Limits of Mathematical Representation

There are fundamental limits to what physicists can represent by mathematics. It would seem that at least three empirical phenomena cannot be entirely represented by mathematics. These three phenomena are 1. qualia, 2. that aspect of time we call the unique 'now', and 3. existence (the fact of whether or not some particular exists, discussed below). These can be canvassed.

1. qualia
Everyone reading this knows that *it is like something* to have internal, subjective experience. We experience the color green, the smell of cinnamon, or a tactile sensation. The atoms (smallest units) of our experience have been given the name *qualia.*

A mathematical model of qualia has been put forward [1]. What does the definition accomplish and what does it not accomplish? It is a plausible mathematical definition of the geometry of qualia (specific informational relationships) that's embedded in a mathematical model of the geometry of consciousness (integrated information). So if we have the structure of consciousness we have the structure of qualia. But only 'structure'.

The theory, sizable advance that it is, can't tell us what it's like to experience green. If we agree on some geometry for red qualia we could calculate that qualia of a different geometry should be associated with something different. But we can't calculate what it is actually like to experience green (supposing green is correlated to the calculated geometry). So, if we know the geometry associated with blue qualia and we know the geometry associated with yellow qualia we might(?) be able to calculate the geometry of green qualia, but we cannot calculate what it is like to experience green qualia. The theory cannot *predict* what it is like, in its specifics. Nor can calculations tell us there should be anything at all it is like to be conscious in the first place.

There's no hope of doing any better in a strictly mathematical representation of the situation.

2. 'now'
It is empirical that there is a unique moment 'now'. All mathematical representations of 'now' can't pick out a unique moment as the one that actually exists. Therefore, math can't model this fundamental empirical aspect of our experience.

Time in mathematical models: time is often represented by physicists by a parameter t in a set T that is isomorphic to the reals R^1. This happens in the ontological-models framework, [5]. Also in string theory there is a variable *tau* on the worldsheet that plays the same role for time, [2]. But there's already a problem. Every element in T exists (in the mathematical sense) as much as every other element in T. None are genuinely unique. Therefore this definition of time doesn't sufficiently represent the fundamental aspect of our experience of there being a unique present.

Stephen Hawking advocates imaginary (by which I will understand complex) time t in T such that T is isomorphic to the complex numbers C^1, [3]. But it doesn't make any difference what manifold T is isomorphic to. (The proof is that the isomorphism is itself a mathematical object.)

Time can also be conceived as a collection D of durations d. But this has the same problems.

Barbour's time capsules suffer from an interesting problem [4]. To model the situation, one must require that *each* capsule finds itself to be the one uniquely existing. But then the theory is wrong. It is

an empirical result that there is only one, unique 'now'.

It does no good to talk about a mathematical variable *t′* that ranges over a timeline (or worldline) parameterized by *t* in T (as in the spotlight theory), because *t′* itself ranges over some manifold, or, at least, ranges over some mathematical object.

Mathematics does not by itself give us the ability to infer 1. which present we are in, and 2. that there is anything like a unique present at all.

3. existence
A mathematical model of a block on an inclined plane, a cup of coffee, or a superstring, doesn't of tell us if the thing modeled actually exists. A mathematical model of the block is the same whether or not a physicist actually has a block on an inclined plane at hand. This can be seen as a limitation of the model.

Attempts to prove something exists using logic fail, because existence proofs can be traced back to an assumption about whether some primitive or first thing exists, such as the null-set or ur-elements, in set theory, or objects in category theory. The existence of a mathematical structure can only be shown relative to other mathematical structures.

Conclusion

Mathematics is the language of pattern. It is patterns that physicists look for in nature, and that's why they use mathematics to describe it. Aspects of qualia, time, and existence are not wholly captured by pattern. Therefore, physicists can't represent these fundamental aspects of our experience completely using only mathematical theories.

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References

[1] Qualia: The Geometry of Integrated Information, Balduzzi and Tononi, ***http://journals.plos.org/ploscompbiol/article?id=10.1371/journal.pcbi.1000462#s2***

[2] Time in Quantum Gravity, Huggett, Vistarini, and Wuthrich, http://philosophyfaculty.ucsd.edu/faculty/wuthrich/pub/HuggettVistariniWuthrich2012_TimeQG_PittArchive.pdf

[3] "When asked what one thing he wished people would understand about his work, Hawking replied: *Imaginary time. People think it's something you have in dreams. Or when you're up against a deadline. But it's a well-defined concept. Imaginary time is like another direction in space. It's the one bit of my work science fiction writers haven't used. Because they don't understand it.*", http://file770.com/?tag=stephen-hawking

[4] Julian Barbour, Timeless physics, http://en.wikipedia.org/wiki/Julian_Barbour#Timeless_physics

[5] Contextuality for preparations, transformations and unsharp measurements, Spekkens,

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