Software Cosmos

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Introduction

To decide the question "It from Bit or Bit from It?" we describe a digital simulation model. After fleshing out our model using ideas from physics, we will consider whether it is consistent with astronomical observation. We describe, and have carried out, a test for the model. The result will give us an opportunity to reflect on the insights available from ancient philosophies and finally to answer the question with another aphorism.

Software for a Digital Universe

Suppose we are given the task to simulate our universe in software. We face an immediate difficulty: we have as yet no unified "Theory of Everything" that can guide our design. So instead we will start with some general software engineering concepts that apply to large scale simulations, assuming they apply here as well. With a rough idea of the software architecture, we will look for physics that can make it mathematically more precise and bring it closer to modeling reality.

Consider a virtual world simulated by a multi-player online game system. In such a system, there are two forms of information related to the game world. First, the visible pixels on a player's computer screen, along with any sounds conveyed, depict the state of the world from the player's point of observation. Each player interacts with their individual view to play the game. The second form of information is the game state in the central server's computer memory. The algorithms that define the game's logic interact with this game state. This is the shared, but invisible, representation of the game world. The moves that players make are communicated through their computers to the server, so it can maintain their presence and actions in the game world. This client-server architecture offloads what could be a costly rendering process to the many client computers, leaving for the central server only the essential computations that update the shared game state. This is the general software architecture we will elaborate with ideas from physics.

Holonomy

> "The universe does not exist 'out there,' independent of us. We are inescapably involved in bringing about that which appears to be happening. We are not only observers. We are participators. In some strange sense, this is a participatory universe."
>
> — John Wheeler

In the 1970s, physicists David Bohm and Basil Hiley described their Holonomy Paradigm. Here the measurable and observable "explicate orders" are derived from another domain, the "implicate order". Hiley suggests that the implicate order has algebraic structure, with the explicate orders being "shadow manifolds" projected from it. We take these ideas, and these terms, for our digital model: the explicate is experienced by an observer at a specific place and time; the implicate, like the server computer memory, is not directly observable, but provides for the shared dynamics of the world.

The transformation of information from implicate to explicate is the act of observation. You might imagine each observer (or their computer) "creating" their own explicate order or view of the cosmos, piecing together evidence of their own "existence" using a series of "now" snapshots. Each "now" is separately extracted from the implicate order, and it is the shared implicate order that makes the views by different observers consistent. This is John Wheeler's "It from Bit", wherein implicate information is transformed into measurable things.

Does "It" come from "Bit" then? On the one hand, each player's view depends on communications from the server to represent the world around them. But on the other hand, the server depends on communications from the players to represent each one's presence and influence in the game world. In fact, the entire population and information content of the game world may ultimately derive from those communications in aggregate. In this sense, "Bit" comes from "It" in a "participatory universe".

A Language for Digital Physics

Elaborating the generic client-server simulation model requires answers to several questions:

1. How is the state of the cosmos represented?
2. What transformation occurs between explicate spacetime and implicate information?
3. What dynamic within the implicate would correspond to the physical laws we observe in the explicate?
4. How to structure the implicate order to be computationally effective?

We will address these questions by looking for relevant physics, but we need to be aware of a second difficulty. A physics model defined using the mathematics of the infinite will not help us if we have only finite resources available to simulate it. Ideally, the physics we find would be expressed in a mathematical language that can be readily approximated by finite means and so translated into software. Luckily, there is such a language. Geometric Algebra (also called Clifford Algebra) provides the physicist and software engineer a common language. With the work of David Hestenes, and his formulation of Spacetime Algebra, the language has been applied to describe Special Relativity, Classical Mechanics and Quantum Mechanics[1]. Chris Doran and Anthony Lasenby have also written extensively about the language and its potential for formulating theoretical physics[2]. Open source software libraries are available that provide the programmer the basic objects and operations[3]. With such a toolkit in mind we will look to physics to see in more detail how we might simulate a digital world.

Representation

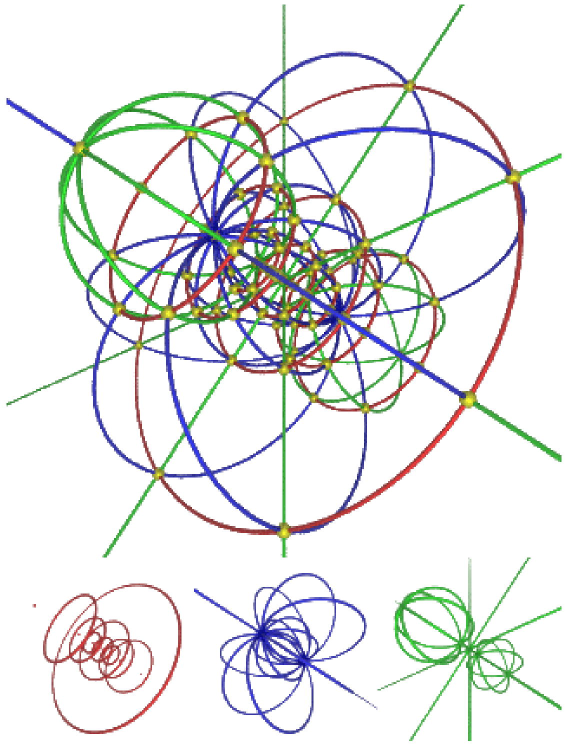
How is the state of the cosmos represented?

In order to manipulate information in a digital cosmos, we must find a numeric representation of the Standard Model particles observed in nature. With his E8 Physics model, Tony Smith has shown how their symmetries and properties can be represented numerically, using quaternions and bi-quaternions. Peter Rowlands has also developed a representation.[4]

Lou Kauffman has described how quantum mechanics can be handled with discrete algebraic models for calculus and differential geometry.[5] But Bell's Theorem proves that no local realistic hidden variable theory of quantum mechanics is possible. Or does it? It turns out that Bell's Theorem only applies to the standard formulation of QM over the complex number field. Quantum Mechanics, however, can be formulated over other number fields, including quaternions. In 2007 both Joy Christian[6] and Carlos Castro[7] were able to construct local realistic theories for quantum mechanics using Clifford Algebra.

Joy Christian's formulation of QM is based on a model that requires a three dimensional hypersphere:

> "I hypothesize that the space we live in respects the symmetries and topologies of a parallelized 3-sphere, which is one of the infinitely many fibers of a parallelized 7-sphere."
>
> — Joy Christian



S3 Hypersphere Fig. 1

We will adopt this approach for quantum mechanics and rely on a representation of the S3 three dimensional hypersphere.

Transformation

What transformation occurs between explicate spacetime and implicate information?

Both explicate and implicate orders are represented by geometric algebra, allowing for mathematical operations between them. In order to reproduce the effects of special relativity we will assume the explicate representation of space-time is Minkowski space. Stored in natural units, separations here correspond to space-time intervals that are Lorentz covariant, and thus consistent with special relativity. We can represent such a Minkowski spacetime quite naturally using the objects of geometric algebra[8].

The implicate representation can be encoded for storage efficiency and for the convenience of the algorithms that compute the dynamics of the world. We will use conformally compactified Minkowski space-time, a space advocated[9] by Roger Penrose. Recently, Arkadiusz Jadczyk has discussed[10] several issues regarding the use of such a space for physics. Conformally compactified Minkowski space can be viewed as an S3 hypersphere, the same structure we found is a useful space for conducting quantum mechanics.

So the transformation between explicate space-time and the implicate order is conformal compactification. Peter Rowlands has also discussed[11] properties of elementary concepts such as space, time, mass, and charge and how they might be represented and combined using quaternions. If conjugate variables are combined in a single value for manipulations and storage, then we can maintain the necessary trade-offs in precision. Rowlands suggests four natural scaling parameters, which we can also use as parameters for the projection.

The conformally compactified implicate structure is geometrically closed, and similar to the Einstein Static Universe model. Indeed even before Einstein's special relativity, mathematicians as eminent as Reimann, Gauss, and Clifford believed the shape of the cosmos was a 3-sphere[12]. However, while the implicate order is closed, the explicate order is neither static nor closed. This hints at the important freedom that results from distinguishing two representations for the universe, an implicate one supporting computation and an explicate one supporting measurement.

Dynamics

> What dynamic within the implicate would correspond to the physical laws we observe in the explicate?

For classical mechanics, the choice of geometric algebra is a natural one. The explicate kinematics of a digital world can be expressed with various operations on quaternions[13]. Doug Sweetser has compiled a list of quaternionic versions of many physical laws. He gives a useful quaternion-valued derivative that contains the time derivative along with the divergence, the gradient, and the curl. Quaternions are useful for rigid body dynamics, such as the modeling of molecular dynamics[14].

But remember that we intend to compute our dynamics in the compactified implicate domain, not in the explicate domain. How do these compare? Let us look first at the effect on time. In *Projective Invariance for Classical and Quantum Systems*[15], Bobylev and Vilasi show that many fundamental physical laws are invariant under projective transformation of the time scale. This means that the same formulas that physics has derived from measuring the explicate world (typically expressed as functions in time) will operate correctly when applied to values in the implicate domain (which has a projected time as well as space). In a later section, we will take up the distortions introduced by projecting space. Here we only note that, for length scales small with respect to the size of the implicate hypersphere, the spatial distortion is also small. Quantum dynamics as well as Classical dynamics exhibits the projective invariance described by Bobylev and Vilasi.

There have been several approaches to quantum dynamics using geometric algebra. Peter Rowlands and John Cullerne have developed a full theory for quantum electrodynamics based on geometric algebra[16]. To be computable, such dynamics must be placed on discrete foundations. Recently, David Finkelstein has developed discrete theories of QED using the language of geometric algebra, called Recursive Quantum Gauge Theory[17] and Simplical Quantum Dynamics[18].

The hypersphere S3 has properties that make it convenient as a space for doing calculations: it is parallelizable, has isometries without an axis, and it corresponds to the unit quaternions. But long range interactions create problems for economical computation. For example, in the explicate picture, to compute the motion of each particle, the relative location and properties of every other particle in the universe must be taken into account. This quickly gets difficult as the number of particles under consideration increases.

The straightforward solution is to represent fields rather than particles in the implicate. Each explicate particle corresponds (under transformation) to the field of influence that it has. In the implicate, these fields are maintained, not individually, but in summary form. Then the implicate dynamic becomes local: only the local gradient of the field is needed to calculate dynamical changes to the field. No long range information is necessary, so the process can proceed in parallel using many computational elements. When an explicate snapshot is needed, the field representation can be translated back to the particle form. Milo Wolff gives an example of this technique with his Space Resonance model, applicable to electromagnetic interactions[19]. It remains to be seen if an analogous procedure can be applied to the full set of Standard Model particle interactions.

There have been formulations of explicate gravity that use the language of geometric algebra. For example, Sweetser has suggested a hypercomplex version of gravity[20]. An alternative is Gauge Theory Gravity[21], developed by Lasenby, Doran and Gull of Cambridge University. But Einstein's general relativity suggests that gravity operates on matter indirectly, via the geometric structure of spacetime. Indeed, John Wheeler's geometrodynamics sought to unify the theories of particle physics and gravity via geometry.

Our software paradigm suggests that the geometry of the implicate (rather than that of explicate spacetime) should play the dynamical role. Lisa Randall and Raman Sundrum have argued[22] that a plausible model of gravity can be constructed for a situation where a 3+1 dimensional space is curved inside a higher dimensional space with large extra dimensions. Indeed, Maurice Dupré and Frank Tipler have shown that general relativity itself can be reformulated as an aether theory on S3, our implicate hyperspace.[23]

Structure

How to structure the implicate order to be computationally effective?

Computing the cosmos would require immense computing resources. One computational approach, used in large scale simulations today, is called adaptive mesh refinement. With this technique, only the parts of the simulation that need to be are elaborated in great detail, those parts that are empty or smooth are simulated with a coarser mesh. If efficiency is an issue, an appropriate design would be a fractal system structure, whereby each level of the system could make use of other systems responsible for finer details.

Indeed, the ubiquity of fractals in nature has led some to wonder whether space and time itself might be fractal, not just the objects within. Laurent Nottale has investigated fractal space-time and developed a theory he calls Scale Relativity, finding that a fractal structure alone can produce quantum mechanical effects[24]. In our picture, the implicate cosmos can be viewed as a dynamical fluid on the surface of a hypersphere, where a fractal structure implies the presence of similar sub-structures. In the way that turbulent fluid flow induces a cascade of ever smaller vortices, large hyperspheres can generate smaller hyperspheres. Information from several of these are combined when the observer generates a explicate view of an unbroken world.

We have argued that a software cosmos can plausibly be constructed using known physics, and outlined its design. Now we take a look at how such a software system might appear in operation.

Black Holes

> "A black hole has no hair."
>
> — John Wheeler

John Wheeler hoped to find a way to determine "mass without mass", i.e. directly from geometry. Nassim Haramein has recently found that the mass of a proton can be determined from its size, assuming it is a black hole in the shape of an S3 hypersphere[25]. Encountering the S3 hypersphere in the guise of a black hole may remind us that in our model we use the S3 structure in the implicate order to compute particle dynamics. According to the Black Hole Membrane paradigm[26], from the outside, a black hole appears to have the properties of a dynamical fluid[27].

Rafael Sorkin finds that black holes have "wrinkles", in fact a fractal structure[28]. So is it possible that protons, or other black holes, are micro-universes? In developing a theory of gravity, Nassim Haramein and Elisabeth Rauscher found a solution to gravitational field equations in the form of a fractal hierarchy of black holes[29]. This leads us to ask about the cosmos as a whole. Justin Khoury and Maulik Parikh have found[30] that imagining our cosmos to be a black hole viewed from inside provides an explanation for inertia.

Quasars

> "If this plane were to crash, we could get a new start on this quasar problem."
> — John Wheeler, boarding a flight along with four quasar physicists in 1964

> "Without predictive theories we have nothing – our best hope for understanding quasars is that extraterrestrials might drop in and explain them to us."
> — Robert Antonucci, "Quasars Still Defy Explanation" in 2013

The "quasar problem" Wheeler refers to is the difficulty of finding sensible theories to explain the energies of distant quasars, and it is still with us, as Robert Antonucci notes.[31] Observations of high energy gamma rays are problematic[32] [33], as well as cosmic rays that exceed the GZK limit.[34] Today quasars are identified as "active galactic nuclei", the supermassive black holes at the center of galaxies. Detailed modeling of black holes finds that magnetic turbulence can account for hard X-rays.[35]

The projective transformation between implicate and explicate in the software cosmos may also supply part of the answer: objects near our antipode in the implicate will appear at enormous distances in the explicate. Structures made up of these objects will appear unnaturally large. Indeed, we find that is true of the quasars of the Large Quasar Group[36] and the galaxies of the Pisces-Cetus supercluster[37]. If the standard interpretation of cosmological distance is wrong, then quasars may be closer than they appear, and thus have more reasonable intrinsic luminosity.

Dark Matter

Dark matter, invisible but responsible for gravitational influence on the matter we can see, appears to be distributed in a fractal pattern[38]. The fractal distribution accounts for the large voids observed[39] as well as the concentrations of matter. A consistent fractal structure is apparent out to about 180 megaparsecs; after that, a lower dimension fractal structure continues[40].

It is natural to identify dark matter with the explicate projection of our implicate hyperspheres, noting that the fractal structure would appear stretched at large distances because of the projection. This model may also be helpful to explain the unexpected structural simplicity of galaxies. Given their varied compositions and histories, the structure of galaxies should be controlled by six independent parameters. But five of them appear to be correlated, leaving only one parameter free, with all others indirectly determined[41]. Perhaps the implicate hypersphere not only binds a galaxy together, but the size or spin of the hypersphere determines its structure.

Dark Energy

The existence of Dark Energy is derived from evidence for accelerated expansion of the cosmos, and the standard explanation is a cosmological constant. In the 1970s Irving Segal assumed a compactified Minkowski space and a non-standard redshift-distance relationship to construct his Chronogeometry theory[42] [43]. Similarly, Dragan Hajdukovic has suggested[44] that one way that the effects of dark energy might be explained is if the Universe had the shape of a hypersphere.

Our implicate model is similar to these, and we may find that there is no mystery once the explicate projection is seen for what it is.

Multiverse

> Wheeler cries out to Copernicus,
> "Remind us that there is no other universe
> Than the universe of mind and man,
> The universe that is our home."
>
> — John Wheeler quoted by Richard Henry

In his book *Just Six Numbers* Martin Rees writes that the universe appears "fine-tuned" to develop life: the values of several physical constants must have precisely the values they do for the universe to develop and function. The anthropic principle suggests that this should be no surprise: after all, if there were no life, there would be no one to ask cosmological questions. But it quickly leads us to the idea of a Multiverse: that there are many universes with differing values for these constants.

Our software cosmos may more satisfying than having to imagine a huge expanse of randomly configured universes, as it is only the explicate observer views that are fine-tuned; the implicate order need not be.

Detecting a Simulation

If the universe is a simulation, can we detect its "hardware"?

There have been several lines of research regarding what such hardware underlying the apparent reality of space-time might be like. The search for a discrete causal structure led Rafael Sorkin to formulate[45] the theory of Causal Sets in 1995. David Finkelstein worked on quantum infrastructure at about the same time and has recently summarized it in *Nature as Quantum Computer*[46].

David Deutsch points out in *The Fabric of Reality*, that due to the universality of computation, if we are in a virtual world, we cannot deduce anything for certain about the "hardware" the software runs on. This does not necessarily prevent us from detecting evidence of its organization: a Planck scale lattice might be detected by analyzing cosmic rays[47]. These tests are quite difficult, due to the extremely small scale of the hypothesized underlying lattice. However, there may be an easier way to tell whether we are living in a simulation. We start by asking a slightly different question:

If the universe is a simulation, can we detect its "software"?

We may be able to see tell-tale side effects of the software algorithms used to simulate our cosmos. One possible way to distinguish virtual and non-virtual worlds is to look for a phenomenon called fractal creasing. Fractal creasing is a statistical effect that appears in software-generated landscapes when using a midpoint subdivision algorithm[48]. The extrema of the landscape will have a statistical association derived from the arrangement used to initiate the fractal subdivision.

Such creasing is, in principle, detectable from within the simulation. So the next question is where to look for creasing. Fractals are ubiquitous in nature and since fractals are self-similar, we can inspect them across a variety of scales. That means we can pick a scale that is convenient and easy to measure. A good model system would be spherical, or approximately so, so that all three spatial dimensions are accounted for. The idea behind the Landscape Test, described next, is to use the whole Earth as a model system wherein to hunt for fractal creasing and evidence that we might be in a simulation.

The Landscape Test

> "In any field find the strangest thing and then explore it."
>
> — John Wheeler

The Landscape Test measures the degree to which Earth's terrain indicates a global underlying geometry. A data set of Earth's land-based terrain highpoints can be derived from public domain Digital Elevation Models[49] by extracting topographic highpoints. Highpoints have the property that they are as high or higher than all the points surrounding them out to a few kilometers radius. If a digital cosmos is computing the landscape recursively, this might be revealed from geometric relationships between highpoints that occur above the level of chance alignment. Without knowing the specific geometric structure that might underlie such a landscape generation algorithm, the best we can do is check for a common property. That would be great circle alignment, which we expect to be present to some extent in projections of most regular polytopes onto the sphere.

Using a Monte Carlo statistical test, we compare alignments seen between real highpoints to randomized data sets. Each randomized data set is derived from the real data set by moving each highpoint independently a few kilometers in a random direction. Note that each pair of non-antipodal points on a sphere determines a great circle, and so we can find the set of alignments by examining the set of circles generated by each pair of highpoints. The general procedure is as follows:

1. For each pair of highpoints, determine the intersecting great circle and inspect the other highpoints (if any) that lie within a fixed distance of that circle. These points determine the figure of merit of the alignment.
2. Tally the number of alignments found, grouping by figure of merit.
3. Calculate this tally for the real highpoint data, and several times for different randomized comparison data.
4. Determine the expected distribution from the randomized data, and then compare the distribution derived from real data to see if there is any significant alignment above random chance.

There are several parameters that can be varied when using this approach. These include: the size of the set of highpoints to test, the figure of merit metric, a minimum threshold for accepting an alignment, techniques to account for different highpoint densities in different regions, methods of filtering resulting alignments to remove duplicates, and others. But in the end, it is the same test for a global geometry underlying terrain that is not accounted for by random chance.

For example, taking as a data set the top 10,000 highpoints (ranked by distance to closest higher point), and using 100 meters as a maximum alignment distance, the real data show 1% more alignments that random comparison data. While there is clearly a great deal of noise, this is good for better than 3 sigma significance. Over 75% of these highpoints have alignments in three or more directions. Along each direction, four or more peaks form a great circle arc, all aligned to better than 100 meters.

To maintain such precise global geometric alignment during movements over geologic time is as remarkable as it is statistically implausible. However, retaining alignments between moving points is possible under projection from a higher dimensional space: such projections can preserve geometric invariants while allowing vertices to move relative to each other. So alignment suggests holography at work along with virtuality in the layout of the landscape.

The Grid

> Behind it all is surely an idea so simple, so beautiful, that when we grasp it - in a decade, a century, or a millennium - we will all say to each other, how could it have been otherwise? How could we have been so stupid?
>
> — John Wheeler

The idea of alignments between landscape markers appears in many ancient traditions. It is worth remembering that the business of traditional cosmology did not concern itself with extragalactic astrophysics. Rather, it was to explain how the Earth came to be, and us on Her. So perhaps it is no surprise that the Landscape Test, conducted at the planetary scale, makes contact with traditional views.

In China the ancient Form School of feng shui pays close attention to the landscape, enabling the practitioner to discern the flow of invisible "energy" in relation to the "green dragon" highpoint and other elements of the terrain.[50] Western scientific investigation of Earth energy dates to an 1870 lecture[51] given by William Black to the British Archaeological Association, appropriately enough, in Hereford's Green Dragon Hotel. This was followed by several scientific articles by Alfred Lewis regarding alignments of stone circles with nearby hills[52] and then popularized by Alfred Watkins in *The Old Straight Track*. The idea of Earth energy has had many proponents, including John Wheeler himself, who practiced dowsing.[53]

Mountain tops have offered a place of contact with the divine. The Biblical account describes Moses on Mount Nebo (also known as Pisgah), the highest point in the Moabite range.

> Then the LORD said to him, "This is the land I promised on oath to Abraham, Isaac and Jacob when I said, 'I will give it to your descendants.' I have let you see it with your eyes, but you will not cross over into it.
>
> — Deuteronomy 34:4; NIV 2011

Whether by dowsing or inner sight, the ability to detect the grid appears to be a capability some people have. An account comes from Black Elk, a seer from the Oglala tribe of the Lakota Sioux nation. Here is his account upon ascending Harney Peak, the highest point of South Dakota:

> Then I was standing on the highest mountain of them all, and round about beneath me was the whole hoop of the world. And while I stood there I saw more than I can tell and I understood more than I saw; for I was seeing in a sacred manner the shapes of all things in the spirit, and the shape of all shapes as they must live together like one being. And I saw that the sacred hoop of my people was one of many hoops that made one circle, wide as daylight and as starlight, and in the center grew one mighty flowering tree to shelter all the children of one mother and one father. And I saw that it was holy.
>
> — Black Elk

Conclusion

> The poetic Wheeler is a prophet, standing like Moses on the top of Mount Pisgah, looking out over the promised land that his people will one day inherit.
>
> — Freeman Dyson[54]

The software cosmos picture answers the contest question in this way: "It from Bit and Bit from Us". In the software cosmos, the explicate world of "It" arises from the implicate world of "Bit", and the content of that implicate information world comes from consciousness, "Us". This picture hints that physics will find the ancients were right and that the cosmos is inherently virtual, holographic, and fractal.

If so, then with "It from Bit" John Wheeler presaged the time when his people, the physicists of the world, would come to inherit the land described by the ancient seers. We can hope, as he surely would, that that inheritance, and the knowledge of the "sacred hoops" it holds, can help us come to "live together like one being" as Black Elk described so beautifully.

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