I

The most fundamental question that I want to pose is: what is the relationship between mathematics and reality?

Many of the things which we take to be real in our daily lives are not, in and of themselves, of a mathematical nature.

Nevertheless, it is so obvious to anyone living in today's world as to go without saying that mathematics is profoundly relevant to the nature of the universe that we are living in.

Mathematics can be thought of as a language; mathematics can be thought of as a tool.

Whether the ultimate nature of reality is mathematical is a question that many have asked, but it is not exactly the question that I want to pose here.

For many, probably most, perhaps nearly all physicists, the answer is an obvious "yes".

Some physicists who would answer the question in this way, like Penrose, are avowed Platonists (though I wonder if his position might be more accurately described as Pythagorean; the two are in any event very closely related thinkers). Far more would no doubt wish to avoid associating the metaphysical, and perhaps as they would see it, mystical associations of such a position; but nevertheless share the assumption that the ultimate description of reality to be a mathematical one.

I take mathematics to be real. I would also take many things which are not mathematical to be equally real.

It is a commonplace assumption among scientists that those things in the world of our ordinary experience which are not obviously mathematical in nature - color, for instance, or any of the subjective or qualitative aspects of experience and of the world of our ordinary experience - are ultimately reducible to things which are of a mathematical nature. Conscious awareness itself, in such a perspective, is judged to be epiphenomenal to the sort of reality that is described by physics. The most extreme formulation of such a position is the philosophical stance known as eliminative materialism.

The question that I want to pose is of a slightly different nature; it is whether or not, to what degree or within what limits, everything that is real is mathematizable.

Mathematization is an operation which we perform upon nature.

We begin to mathematize as soon as we begin to count; so the question of the relationship of mathematics to reality begins at this concrete a level.

It goes without saying, therefore, that the universe is pervade by mathematics.

It goes without saying also, however, that there is no such thing in nature as a circle, a line, a square, an ellipse, or a triangle.

A layman or anyone who is thinking of these matters in layman's terms might wish to add preface the words "a perfect" to "square", triangle, etc. However, these words are unnecessary, as geometrical "objects" are by definition "pure" or "perfect".

Geometrical "objects", then, are not like physical "objects", and are not to be found in nature - if we take "nature" to mean the things of physical experience.

Though it is a matter much less frequently contemplated, numbers are not to be found in nature, either.

We are familiar with the symbols (signifiers, in linguistic parlance) "2", "3", etc. But the signifiers are only representations of a signified concept. The signified concepts, however, are not things which we can experience through the senses.

I can directly apprehend two apples, three pears, or five cigarettes; but two, three, and five, are not things which I can apprehend through the senses.

Number itself, for Pythagoras, was a deeply mysterious matter, as were geometrical "objects".

II

Mathematics is a tool which all scientists reach for when they begin to analyze reality.

The nature of the tool itself, however, remains an unasked question.

We know that all of those things which we most properly call things can be counted - although those things which we refer to as qualities, or by other similar terms, cannot obviously be counted.

We know that geometrical "objects" are relevant to the study of nature, because the shapes and the movements of things found in nature often "approximate" the shapes of geometrical objects.

III

When I was an undergraduate, interested in physics, mathematics, and philosophy, I attended a guest lecture sponsored by the physics department, which addressed the question what the ontological meaning (I am not sure if the word itself was used, but it seems appropriate) of quantum mechanics might be.

Discussions of this sort are rather old hat to physicists. Niels Bohr wrote long philosophical treatises on this question; physicists like Fritjof Capra wrote philosophical or wonder books on the same subject for laymen, who still find the question mystifying. But the apparent paradoxes posed by the Heisenberg uncertainty principle and the apparent wave/particle duality are no longer of much interest to many physicists, who have grown quite used to them; and in any event such apparent paradoxes pose no obstacles to the efficacy of their theories.

The answer given by this lecturer was that the ontological question - what do quantum uncertainty and the apparent wave/particle duality actually mean about the nature of matter - should not or need not even be asked. The equations work, and that is what is important.

The lecturer's answer was actually more profound than that; he pointed out that for more a couple of centuries, incorrect extrapolations were made from Newton's theories, and the success thereof. It was assumed in all fields of inquiry and to a great extent in ordinary life that all the world was deterministic in nature, and wild metaphysical extrapolations were even made, such as that the universe was a kind of clockwork that may have been set in motion by God, but that once set in motion, no longer required His intervention (I am not a Theist and do not wish to accidentally imply the contrary of this thesis, but these were the extrapolations that were frequently made during the time in which Newton's theories were predominant or hegemonic). If we make similar extrapolations from the state of today's theory, the lecturer explained, we are just as likely to mistaken as the thinkers from all walks of life in that era of history were at the time. It was better, he suggested, not to make such extrapolations at all.

Well, I was on the track of ultimate reality, so I switched to philosophy.

I found his arguments very cogent.

IV

Physicists today might be said to be in many different camps, or to adhere to many different schools of thought.

The answer which the lecturer gave is probably of greatest satisfaction to experimentalists.

Physicists, after all, are divided (specialized) into experimentalists and theorists. The physics of the present day is so complicated, and expensive, that it is more or less impossible for one person to be both at the same time.

I think that it is fair to characterize this interpretation (or refusal to interpret?) as pragmatist in nature - from a philosophical perspective, or in the commonsense meaning of that term.

It has frequently been observed that few physicists of the present day have the broad philosophical backgrounds of the physicists of an earlier generation, such a Einstein and Bohr, to take but two examples. Few, but not all, of course. Whether this is to the detriment of physics, or not, is anyone's guess or conjecture. All branches of science today require a degree of specialization which would have been unimaginable in previous centuries; scientific knowledge has grown exponentially - literally exponentially, and not merely in the watered down common parlance which that term has acquired!

Physicists can be readily divided in experimentalists and theorists.

But what is theory?

Many physicists wish to avoid metaphysics, and even philosophy, altogether. Even many theoretical physicists wish to do so.

So "theory" in physics does not have anything like the meaning that "theory" has in philosophy.

There is a camp of theoreticians who believe that philosophy is avoidable - that it can or that it should be avoided.

For these physicists, I would take it that "theory" means the attempt to reconcile different mathematical models - the attempt to create a unified theory, or a final theory, or in the most extreme expression which is commonly used, a theory of everything. As theorists, they are doing this by working from already existing theories, while relying on the experimentalist to do the nitty-gritty work in the laboratories.

For a few theoreticians, such a Penrose, "theory" still implies also philosophy, or an engagement with philosophy.

V

We all start from reality.

There are not physicists, I would assume, who do not take themselves, their family members, the societies that the live in, and so on, to be real.

There are a few religious or philosophical traditions which in some instances may describe the world of ordinary experience as illusory; but this is a largely rhetorical statement, which is not to be taken at face value.

That is to say that we all live in the real world.

The journey of physics, however, is motivated by the quest to find ever deeper levels, ever deeper structures or patterns, of reality.

That mathematics pervades the universe that we live in is obvious.

To anyone who thinks about it, it is equally obvious that the realities described by biology, chemistry, and physics, for example - and more or less in that order - are in some ways more "fundamental" realities, underlying the realities of our ordinary experience.

In some ways, though not necessarily in every way.

Reality is where we all, layman and scientist alike, live.

VI

From a physics perspective, these are only prefatory remarks.

From many physicists' perspectives, or from the perspective of most laymen, they are the primary substance of the essay.

VII

Questions for physics:

1) Is a final theory, or a theory of everything, possible? Or, might it be that a final theory is not possible?

2) If we live in a universe in which a final theory is not possible - what might that mean about the nature of that universe?

3) What are the limits, if any, of mathematizability (of, that is to say, the mathematizability of the universe in which we live)?

4) a) Rather than asking, "what is mathematics" -
b) which is itself an unavoidable preliminary question to asking, what are the limits, if any there be, to mathematization;
c) which is itself a necessary preliminary to asking the (practical!) question of, how might any such limits, in concrete terms, be manifested in the actual problems facing physics today -

rather than asking "what is mathematics", I would prefer to ask - what is mathematization?

Lemma to 4): (given that (a) the universe has a mathematical structure; or, (b) that mathematical structures pervade the order of the universe): Might it even conceivably be said that mathematics, or the order of mathematics, preceded the universe itself?

On the presumption that the order of mathematics did not precede the universe - did it come into existence with the coming into existence of the universe?

Mathematics is reached for as a tool, whenever scientists wish to do science. What is the nature of the tool?

Is mathematics nothing more, or nothing more essentially, than an ordering principle of things?

It would be difficult to conceive of the existence of mathematics in a universe in which there were not things (or beings, to speak more generally, in philosophical terms, since one must work within the limits of one's language). It would likewise be difficult to conceive of the existence of mathematics before the coming into existence of the universe - or of all possible universes - to speak of any sort of 'multiverse' is only to set the question off to a higher level of abstraction or theory.

Propositon 1): Mathematical objects are not to be found in nature - taking "nature" to mean physically perceptible beings.

I would take it to be compellingly likely, therefore, that mathematics is an ordering principle, or a (a set of) relationality principles, among things which are - among physical things that exist.

VII

Practical questions for physics may follow which are more or less obvious, if one has followed and contemplated the argument thus far.

Let us begin with the Heisenberg uncertainty principle, the apparent wave/particle duality, and the (problematically, at least for Einstein, probabilistic) Schrodinger equation.

One models particles, real things that actually exist, as mathematical points.

This is problematic even mathematically - even within the terms intrinsic to physics.

Mathematical points do not exist in nature, any more than do circles, lines, spheres, or squares.

Even Democritus knew that particles are very small.

In a way, a point is simply the ultimate mathematical extrapolation of the quality "small".

From another perspective, a point is simply an abstraction of the concept of location.

Location is a matter of relationality among things.

As Mach pointed out, any object existing alone in is perceivable universe would not be able to construct any concept of space or of location.

Is space merely a relationality among things?

In a process of mathematization - in a process of theoretical abstraction - one proposes to a view a particle - a "very small" thing - as a mathematical point.

This way of modeling a particle is directly contrary to all of the facts of ordinary experience, in which every things takes up space, occupies a volume.

Might the (to some physicists, problematic) probabilistic nature of the Schrodinger equation be nature's response to the theoretical idealization of a particle as a point-mass?

Physics, of course, proceeds by processes of abstraction. There are no frictionless surfaces, though every freshman physics student is familiar with such hypothetical non-existent objects as frictionless surfaces; the laws of motion are deduced from them.

To be utterly precise in one's use of language, what one is really saying by resorting to such hypothetical inexistents is - "*if* there were such a thing as a frictionless surface, an object resting on it would be expected to move in such and such a manner; *therefore*, one may suppose that matter *tends* to move in such a way".

Correspondingly, one might conceivably reconstruct the actual language of the idealization of particles as mathematical points somewhat along these lines - though this is only a first attempt at approximation: if location existed in a universe whose ultimate constituent parts were ultimately small, *and* one such constituent part were not constantly in interaction with other constituent parts, that constituent part might be hypothesized to exist as a mathematical point.

When one proceeds from such a theoretical starting point, and attempts to measure the behavior of a single particle in a laboratory, nature responds by saying - *but*, *if* that is so, then such a constituent part has a location and a momentum which are immeasurable to exactly such and such a degree.

Is it measurement itself, or space itself, or location itself - or even mathematization itself - which "breaks down" when one attempts to study single minute constituent part of the universe at a very small scale?

If space (itself, as one conceives it in physics, a mathematization) is only a relationality among things, it stands to reason that space would begin to "break down" as one begins to study a single thing more and more in isolation from other things.

A single thing alone in its perceivable universe could not construct any concept of space.

A single things, being, or person alone in its perceivable universe with a single other thing could perhaps construct a concept of distance - but relying on what as a measure? And it would not have any means by which to construct any understanding of a geometrical objects - which is to say, space as we normally understand it.

Single things do not exist, any more than do frictionless surfaces.

A point-mass, I ultimately mean to say, is a theoretical construct. In response to the positing of such a theoretical construct, nature responds with probabilistic equations.

A point is a theoretical construct because no points, circles, spheres, lines, planes, ellipses, etc., exist in the world of actual experience.

Is the probabilistic nature of such equations, then, itself, a kind of artifact of theory?

VII

These are only a few questions, which may be more or less interesting, posed by a non-physicist to physicists.

It is the general framework which I think may be perhaps be of value. It should be capable of posing generating any number of such questions.

Mathematics may perhaps be an ordering principle among things.

Mathematization is an operation which we perform upon things.

It is possible that mathematization has its limits.

It is the correlation between mathematical constructs (models) and things which is the relevant part of physics.

Thank you.