Reality and the Universe: Is reality digital or analog?
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Throughout history, humans have sought to understand, describe, and predict behavior in the universe. Initially, they employed only their senses, especially sight, and their intellect. With time, these were augmented by tools - documentation, mathematics, instruments, computers, communication networks, and finally vehicles that traveled beyond Earth. The products of the employment of all of these have produced theories, aspirations, conjectures, and questions concerning the real nature of the macro and micro universe of the cosmos and of matter, but certainly it is still incomplete. Reality is probed, and the answers questioned.

What is real? Is reality digital, analog, or both?

The accepted reality has varied throughout history as has the means of description. In the present day, there is a heavy reliance on mathematics as a standard method of description which is independent of vernacular nuances of meaning. There is a surfeit of data and of theories connecting some of this data; but to date, there exists no uniform theory to cover all aspects of the behavior of the matter which constitutes the fundamental building block of nature. We are not even sure of the relationship of space and time. The problem essentially is one of the limited capabilities of mathematics to describe nature. The models presented to date have changed with time, often in conflict with theories accepted for centuries. Nature is simple. Our descriptions, especially mathematical, are not. Indeed, our models are often ambiguous and cloud the reality of nature.

This paper will examine reality, with particular attention to its nature as digital or analog, and the theories presented to describe this reality. The objective of this paper is to consider the inadequacies of the theories and their mathematical descriptions, and to propose an empirical approach to a Unified Field Theory[1]. The nature of reality as analog, digital, or both is examined, and the paper closes with conclusions to this question.

Reality[2] is the state of things as they actually exist, rather than as they may appear or may be thought to be. This paper argues that it is both or neither, especially in modern physics. In its widest definition, reality includes everything that is and has being, whether or not it is observable or comprehensible.

To understand and accept this proposition, certain lemmas are required. One of these is the historical development of physics up to the present day. Classical physics developed from the application of three forces - human curiosity, the intellect, and observations tied to our five senses. Experiments, observations, and measurements were simple and within the realm of the calculus of the time. Models of physical behavior were based essentially on measurements and projections. Classical physics dealt with a continuum analyzed intermittently via digital measurement, and accepting such measurements as valid.

Mysteries were accepted. For example, behavior of light as both corpuscular and wave was accepted without a universal explanation or theory. In a sense, nature was essentially considered to be a continuum. The models developed were simple. Classical physics could certainly be treated as analog.

Somewhat abruptly in the latter part of the 19th Century, the simple laws of classical physics came under great scrutiny. Experiments and findings such as radiation, the Michelson-Morley experiments, and Rutherford's studies of the structure of matter created new realms for physics. The analog representation suitable for classical physics could not adequately encompass these new theories and findings. Relativity and Quantum Theory[3] provided new insights and tools to go further. The application of mathematics to physical phenomena was extended to include such properties as n dimensions, new concepts linked to complex numbers, considerations of infinity, and the space-time relationship. Digital representation of physical phenomena provided new considerations, posing the option of using digital-analog representation in theories of physics.

Quantum Theory successfully combined the digital and the analog representation of some aspects of nature. The result, especially with extensions in the latter part of the 20^{th} Century, was to unite all aspects of nature except gravity into a Standard Model. Despite these significant advances, Einstein's dream of a Unified Field Theory continued to be unfulfilled.

Nature is simple even if our description of nature is complex. Our inability to develop a Unified Field Theory, or even to fully describe and understand black holes, antimatter, the structure of the nucleus, and the relationship of space and time, is a failure essentially of our descriptive capability.

An example will illustrate this point. A sunset as described by a poet is probably totally different from a sunset described by an astrophysicist. Furthermore, the vernacular used by different poets will contain various nuances associated with language. The nature of a description often clouds the nature of what is being described. In particular, mathematics can be used to distort these representations by presenting diverse solutions for the same data. Consider the current discussions of multiple universes where the number of such universes is in flux.

How can we describe reality? We normally assess reality based upon our five senses - sight, sound, feel, smell, and taste. We describe any reality sensed in the hope that others will come to accept the reality described just as we do. The difficulty is that reality is often beyond our five senses and it becomes necessary to look for shadows or manifestations of reality in order to decide whether something is real, and if so, what it is. Then we attempt to link it to other aspects of reality we know and/or accept, and then describe what we believe to be real in a way that is fully coordinated with other phenomena, complete in its description, and fully comprehended by others regardless of vernacular or discipline.

Consider a piece of string. Assume it has been broken and connected with a knot of some kind. Furthermore, assume it is recumbent upon itself, with loops, sharp bends, twists, and even additional knots. Consider that the string has no definitive pattern as it lays on a surface. Now pick it up. Except where constrained by knots, the string will straighten out. That is the reality of the situation.

How can this reality be described? Mathematically each type of bend, twist, loop, or knot in the piece of string can be represented by a function, even if discontinuous. Whether in the real plane or in a single or multiple transformed plane, these discontinuities can be removed so that the string can be represented by a continuous function. Even a Heaviside step function can be replaced. Thus, mathematically, the string will straighten out when picked up. The mathematics of this solution are quite complex, yet the function of picking up the piece of string is quite simple.

What of the basic question of whether the string is analog or digital? It is both. It is digital at the discontinuities and analog where it is a continuous piece of string.

As another example, consider a person walking in the desert. Let us assume that person sees a mirage of an oasis. It appears to be real, but there is no water to drink. This visual manifestation occurs due to the bending of light rays due to a temperature-related density change to the atmosphere. In other words, the mirage is found in a desert in the same way that a straight rod visually bends below the surface of a glass of water. The mirage can even be captured by a camera. What is reality in this case? Is the mirage digital or analog? The real oasis and the real rod are analog, but the mirage and the bent rod image are digital.

Next, consider the Newtonian question when an apple falls from a tree. Newton deduced that there is a force called gravity which pulls the apple to the earth, but gravity itself is beyond our five senses. We can only measure the affects of that force. The concept of gravity is a conjecture based on observation and reasoning. The affects can be measured but the force itself cannot be visualized.

Is the force of gravity digital or analog? Gravity is certainly analog on Earth and probably digital in black holes.

Let us take a final example. We can take a magnet and sprinkle iron fillings on it. We will see the lines of force surrounding the poles of the magnet. This is a direct visualization once again of the affects, but the forces of magnetism cannot be "seen".

Is the magnetic force digital or analog? In this case, it is apparently analog.

We see because of the phenomena of light. Light is certainly real but even with today's advanced science and mathematics, we cannot be absolutely certain that we know how it works despite the existence of the accepted quantum explanation. We know that the whole world, and indeed the universe, "work", but we are not able to describe the work and relationship of gravity, electromagnetic radiation, light, and space-time relationships into a single integrated description which we loosely call a "Unified Field Theory". We may define a Unified Field Theory as a single set of mathematical equations, or a string, that includes considerations of gravity, electromagnetism, weak nuclear forces, and strong nuclear forces. The latter three are now included as the Standard Model of Quantum Physics. To date, we do not know how to include gravity in the Standard Model, or in a variation. Hence, we can describe elements of nature separately, but not as a cohesive entity.

It is almost as if we can define each of the organs of the human body but cannot encompass the reality of life into a single explanation. We know that the mind controls the body, that the heart pumps blood with oxygen to all the parts of the body, and that our brains can conjure up visions, memories, and thoughts, but we cannot fully define what we mean by personhood, intuition, personality, and free will. Is life, then, digital or analog in nature? Life is certainly real. Elements of the body are analog and elements of the brain are digital, but life is neither.

How do we measure and describe what is real and what is not?

Our descriptions of what is real and what is not real may themselves not be real. For example, the movement of electricity and its behavior are governed by Kirchhoff's Laws[4]. A fundamental element in these laws is a mathematical entity called "j", which is $\sqrt{-1}$(the square root of minus one) which does not exist. Hence, we invent something that is not real in order to create means of describing what is real. In complex number theory the same quantity, $\sqrt{-1}$(the square root of minus one) is represented by "i", the imaginary number.

In the same fashion, we are taught that a point is the intersection of two lines, and that a line joins two points. Careful analysis of these definitions will show that they are interchangeable; and are even interchangeable in three dimensions using the terms "spheres" and "circles". The foundations of mathematics[5] itself require us to assume the truth and validity - or the reality - of certain axioms upon which mathematics is built. These are only two of the seven or more axioms that form the foundations of mathematics.

So is something real if it can be defined mathematically? Not necessarily. Life is real but to date, no one has advanced a set of equations for life.

Mathematics handles great complexity and great simplicity. It makes the simple complex in the hopes of making the complex simple. The simplicity of Euclidean Geometry[6] and the discovery of π (pi) certainly created significant capability in describing physical phenomena. On the other hand, the mathematics of Plane Geometry[7], especially as applied to the concept of parallelism, leads to situations where the sum of the angles of the triangle are less than, equal to, or greater than π radians, or 180 degrees. This is termed hyperbolic, parabolic, and elliptical geometries. Similarly, Transform Theory (LaPlace[8] and Fourier[9]) significantly simplifies mathematical descriptions leading to solutions in closed form in the transform plane, but not necessarily in the real plane.

The example of the piece of string is significant since the reality is that the string will straighten out. Is the string digital or analog? Prior to being picked up, the string can only be represented by a series of functions describing each section of the string, whether continuous or discontinuous. If attempts are made to measure the thickness of the string along its length, these measurements would differ at a cusp, at a knot, break, loop, or whatever shape taken by the string. On the other hand, if thickness measurements are taken of a piece of string that is continuous without a break, and without any loops or cusps, then the measurements should be approximately the same for the entire length. Thus, in one case the representation would have to be digital, and in the other, analog is possible. And yet the digital or analog representation can be applied to segments of the piece of string either in the normal plane, or in multiple transform planes where it is all analog.

Can we, then, think of reality as either digital or analog? If we take the case of an automobile traveling down the highway, the movement is real since there is a change from one place to another. The speed of that change can be measured in many different ways- via radar, by translating the rotation of the wheels in a timed interval, by watching it go by a measured distance, by taking GPS readings at an interval, etc. The display of that speed can be digital or analog with a needle on the dial. The mechanism of the display can also be varied. All of that is incidental to the fact that we are putting a measure on the rate of change of distance with time. That rate of change we assume is independent of the location of the observer. We can extend this situation to placing an observer in the car, in an airplane above the car, or on another planet. For someone in a car, there is no change in speed; for someone in the airplane, it will depend upon whether the speed of the aircraft affects the measured speed of the car or if the car speed is established by the timed interval to cross two lines painted on the highway; and for the observer in space, the rotational speed of the Earth, and the orbital speed in the universe will have to be taken account of the speed of the vehicle. And speed for an observer in the car is analog. Depending on the location of the observer, and whether or not acceleration is intermittent, speed can also be digital.

To extend this concept, consider a merry-go-round. Assume a person walks from one location to another as the merry-go-round is turning. An observer on the merry-go-round would see that person walking a straight line. An observer off the merry-go-round looking at it would see the person in rectilinear motion. Finally, an observer above the merry-go-round would see the path the person travels as a spiral. All three descriptions would differ for the same physical event because of the location of the observer.

Yet the person walked a straight line on the merry-go-round. Hence, the location of the observer is certainly an important element in any measurement, and the measurement must also be calibrated with regard to the impact of the observer on the observation made. The Heisenberg

Uncertainty Principle[10] proposes that the measurement by an observer actually affects the readings of what is observed.

Things are not always what they seem to be. So what is real?

We can use digital means and analog means to describe a phenomenon, but we cannot say for certain that we have truly described it for what it is. Reality is more a case of acceptance by one or many. We all know what the color green is but we cannot describe it. Scientifically, it is one of the colors of the spectrum of light with specific wavelengths for each shade of what we call "green". Perhaps we all sense green differently while calling it "green" because we have learned to call it green. Those who are colorblind can see select colors, no colors, or perhaps colors in a different way. As a result, the manner in which we truly perceive it may or may not be the same for all of us; or indeed for any two of us. Furthermore, the descriptions, whether in the vernacular or in mathematical form, will undoubtedly differ. Yet, green is a real color!

Our attempts to define the universe have advanced significantly. In the period before the Common Era (BCE) and into the first century, Greek philosophers had progressed to the point where Ptolemy presented tables that predicted the behavior of the stars, sun, and moon. These tables were built around the concept of Earth as the center of the universe - the Epicentric Theory. The work of Copernicus, reinforced by that of Kepler and Galileo, produced the Heliocentric Theory which stipulated that the planets traveled around the sun. While the latter is now universally accepted, the fact of the matter is that both produce valid results. But we still question how the universe came to be and what it is[11]. Current theories in mathematical proposals include the Big Bang[12], Superstring Theory[13], Multiple Universes, Planck Time[14], and the Expanding-Contracting Model of the Cosmos. All are mathematical attempts at correlating observations, conjectures, theories, and questions. There is no certainty in the theories as truly representing reality; and yet the reality of the universe if evident. Let me suggest an empirical result in resolving some aspects of this problem.

My personal involvement with imponderable conflicts between reality and mathematical theory began with my doctoral studies. The research proposed for my doctoral thesis was to examine the heating effect of the reentry of space vehicles returning to Earth from outer space. At that time, the early 1950s, the Navier-Stokes equations[15] were accepted as determining motion through the continuum of the normal atmosphere. The Maxwell equations[16] were accepted as deterministic for travel through the regime of free molecule flow in outer space. There is no similar solution of the Boltzmann equation[17] the motion of the slip regime from outer space to sea level. The proposed solution was called the Rayleigh Method[18], which employed the Navier-Stokes equations with boundary conditions attempting to account for molecular collisions of the vehicle. The determining factor was the ratio of the vehicle size through the mean free path of molecules in a regime through which the vehicle was moving. This ratio is called the Knudsen number[19], Kn, and is defined as:

Kn = (mean free path of molecular collisions)/ (vehicle dimension)

where the mean free path is the mean distance travelled before a molecule collides with another molecule.

In free space (a regime termed free molecule flow), the mean free path is large because molecules are relatively rare. At sea level, this dimension is quite small as molecular density is significant. Hence from outer space the Knudsen number is large, whereas in the normal atmosphere it is quite small, approaching zero.

My objective was to determine the heating effect on a space vehicle entering the atmosphere from outer space to establish criteria for heat shields.

The results of extensive mathematical analysis and computerized numerical analysis established that the Rayleigh Method proved inconclusive and provided results that were incorrect.

The results I found are shown in Figure One. This figure is taken from my Thesis[20]. Note that in this diagram the curves are not connected. Hence, the Rayleigh Theory predicted discontinuous results for a known continuity in the region between sea level and outer space. This was an important discovery. A solution was still required to design the heat shields for space vehicles.

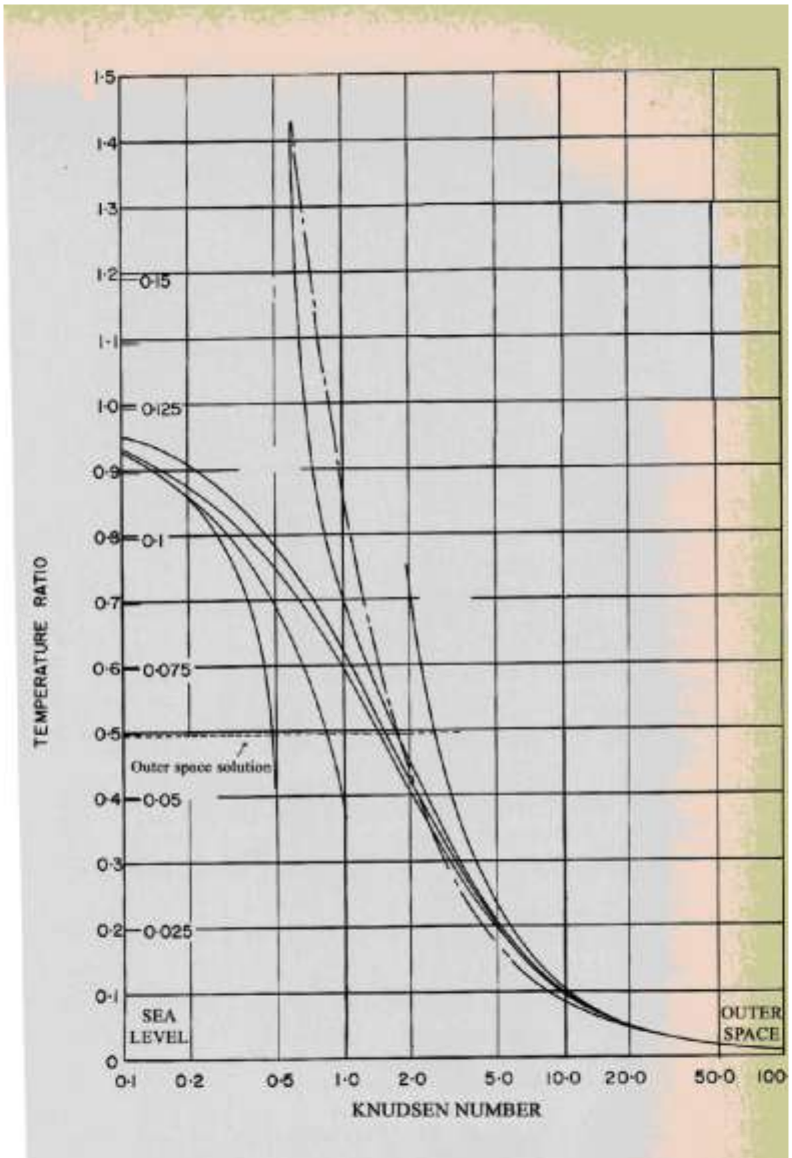

FIGURE I

VARIATION OF TEMPERATURE RATIO WITH KNUDSEN NUMBER FOR $M_{\infty} = 4.0$

Using pure logic, I proposed an empirical solution. The results are shown in Figure Two. The curves produced fit all the experimental evidence available at that time. This curve and approach have been validated with increased observations since. Figure Two is also taken from my Thesis and report.

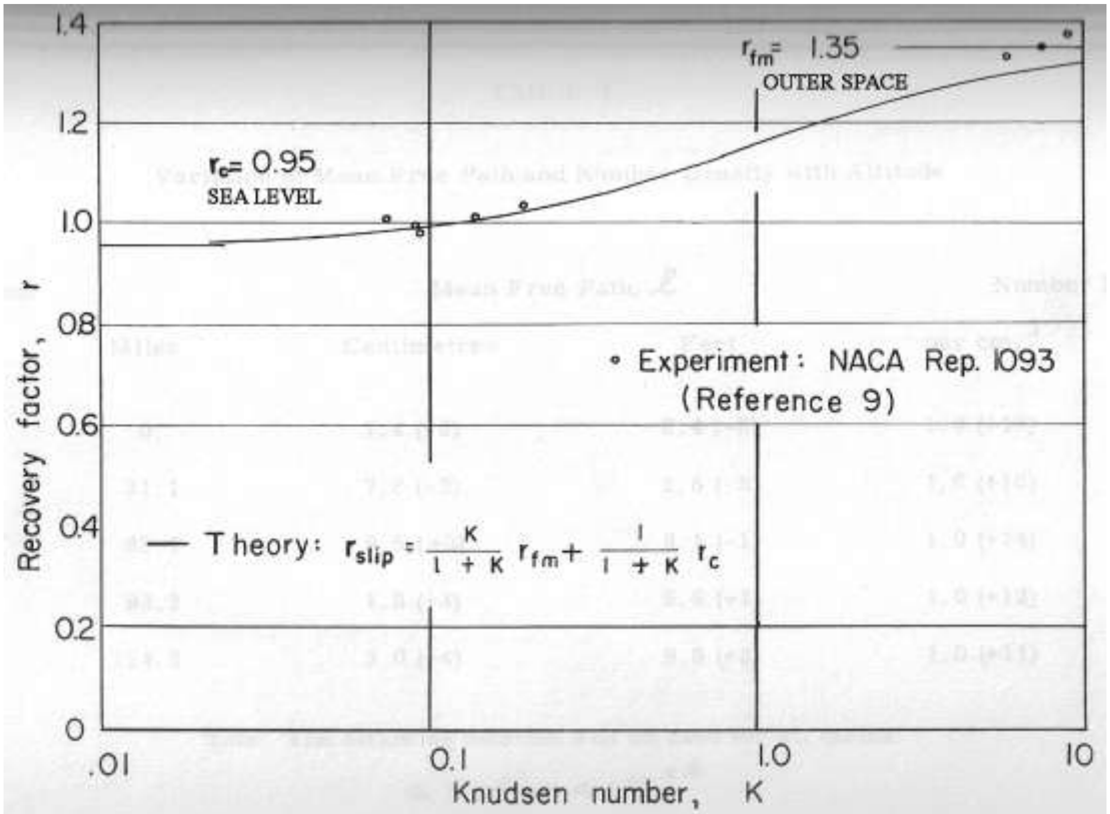

FIGURE 2

EMPIRICAL VARIATION OF RECOVERY FACTOR WITH KNUDSEN NUMBER

The basis of this curve is the idea of generating a new equation for space travel that would use the equations at both extremes, with factors that would eliminate one or the other in free space or at sea level. In between, the factors would associate varying degrees of the results according to the relative importance of the continuum or of the near vacuum of outer space. The approach was to use the fundamental factors associated with space flight – the size of the vehicle versus the distance a molecule would travel before striking another molecule. At sea level, molecules collide very frequently so the distance traveled is small. In outer space, with fewer molecules, the distance traveled is large, so collisions between a space vehicle and molecules are low.

The parameter measuring this difference is the Knudsen number which is the ratio of the size of the body traveling in space and the mean free path – the average distance traveled before molecules collide.

The Knudsen number is:

Kn = mean free path/size of body moving through it

where Kn is close to zero at sea level and large in outer space.

Hence the ratio:

Kn/(1 + Kn) is very small at sea level and close to one in outer space.

Hence I proposed the formula:

Desired result = (Sea Level Equations)*{Kn/(1+Kn)Kn}

+ (Outer SpaceEquations)*{1/(1 + Kn)}

In my report this was stated as:

Theory: $r_{slip} = r_{fm} + r_c$

where the subscripts fm and c respectively refer the free molecule flow regime and the continuity at sea level. The heat recovery factor, "r", was the important element in this study.

Essentially, I considered the slip regime as a combination of the two extremes. My solution produced the results for sea level and outer space by taking into account the Knudsen number which was indeed the qualifying aspect of the relative effect of molecules hitting a body or missing it. In the rarified atmosphere of outer space, molecules did not collide with a space vehicle; whereas in the continuum of sea level such collisions were significant. Hence, the speed of the vehicle would produce intense heat in a space vehicle re-entering the atmosphere. Various techniques were considered, but the one established was to create a heat shield -hence, the importance of my work in determining that heating effect.

This work is important not only for its contribution to the design of the heat shields for the space program, but also as a starting point for concepts of creating unified theories to be applied to science, truth, and our concepts of reality in the universe.

I advance this explanation because it is a case history in my approach to generating means of linking diverse technologies to solve a new problem. Essentially, as I did, the solution is to take known technologies and to link them with factors controlling the relative impact upon the entire problem. To summarize from my doctoral dissertation, I took the Navier-Stokes Equations and multiplied them by a factor associated with the Knudsen number; and then added the impact of the Maxwell equations multiplied by a different function associated with the Knudsen number. At sea level, only the Navier-Stokes Equations applied since the multiplicative formula would turn to one for the Navier-Stokes Equations and zero for the Maxwell Equation. On the other hand, in outer space, the Maxwell Equation would apply as the multiplicative formula. The Maxwell Equation would become one and that for the Navier-Stokes Equations would become zero.

In this paper, I propose an extension of this empirical approach to any movement or flight characteristic to be determined by the relationship:

C (Kn) [Navier-Stokes Equations] + F (Kn) [Maxwell Equations]

Furthermore, as the first approximation, I propose that:

C (Kn) = 1/ 1 + Kn

and

$F(Kn) = Kn/ 1 + Kn$

For sea level, where Kn goes to zero Lim C (Kn) = 1 and Lim F (Kn) = 0.

In free molecule flow for outer space, where the Knudsen number is large,

Lm C (Kn) = 0 and Lim F (Kn) = 1

It may be argued that solution is simplistic. Perhaps it is, but to this day, there is no better solution. Furthermore, as shown in Figure Two, the predictions of this empirical theory fit the observations.

What is most significant is that in outer space nature is digital - discreet and not necessarily continuous since a body moving through space does not affect the nature of the regime. Observations are obviously dependent on the location of the observation, and on the location of the observer. In the farthest reaches of space, where the molecular density is quite rare (ultra small), it might even be argued that the Heisenberg Principle can significantly affect the validity of observations if we are concerned with the nature of the medium.

Can such an empirical approach be applied to create a Unified Field Theory? I propose the following empirical relationship:

$$F_1 (s,t,e,m)^* \left\{ \begin{matrix} \text{Gravity} \\ \text{Equations} \end{matrix} \right\} + F_2 (s,t,e,m,)^* \left\{ \begin{matrix} \text{Standard Model} \\ \text{Equations} \end{matrix} \right\}$$

where "s" is space, "t" is time, "e" is energy, and "m" is matter

and where F_1 (s,t,e,m) and F_2 (s,t,e,m) are unknown functions to be determined with the conditions that in the case of gravity alone,

Lim F_1 goes to one and Lim F_2 goes to zero;

and where in the absence of gravity,

Lim F_1 goes to zero and Lim F_2 goes to one.

The limit case is for mathematical completeness only since all forces are necessarily active at all times.

This is a conjecture with more than idle thought behind it. Gravity and the forces included in the Standard Model are certainly universally present in the real world. They are also linked in a continuous fashion which we have not yet discerned nor described in mathematical form. It is suggested that this empirical approach can be the starting point for establishing an accepted mathematical set of representations of Unified Field Theory. Furthermore, particle attraction in the macro and micro milieu may very well be linked to forces akin to gravity as we believe we know it.

So now we return to reality.

Descartes said it all with his famous comment, "I think, therefore I am"[21].

Reality is "what is". It can be digital, analog, or both.

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