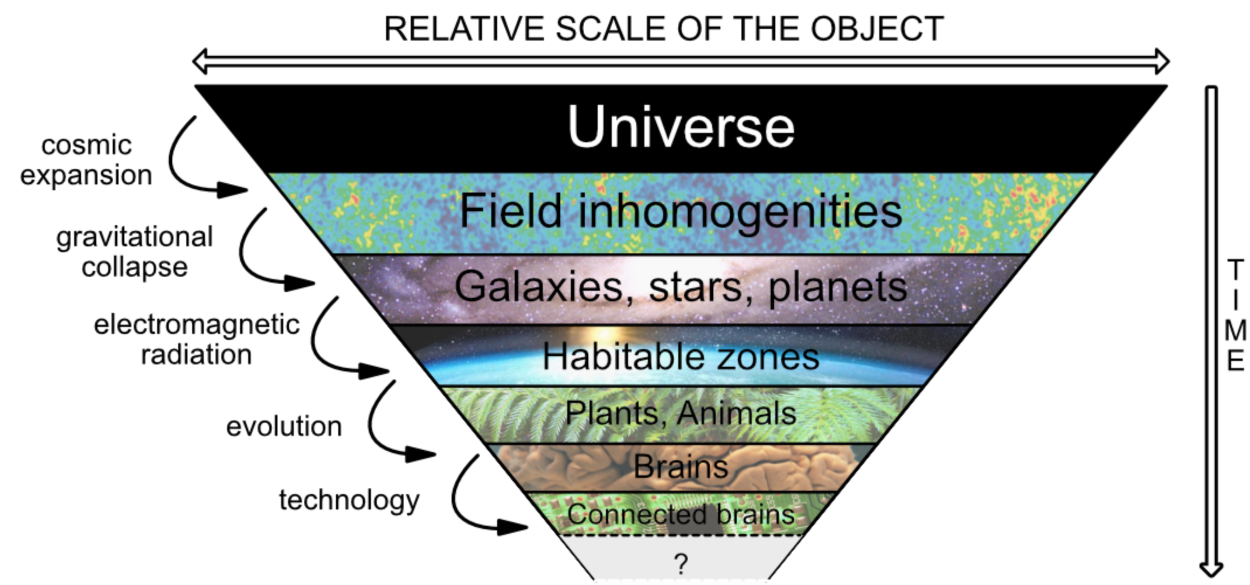

Figure 1. Top-Down trend of the complexity growth.

Therefore, if quantum mechanics applies, the Universe as a whole is described by a pure state $|\Psi\rangle$ satisfying the timeless Wheeler-DeWitt equation $\hat{H}|\Psi\rangle = 0$, such that there is no evolution of the state with respect to some external time parameter[1]. However, *internal times* are possible to introduce, which encode relative evolution between

[1] This is just a reflection of the fact that there is no environment with respect to which the evolution of the system could be measured.

subsystems of the Universe.

Since the state $|\Psi\rangle$ is pure it contains all information available in the universe. Consequently, for such a state with density matrix is $\rho = |\Psi\rangle\langle\Psi|$ the von Neumann entropy $S = -\text{tr}\,(\rho \ln \rho)$ remains constant. In the information theoretical picture, the von Neumann entropy can be considered a measure of our lack of information about the system. Therefore, for a fictitious observer, with an access to the whole state $|\Psi\rangle$ the total amount of information in the Universe might be considered a conserved quantity.

However, in general we deal with situations in which the observer can collect information only about a limited number of degrees of freedom. Such subset of the Universe we call the *system*. The rest we call the *environment*.

In the cosmological context, the Universe can be decomposed into background (the system) and inhomogeneities (the environment). As the studies for perturbative inhomogeneities suggest, in such a case the value of entanglement entropy and the value of the scale factor a are positively correlated [1, 2]. Consequently, in the present phase of the Universe expansion the entanglement entropy S is increasing, satisfying the second law of thermodynamics. This, however, does not mean that the Universe will become homogeneous in the whole space, on the contrary islands of complexity are indeed possible as will shortly become clear.

At the end of the radiation epoch the inhomogeneity of the Universe was at the level $\Delta T/T \sim 10^{-5}$ and baryonic matter and radiation remained in the thermal equilibrium forming a primordial plasma (see Fig. 2a). Then, as a consequence of the temperature drop, the recombination process (formation of neutral matter) occurred leading to departure of the system from the thermal equilibrium. This process can be perceived as a decay of the system into two equilibrium states each characterized by a different temperature (thermal decoupling). For radiation, temperature gradually decreased following the $T_r \sim 1/a$ trend, which was valid also for the relativistic plasma before the recombination.

The cosmic expansion enhanced inhomogeneities of the matter density, which ultimately became unstable initiating the process of gravitational collapse. This process lead to formation of dense inhomogeneous structures (stars) in which gravitational potential energy was transformed into the kinetic energy, increasing their temperatures. Quite often the details of this process (concerning both the formation of stars and planets) are neglected and lead to confusion whether matter localized as stars or planets represents a state of higher entropy. Therefore, let us articulate this clearly - in case of non-interacting particles the state of maximum entropy is a homogenous distribution. However, in case of gravitationally interacting particles the entropic "price" for localizing particles in one place is "paid" by the heat emitted to the environment, such that the total entropy in this process also rises. For stars the accumulated energy due to this process leads to ignition of the nuclear reactions increasing further the temperature of stars.

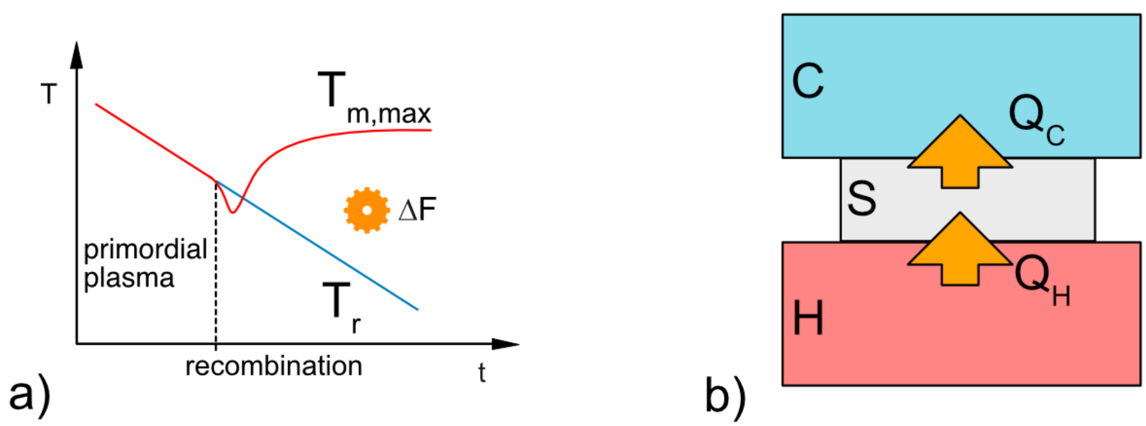

Figure 2. a) Thermal decoupling of radiation and matter. b) Schema of a heat engine consisting of the system (S) and its environment in thermal equilibrium (Heater H, Cooler C).

The maximal temperature of localized matter systems (stars) $T_{m,max}$ eventually became much higher[2] than the temperature of the background radiation T_r filling the Universe (See Fig. 2a).

This difference of temperatures between the hot spots (stars) and cold surrounding allows for work or the free energy ΔF (depicted as cogs in the illustrations) to be extracted

[2] Nevertheless, the averaged matter temperature remained below the temperature of radiation.

as it happens in case of a heat engine (See Fig. 2b). The stars are the heat reservoirs here while the cold space plays the role of the cooler. Objects existing close to the stars can obviously benefit by absorbing the incoming radiation and transforming it into usable work. These objects are called planets.

The planets or rather planetary atmospheres are open systems resembling heat engines powered by solar radiation. Simple calculation yields that incoming solar photon may give rise to about 20 outgoing terrestrial infrared photons. Each photon has approximately the same entropy of order - one bit, so the entropy carried by the photons increases 20-fold which is more than enough to allow some decrease of the entropy of the life forms. The total entropy will go up even if the organisms manage to reduce their disorder. This of course only settles the consistence of the second law of thermodynamics, but does little to answer the question of why we observe the emergence of complex structures. This is why we will now perform an analysis of a simple non-equilibrium model that resembles the systems described in previous paragraphs and demonstrate that a system within that model lowers it's entropy.

Consider a system S, stacked in between a heater H and a cooler C, much bigger than the system and with constant temperatures T_H and T_C (See Fig. 2b) such that a steady amount of heat flows from the heater through the system to the cooler.

Following Prigogine's approach [3] we will analyze it from the perspective of internal entropy produced (i) and external entropy transferred (e) *to* the system of interest S. Of course the entropy change of the system dS_S would be the sum of those two contributions:

$$\frac{dS_S}{dt} = \frac{dS_i}{dt} + \frac{dS_e}{dt}.$$

As we have mentioned, in our analysis we limit ourselves to the scenario of a *stable state* in which the same amount of heat that goes in also goes out. In other words $dQ_C = -dQ_H$, using which we get the following equation

$$dS_e = \frac{dQ_H}{T_H} + \frac{dQ_C}{T_C} = dQ_H \left(\frac{1}{T_H} - \frac{1}{T_C} \right) = dQ_H \left(\frac{T_C - T_H}{T_H T_C} \right) < 0.$$

Therefore, the heat flux transfers some of the entropy outside the system (or negative entropy in). Now, since we require the state to be stable and the system's entropy can't grow ad infinitum we have $dS_S = 0$ and thus the external negative entropy flow balances the internal entropy production $dS_i = -dS_e > 0$ (In some sense we could say that the rate of internal entropy production dS_i is a function of the entropy inflow dS_e).

Our system is peculiar in one simple manner, namely, thanks to the constant heat flow, the system entropy decreases[3]! Non-equilibrium conditions indeed let us escape the tyranny of the entropy.

When we turn on the driving force (here temperature difference) the entropy smoothly lowers to minimum entropy S_{min}. Moreover, this model does not store energy, hence internal energy stays constant, which means that the free energy, defined by $F = U - T\ S$ gets maximized. Free energy allows us to perform work which can be realized in different ways depending on the nature of the system. It can mean the formation of stars, emergence of complex behaviour in the atmosphere or circular convective motion in the Benard cell.

In fact, Chaisson hypothesized [4] that for an object of mass m the *free energy rate density* $\frac{1}{m}\frac{\Delta F}{\Delta t}$ is a universal indicator of complexity of the system and that cosmic evolution is correlated with the growth of localized free energy rate density. Indeed, this trend is visible from our top-bottom approach in which cosmic expansion leads to thermal decoupling (formation of a heat engine). Then, going to smaller and smaller scales this free energy is lent and localized in the subsystems resembling a bifurcation phenomena (See Fig. 3a). First the free energy is passed from the solar systems to planetary atmospheres and then from planetary atmospheres to the biospheres on human scales. This trend does not seem to stop here and the free energy continues to get localized in human societies.

At first it may appear counterintuitive, but the free energy rate density is indeed smaller for stars than it is for planetary atmospheres, and it is smaller for planetary atmospheres than it is for animals. The XXI century inventions, like engines and computers also have their place in it and might be considered a part of our modernised

[3] To see this, Taylor expand $\frac{dS_i}{dt} = j_i(S_S) = j_i(S_0) + (S_S - S_0)\, C_1 + \mathcal{O}\left(S_S^2\right)$ and solve differential equation $\frac{dS_S}{dt} = j_e + j_i(S_S) = j_e + \frac{S_0 - S_S}{\tau}$, getting $S_S(t) = S_0 + j_e \tau \left(1 - e^{-t/\tau}\right)$, where j_e is a negative constant. In the $t \to \infty$ limit the system's entropy falls from the initial value $S(t=0) = S_0$ to the minimal value $S_{min} = S(t \to \infty) = S_0 + j_e \tau < S_0$.

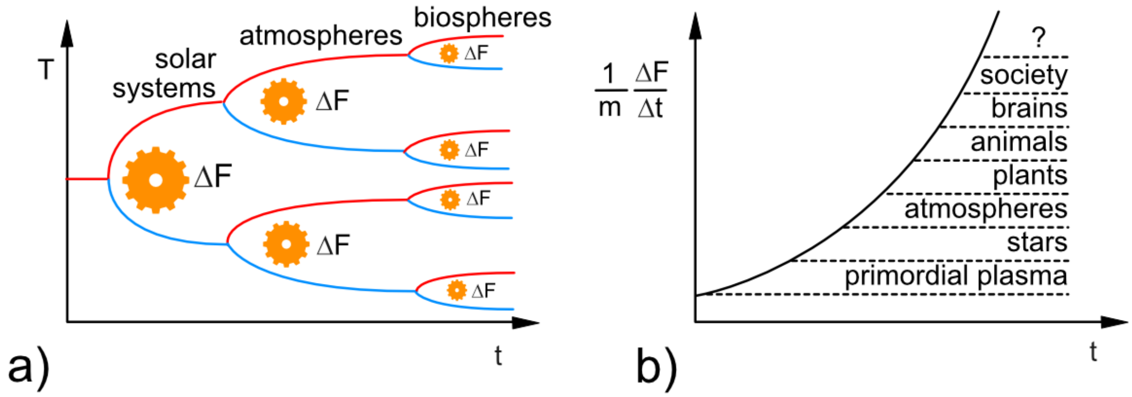

Figure 3. a) Emergence of heat engines powering complexity growth as a result of departure from equilibrium. b) Timeline of free energy rate density growth adopter from Ref. [4].

society, amounting to our complexity. Human brains take the top places in this ladder of complexity and continuously climb it; through augmentation with technology.

We did not prove the association made by Chaisson by any rigorous way. At the end some further improvements might be possible, but the trend is clearly visible, almost as in Darwinian evolution. The inflows of free energy localize on smaller and smaller scales in tandem with growth of complexity.

On the other hand, one shall not be tempted to try to find the explanation in terms of teleology, a way which has a certain appeal for human beings. A great example can be given by trying to explain the convection currents in Benard cell. The sentence "Benard cell activates another mean of transport to efficiently transport heat from the heater to the cooler" implies an idea proposed by the Maximum Entropy Production (MEP) principle which validity is under recent theoretical investigations [5]. Precisely because the MEP principle is currently teleological in nature it does not get a very wide approval among scientific community. Laymen might think that it's some close-minded thinking of the scientific community, but this insistence on causal explanations is one of main reasons of triumph and an enormous success of science in explaining natural phenomena. Advancement in our understanding of say, gravity, was made by the retreat from teleological explanations. For example the ideas of ancient greeks who believed that the cause of the downward motion of heavy bodies on Earth was related to their nature, which caused them to move downward, toward the center of the Universe, which was their natural place. Conversely, light bodies such as the element fire, moved by their nature upward toward the inner surface of the sphere of the Moon [6, 7].

Nowadays we recognize the naivety of those explanations and the superiority of Newtonian gravity. Even though the benefits of adopting Newtonian ideas were enormous as they allowed us to predict various events, they still left some room for improvements (force had instantaneous effect on distant bodies) as Newton himself noted [8]. This and more weaknesses were brilliantly noticed by Einstein, whose theory not only opened new areas of inquiry (black holes and studies of the beginning of the Universe) and made gravitational predictions more precise (allowing us to develop GPS systems etc.), but also strengthened the *causal* nature of gravity.

It's not as to say that teleology can not be a vestibule to physics. If the idea is further developed, a teleological explanation may be untangled to "theory of principle" or a "constructive theory", classification proposed by Einstein in 1919 [9]. With enough work, MEP principle (in one form or another) might become a part of a valid scientific framework leading us to a correct and complete description of non-equilibrium phenomena.

So far we have learned that teleological explanations have little place in physics, but what might be less obvious is that it is also the case for evolutionary biology. To see this, let's try to discriminate the content of various statements. The everyday statement, we usually consider true "birds have wings *to* fly" has little scientific relevance. A slightly modified version of this statement "birds evolved wings to fly" is much better, because (even though it contains the grammatical particle "to") it underlines it's causal ingredient, namely evolution. Natural selection is all about discovering the evolutionary pressures that led to formation of complex features; there is no goal in this process. Of course we talk about "survival of the fittest", but this is just a poetic way of the description and the ultimate end state (goal) is not known. This feature of biology convinces us that it can, and will be ultimately described by physics and chemistry.

To make the case stronger, just recently J. England [10] proposed a physical process by which evolution can take place at the microscopic scale. In fact his approach is another facade of the non-equilibrium theory, taste of which we experienced in the previous paragraphs. In his paper, J. England employs a peculiar family of laws known as Fluctuation Theorems. In contrast to known laws of Statistical Mechanics those laws take into account the history of the process and discriminate between the more likely histories according to the entropy transferred to the environment. The more entropy (or in other words heat) is produced the more likely a microscopic path ($\Rightarrow$) is relative to it's reverse ($\Leftarrow$):

$$\frac{\Rightarrow}{\Leftarrow} = \exp \Delta S_{env}.$$

Once again, the end states are determined by microscopic evolution, the end goal is not set. Moreover, this last equation can be easily expanded to macrostates which give us a generalized form of second law of thermodynamics

$$\Delta S_{tot} + \ln(\frac{\Leftarrow}{\Rightarrow}) > 0,$$

(note the change of direction of the arrows) where ΔS_{tot} is the sum of entropy change of the environment and the system under consideration.

What this tells us is that the more irreversible a transition is, the less entropy has to be produced *inside* the system (in a similar way to our previous considerations). However, the form of this equation makes the link between irreversibility and entropy much more evident and fruitful, thus we will mention here a few results to which it leads.

If we apply it to a system under periodic driving force, then it was shown [11] how the system can transform or remodel to a form in which it dissipates energy more efficiently. This result gave a pioneer example of evolution taking place at the microscale. On the other hand applying this equation to self-replicators [12] an inverse relation between the maximal growth rate of a self-replicator and it's durability was found. This implies that there must be a biological tradeoff between the pace of reproduction and durability of self-replicators. Those advancements push us towards understanding the origin of biological complexity and it is likely that the next step, with the help of information theory, will unravel the emergence of agency. In fact, purpose directed behaviour could be seen as a result of information processing and storage which in itself relies on availability of free energy as shown by Landauer [13]. This accumulation of knowledge, being synonymous to the *free energy rate density* growth is also expected to follow the double exponent rule [14]. Those and other insights guide as in understanding of complex phenomena and allow us to extrapolate the trends and prognose the future.

To sum up, we have presented a raw mechanism, by which localized complexity (stars, atmosphere, life, technology) emerges from physical laws of Nature. The crucial role was played by the free energy rate density, given by the chain of spontaneously forming heat engines initiated by cosmic expansion. We then argued that teleological reasoning can also be thought as an imperative result of the growing complexity which at about XVI century has been superseded by modern, causal, scientific reasoning in a similar way in which walking on land superseded swimming in sea for our predecessors. Obviously we still use it (as we still enjoy swimming), but it's purpose is different. Finally, thanks to recent advancement in non-equilibrium statistical mechanics, our belief in the causal structure and the illuminating power of modern scientific theories was transferred to the smaller scales governed by biology. There, adaptation took place under the influence of an external periodic force, in case of Earth - solar radiation.

The upcoming years will certainly bring us closer towards understanding the emergence of life and agency, in the context of causal frameworks of physics. Therefore, the insistence on finding purpose in the laws of Nature is illusionary and futile. This, however, should not be perceived negatively or discourage us, as we believe that through ingenuity and the pursuit of truth we will find ourselves a purpose.

"The significance of our lives and our fragile planet is then determined by our own wisdom and courage. We are the custodians of life's meaning. ...If we crave some cosmic purpose, then let us find ourselves a worthy goal." — Carl Sagan, 1997 [15]

[1] H. D. Zeh, *The physical basis of the direction of time*. Springer, Aug. 2007.

[2] C. Kiefer, "Does Time Exist in Quantum Gravity?," in *Towards a Theory of Spacetime Theories*, pp. 287–295, New York, NY: Springer New York, 2017.

[3] I. Prigogine, "Time, structure and fluctuations," *Nobel Lectures in Chemistry 1971-1980*, 1993.

[4] E. J. Chaisson, "Cosmic evolution: the rise of complexity in nature." Harvard University Press, 2002.

[5] H. Ozawa, "The second law of thermodynamics and the global climate system: A review of the maximum entropy production principle," *Reviews of Geophysics*, vol. 41, no. 4, pp. 1018–24, 2003.

[6] O. Pedersen and M. Pihl, *Early physics and astronomy: a historical introduction*. London: Science History Publications - Neale Watson Academic Publications, 1974.

[7] E. Grant, *The foundations of modern science in the middle ages*. Cambridge University Press, Aug. 2009.

[8] A. Janiak, "Newton and the Reality of Force," *Journal of the History of Philosophy*, 2007.

[9] A. Einstein, *Ideas and opinions*. London: Crown Publishers, 1954.

[10] J. England, "Dissipative adaptation in driven self-assembly," vol. 10, pp. 919–923, Nov. 2015.

[11] N. Perunov, R. A. Marsland, and J. England, "Statistical Physics of Adaptation," *Physical Review X*, vol. 6, no. 2, 2016.

[12] J. England, "Statistical physics of self-replication," *The Journal of Chemical Physics*, vol. 139, no. 12, p. 121923, 2013.

[13] R. Landauer, "Irreversibility and heat generation in the computing process," *IBM journal of research and development*, vol. 5, no. 3, pp. 183–191, 1961.

[14] R. Kurzweil, *The Singularity Is Near*. The Penguin Group, Feb. 2007.

[15] C. Sagan, *Pale blue dot: a vision of the human future in space Sagan*. New York: Random House, 1994.