

Revisiting Our Basic Physical Assumptions: Our *Density* Concept vs. Riemann's *Dichtigkeit*

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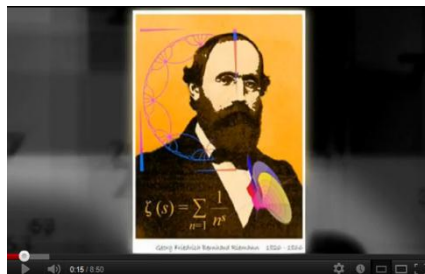
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Abstract: In this essay I argue that we need to revisit and re-conceptualize our basic physical assumptions within a wider scientific context. I exemplify my argument with Riemann's concept of *Dichtigkeit* (density).

1. “Looking back over the history of physics we can identify a number of places where thinkers were “stuck” and had to let go of some cherished assumptions to make progress. Often this was forced by experiment, an internal inconsistency in accepted physics, or simply a particular philosophical intuition. What are the tacit or explicit assumptions we are making now that are ripe for re-thinking?”

One word from the 2012 FQXi's Essay Contest's question above stuck with me: “stuck.” The word made me wonder if those places where *physicists* are stuck, such as the quest for physics' Holy Grail, a theory of quantum gravity, bear any similarity to those places where *mathematicians* are stuck, such as the quest for mathematics' Holy Grail, a proof of “Riemann's hypothesis.” I explored the former quest in an article on a forgotten controversy at the foundation(s) of quantum physics. [1] At the time, I knew that Bernhard Riemann developed much of the mathematics underlying the foundation(s) of general relativity. But I never thought that the latter quest, artistically explained below, or anything else in pure mathematics had anything to say to foundational physics. It hit me. I was wrong. I was ... “stuck.”



Vid. (a) <http://www.youtube.com/watch?v=MsBUTuYI62k> ¹

Take the conclusion to the famous number theory paper containing the celebrated hypothesis, titled “On the Number of Prime Numbers Less than a given Quantity (1859)” :

... the rarefaction and con-densation (*Verdichtung*) of primes from place to place, on the periodic terms has already excited attention, without however any law governing this behavior having been observed. In the future it would be interesting to keep track of the individual periodic terms' influence in the expression for prime numbers' density (*Dichtigkeit*).[2] p 185 ²

Compare the “mathematics” passage above to the “physics” passage below from Riemann’s “*On the Propagation of Planar Air Waves* [2]p 207. Forced by W. Thomson, aka Lord Kelvin, and P. Tait’s experiments on relative density (specific gravity), our thinker, taking as “foundation the law that gas pressure (*Druck*) increases in proportion to the density (*Dichtigkeit*),” sought an expression (*Ausdruck*)

... [for] greater densities (*Dichtigkeiten*), [which] move with greater velocity. From this we can deduce that rarefaction waves –those parts of waves in which the density’s dilation occurs in the direction of movement – increase in altitude in proportion to the time [while] con-densation (*Verdichtung*) waves decrease; becoming shock waves/density waves (*Verdichtungstößen*)...

First we note that both papers share a date, 1859, and a theme, density. Do different contexts indicate different concepts? Not necessarily. We find both when Riemann attempts to find the above expression in his tentative response to the 1861 Paris Academy “Essay Contest” ’s Question on Fourier’s wave approach to specific heat density [2] p423.³ Required to open with a motto, Riemann opens with Newton’s: “From these principal concepts, through dilation, we move towards greater things.”[2] p391.⁴ What if we follow suit? What if we extend our concepts towards greater, wider contexts? Would we be able to escape our wrong assumptions? Can we unstuck ourselves from misconceptions about concepts?

Here I argue that we need to re-conceptualize our basic physical assumptions within a wider scientific context. I exemplify the argument with Riemann’s density concept within “Natural Science, [the] attempt to conceive Nature through precise concepts.” [2] p 553. I begin Mazur and Stein’s “*What is Riemann’s Hypothesis?*”

[Ancient Greek] Pythagoreans thought that the principles governing Number are “the principles of all things,” the concept of Number being more basic than earth, air, fire, or water, which were according to ancient tradition the four building blocks of matter. To think about number is to get close to the architecture of “what is.” So, how far along are we in our thoughts about numbers?

The French philosopher and mathematician René Descartes, almost four centuries ago, expressed the hope that there soon would be ‘almost nothing more to discover in geometry.’ Contemporary physicists dream of a final theory. But despite its venerability and its great power and beauty, the pure mathematics of numbers may still be in the infancy of its development, with depths to be explored as endless as the human soul.... [3] p8

Inspired by the mathematicians’ passage, I propose as foil to a stuck self, Bernhard Riemann, “collaborator” with Kelvin, Tait, Fourier, Newton and Descartes. Riemann, forced by experiment, noticed inconsistency within physics and turned to philosophically intuitive self-reflection. His questioning points us towards our dreamed final theory explaining perhaps quantum gravity, perhaps even the human soul.

2. “What are the basic physical and mathematical postulates in our fundamental physical theories? What are the implicit assumptions we tend to forget we have postulated?”

Riemann answered that not exactly Pythagorean numbers but what Mazur and Stein relate analogically to the geometry of atoms in nature, prime numbers, or more accurately, prime *elements*. What if this concept of primes as atoms is closer to reality than we assume? Many of Riemann’s contemporaries seemed to have believed so. But we have forgotten their assumptions concerning basic physical postulates. In the interdisciplinary, international arena of Natural Philosophy university faculties thinkers discussed and exchanges of ideas regarding a basic physical assumption of the time, “mathematical elementarianism.” According to the theory, Euclid’s geometry, postulating elementary points, lines, and surfaces, points towards how “the world consists of volume elements whose actions accrue to become natural phenomena.” [4] p 62. Forced by recent developments related to Riemann’s number theory and recent experiments involving smoke rings Riemann and others postulated for a while a candidate for prime elements constituents of matter, a candidate theory that has returned today as an exciting approach linking physics, cosmology, arithmetic and topology. W. Thomson, Lord Kelvin’s atomic “knots.” [5–7].



http://en.wikipedia.org/wiki/Smoke_rings , <http://www.youtube.com/watch?v=-VL0M0jmu7k>

Mid-19th century, Kelvin, and later, Riemann, wondered whether P. G. Tait’s smoke ring experiments, enjoyable experiments we can watch today online (above), could be explained by Greek atomic and Cartesian “vortices,” helical whirling strings or “knot points.” Inspired in part By Riemannian mathematics, leading physicist Kelvin and later knot theorist P. G. Tait’s wrote *The Unseen Universe*, partly as a call to re-conceptualize, in light of their experiments, the elementary atom

[as postulated by Democritus-] Lucretius, —much rather yielding to the least external force, and thus escaping from the knife or wriggling round it, so that it cannot be cut,—not, however, on account of its hardness, but on account of its mobility, which makes it impossible for the knife to

get at it. This is the vortex-atom theory of Sir W. Thomson, made distinctly conceivable in very recent times by the hydrokinetic researches of Helmholtz. Helmholtz, in 1858, first successfully attacked the equations of motion of an incompressible frictionless fluid, [and] that those portions of the fluid which at any time possess rotation preserve it forever, and are thus as it were marked off from the others; also that these portions must be arranged in filaments whose direction is at each point the axis of rotation, and that the filaments are either endless, i.e. form closed curves (whether knotted or not), or terminate in the free surface of the fluid. Hence Sir William Thomson's idea that what we call matter may consist of the rotating portions of a perfect fluid, which continuously fills space ... [for] to cut a vortex atom, it would be necessary to give a free surface to the perfect fluid which on this theory is supposed to fill space, i.e. virtually to sever space itself! [8] p 104

Forced by Tait's, Helmholtz', and others' experiments, Kelvin called to get rid of cherished assumptions about the atom concept. Riemann did too. He wrote notes on "proto-knot theory" taken while engaged in applied physics involving smoke and "color spectrum" ring experiments.[2] [9] Now, it has been claimed that Riemannian overturned the concept of mathematics by introducing concept itself as a foundational approach. [10], [11]. I suggest next that Riemann introduced a specifically Lucretian concept of atomic element into Kelvin's theory. Kelvin admitted he was not fully "acquainted with Riemann's remarkable researches on this branch of analytical geometry to know whether or not all the kinds of "multiple continuity" now suggested are included in his classification." But in one move, Kelvin integrates philosophical intuitions about Newton's color spectrum with Riemann's analysis of *number*, discrete multiplicity, and *space*, multiple continuity. Knotted atoms can be hypothetically identified through their specific spectrum signature and identifying number, their molecular mass, now explained through their volume extension. We thus arrive at the final knot, Riemann's density concept.

It should not surprise us that Mazur, Stein and others, tentatively suggest analogies between density in the context of Riemann's hypothesis, density in the context of knot theory and density in the context of Fourier's theory." [3] p 58 Riemann found in Fourier how to subsume Pythagoras' "arithmetical" elements and Euclid's "geometrical" ones elements, together with the four Greek building blocks of matter, Lucretius' "physical" elements, fire earth, water and air. As Fourier did with Plato, Riemann will do with Lucretius. Before Riemann, Fourier wrote a "natural philosophy," his classic *Theory of Heat* which opens with the Latin motto "(Numbers rule Fire).⁵ He owes more to Pythagoras and Plato; specifically the latter's dialogue, *Timaeus*, which claims that:

fire and water and earth and air, although possessing some traces of their own nature, were yet so disposed as everything is likely to be in the absence of the demiurge, and inasmuch as this was then their natural condition, the demiurge began by first marking them out into shapes by means of forms and numbers. [12]

Riemann read the above through a Pythagorean lens. He did so even more through a Lucretian one. Fourier's passage from Plato gives us the setting for Riemann's own Latin motto. It does so because with the above Fourier gives us the "numerical" context to read Riemann's Lucretius, specifically the context elucidating how natural processes, both physical and mathematical, arise out of the rarefaction, and condensation of elements, elements rarefied throughout the infinite volume of the cosmos and condensed not only in "condensed matter" but even in the compression of a mathematical expression.

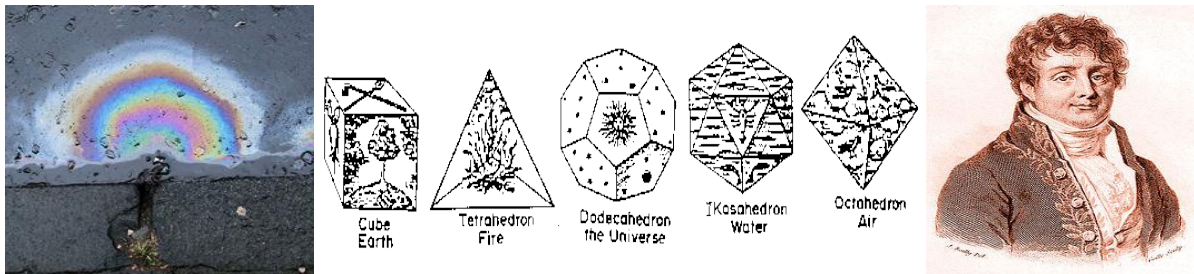


Fig. c Newton's "color rings" spectrum Fig. d. Pythagorean-Platonic solids Fig. d Jean Baptiste Fourier

3. Which assumptions have become so ingrained that they have become unquestioned dogma? Which assumptions in physics and in cosmology are interdependent or required for self-consistency?

Riemann's density concept as postulated in his immensely dense notes of a final theory of *Naturphilosophie*, tentatively titled "Research into a Theory of Mathematical and Physical Basic Concepts (*Grundbegriffen*) as Foundations (*Grundlage*) towards an Explanation of Nature." [2] pp. 539-570. Riemann believed this "title could awaken in most readers an ingrained prejudice." [2] p 560. So with an opening motto appropriated from Lucretius' cosmology, *On the Nature of Things*, he cautions readers against unquestioned, inconsistent, dogma. I translate in boldface the original. [2] p 553. ⁶

*Summon to judgments true,
Unbusied ears and singleness of mind
Withdrawn from cares;
these my gifts, arranged
For you with eager service, do not disdain
Before you comprehend them: since for you
I prove the supreme divine cosmic laws,
And the prime germs of things unfold,*

*Whence Nature all creates, and multiplies
And fosters all, and whither she resolves
Each in the end when each is overthrown.
This ultimate stock we have devised to name
Procreant indivisibles, [prime] matter,
seeds of things,
Or prime bodies,
primordial to the world. [13]*

Lucretius' motto illuminates why Riemann often changed the title of his final theory, trying to achieve true consistency in *all* natural laws. He believed a mathematically *consistent* proof (*Beweis*) requires *certified* proof (*Nachweis*), a theoretically devised proof later explored experimentally within a wider "cosmological" world. And viceversa. Riemann begged readers not to disdain his reciprocal application of number theory and geometry to physics and cosmology, an application moving in both directions. He begged readers to first understand his intuitive move, fearing their reaction to his overthrowing ingrained assumptions, such as Euclidean flat "homaloid" space assume to be intimately connected to Newtonian absolute, "sensorial" space. He worried controversy could be an obstacle towards acceptance of his theory of indivisible ultimates, his final theory on the density of "everything that is." And he was right. After his death, mathematicians W. Clifford and F. Klein reminded audiences at every International Congress for the Unity of Science, Riemann's colleagues accused him of lack of rigor. [14] True, in his time, and ours, Riemann's "Natural Philosophy" can be a source of puzzlement and incomprehension, especially if we forget its "Lucretian" *inspiration*. Lucretius' cosmos or World exemplified for Riemann why internal inconsistency in physics requires external consistency. Physics must be consistent with knowledge gained by the other sciences of Nature so as to form a Whole, *natural* philosophy explaining the World. But today speedy specialization sacrifices interdisciplinary collaboration. Ingrained assumptions about disciplinary rigor sometimes make us forget that multiplicity of meaning does not require sacrificing simplicity of unity.

Riemann's density concept exemplifies for me how assumptions restricting density to the physical, as if unrelated to other field, could restrict us to unoriginal theories that fail to challenge assumptions concerning intra-disciplinary rigor. Through his research into the foundations of natural science Riemann sought to explain how Lucretius' elementary rarefaction and condensation processes explain how the seeds of everything, living and nonliving, form the whole we call world. Riemann identified the Greek four basic building blocks, earth, air, fire, and water are with four "ultimately condensed [*Druckverhältnisse*] organic elements" eerily reminiscent of a fourfold DNA sequence [2]. Even these can hypothetically be condensed and then, as prime numbers, decomposed into another kind of "primes," in turn re-composed in insightful notes ranging from astrobiology to philosophy of physics.[2] p 544ff. Riemann's intuition rarefied into a spectrum ranging from elementary arithmetic to obscure philosophy. Indeed, just as Kelvin explored density by submerging and extracting an "atomic mass" in and out of a compressed volume to forgotten speculations by Descartes, who imaginatively submerged and extracted a sponge in and out the depths of the sea.

Decades before Einstein and Planck developed their theory of quanta, Riemann defined elements “distinguished by some characteristic feature or boundary as *Quanta*, for we compare them quantitatively in the discrete, numerical, temporal, case, by counting, and in the continuous, spatial case, by measuring.” [2] p306. Riemann’s definition, from “On the hypotheses serving as Foundation for Geometry,” integrates Descartes’ *Geometria* and *Principia Philosophiae*. Riemann read in Descartes:

Some say that rarefaction [and condensation] is the result of the augmentation [and diminution] of the Quantity of body, rather than to explain it on the principle exemplified in the case of a sponge [in the sea]. ...[But] Quantity differs from extended substance, and number from what is numbered, not in reality but merely in our thought; so that, for example, we may consider the whole nature of a corporeal substance which is comprised in a space of ten feet, although we do not attend to this measure of ten feet, for the obvious reason that the thing conceived is of the same nature in any part of that space as in the whole; and, on the other hand, we can conceive the number ten, as also a continuous quantity of ten feet, without thinking of this determinate substance, because the concept of the number ten is manifestly the same whether we consider a number of ten feet or ten of anything else [15] p29

Descartes’ Quantity concept illustrated for Riemann a primordial, cosmological principle. A sponge immersed in water expands and one out of water contracts. Expansive rarefaction is to continuity (Space) what contracting condensation is to discreteness (Number). No matter how much a sponge submerged in water stretches, expands itself, like an inflated balloon, by some sort of spatial rarefaction, or snaps back, contracts itself, like a deflated one, through a reverse process of condensation, it remains, according to Descartes, and Riemann, *one* discrete sponge. Riemann expands the concept of the number one, Unity, into the concept of the number ten, or any other number, thus enabling Quantification. Quantification, as a numerical method, enable in turn our one ‘physical’ act of density measurement. And, as we will see now in the conclusion, for Riemann all Quantification act becomes condensed, simplified, and unified, not only as a sponge within a sea of general Gravitation but also as, let’s say, the often wished-for final theory equation so compressible that it fits on a T-shirt.



4. “Are there “meta”-assumptions or criteria (e.g. ‘utility’, ‘simplicity’, or ‘beauty’) that can or should underlie some current ‘fundamental’ assumptions?”

Riemann clarifies our opening definition of Natural Science whenever he questions scientific foundations throughout his unpublished writings (200+ pages!) on *Naturphilosophie*. Posthumously published in 1871, they reveal Riemann’s final thoughts on meta- assumptions. For example, he claims

[t]he Explanation of Nature requires the clarification of two elements: “1) The [Newtonian] Laws of Motion of “point-mass” substances that must be assumed in order to explain phenomena. 2) The causes which explains these motions. The first task is mathematical one; the second, a *metaphysical* one (italics mine).” [2] p 566

Lucretian-Cartesian prime numbers, later Quanta and/or Knots (*Knottenpunkte*), now point-masses (*Stoffpunkte*), become foundational for Riemann’s questioning of Newtonian assumptions, In order to clarify these physical assumptions he needs *meta-* physical assumptions, assumptions *beyond* physics. For such future Science’s task: “to penetrate inner Nature, to go beyond the foundations of astronomy and physics as laid down by Newton... [even if] these speculations have no immediate practical utility” [2] p 560. So we see he assumes here Newton’s First Law of Motion, the Law of Inertia, and extends it; inertia impelling Riemann in his path, compelling him to leave behind the secondary utility requirement, exchanging it with the simplicity required by a truly general (*allgemeinen*) gravitation theory. Such a move cannot be extricated from a shock/density wave’s movement, for through it natural science reaches an even greater density, an infinite density of *meaning*.

What aspect of Riemann’s legacy for us today can we exemplify with an expanded density concept? We know that, decades before Maxwell, Riemann sought a key to unify electromagnetic phenomena in a single expression encompassing infinite variations of pressure and density [10] p lxviii. But, as divers know, fluid density and water pressure increase proportionally to sea depth. Riemann knew this. He studied deeply the mechanics of everything from celestial gravitation to the “electric fluid” postulated by Benjamin Franklin after his experiments involving a kite and a lightning storm. A collaborator recalls the latter as a crucial element of what became Riemann’s “infinitely dense” medium [4] p828ff. I will expand what his collaborator called infinite density and call it *meta-density*, for Riemann’s much more ambitious goal was not Maxwell’s electromagnetic theory but what he called a “mathematical theory complete in itself, preceding from the elementary laws without having to distinguish between whether we are concerned with gravitation, electricity, magnetism, or heat.” This leads him to seeks a density expression not only of “general gravity’s action” but even the action of an observing “soul.”

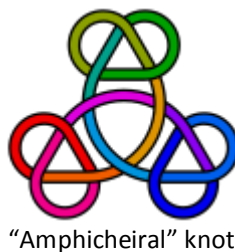
Riemann speculated that prime elements as point-masses become infinitely dense points. These would move towards infinity through the processes of rarefaction, physical compression (i.e. of a compressible gas) and scientific expression (i.e. of an equation describing compression). He concludes that

“[t]he foundation of Newton Principia’s motion laws for [point-masses] must be found in their inner state. We infer their properties by observing our own inner state... our soul, under which every one of its act something permanent lies as foundation...From such hypothesis we can infer general gravity’s action. [2] p 560 ⁷

We can clarify Riemann with his own example, seeing red. Just as “the sight of a red surface the mind-masses, begotten in a multitude of distinct primitive fibers, combine into one single compact spirit-mass, which appears at the same instant in our thought, so the spirit-masses combine into a *Gesammteindruck*.” [2] p 523 So just as sense perceptions results in a single whole observation about density and/or pressure (*Druck*), many scientific observations unite in a *Gesammteindruck*, a simple, single, whole general expression. One process encompasses all of the above from elements to minerals, plants, and beyond to the “soul-filled stars” and the whole cosmos. [2] Humans add just another “spiritual” element, one observation able to recognize the foundations that underlie nature, able to recognize that when simplicity compresses the harmonious whole, natural science reveals *beauty*.

Immersed in the depths of scientific beauty, Riemann endured much pressure and penury, indigence and illness.[2] 821ff. Yet, he also experienced joy. He spent his last days in Italy surrounded family, friends and frescoes, which inspired him to write that a future scientist ought to create theories of “ineffable sublime” beauty like past “great poet[s] painted, with prophetic enthusiasm” [2] p515 Scientist/poets trace Nature’s *rhythm* via Number (Gk. (a)*rythmos*). Like his cherished Greeks, Riemann believed that the beauty of the whole lies in the harmony of its parts. So I leave you with the prophetic poem Maxwell wrote on his own last days (1878). [16] It encompasses our discussion’s elements within a rhythmic harmony. Maxwell prophesies the sublimation of the “philosophical theology” of Kelvin and Tait, within the mathematical physics of Klein and Clifford, best interpreters of Bernhard Riemann.

*My soul's an amphicheiral knot
Upon a liquid vortex wrought
By Intellect in the Unseen residing,
While thou dost like a convict sit
With marlinspike untwisting it
Only to find my knottiness abiding,
Since all the tools for my untying
In four-dimensioned space are lying,*



*Where playful fancy intersperses
Whole avenues of universes,
Where Klein and Clifford
fill the void
With one unbounded,
finite homaloid,
Whereby the infinite
is hopelessly destroyed.*

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TECHNICAL NOTES

¹ A more technical statement of Riemann's Hypothesis is as follows. The Riemann zeta function is defined, for $\Re(s) > 1$ by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s},$$

Riemann's hypothesis states that the nontrivial zeros of the Riemann zeta function lie on the line $\Re(s) = \frac{1}{2}$.

² *Mathematical (numerical) density*. Let U be some specific (infinite) collection of objects U , together with a choice of size function $s : U \rightarrow \mathbf{R}_{\geq 0}$, where a size function means that for every $X \in \mathbf{R}_{\geq 0}$, the number of objects in U of size less than or equal to X is finite. Property P occurs with density r in U if:

$$\lim_{X \rightarrow \infty} \frac{\# \{u \in U \mid u \text{ has property } P \text{ and } s(u) \leq X\}}{\# \{u \in U \mid s(u) \leq X\}} = r$$

Physical (relative) density, or specific gravity. Define density ρ as mass m divided by volume V . $\rho = \frac{m}{V}$

Let ρ_{gas} be gas density and let M be molar mass. Then, for relative density RD and specific gravity SG , measured in respect to air:

$$RD = SG = \frac{\rho_{gas}}{\rho_{air}} \approx \frac{M_{gas}}{M_{air}}$$

³ Riemann reformulates the Paris Academy's question as "What must be the properties of a body that determine its conductivity and caloric distribution so that a linear system, remaining isotherm, exists?" Beginning at the general foundations he then moves towards a solution of the specific, given, problem. "If u denotes the temperature, at time t , at point (x_1, x_2, x_3) , the general (*generalem*) equation, according to which the function u varies, takes the form

$$\begin{aligned} & \frac{\partial}{\partial x_1} \left(a_{1,1} \frac{\partial u}{\partial x_1} + a_{1,2} \frac{\partial u}{\partial x_2} + a_{1,3} \frac{\partial u}{\partial x_3} \right) \\ & + \frac{\partial}{\partial x_2} \left(a_{2,1} \frac{\partial u}{\partial x_1} + a_{2,2} \frac{\partial u}{\partial x_2} + a_{2,3} \frac{\partial u}{\partial x_3} \right) \\ & + \frac{\partial}{\partial x_3} \left(a_{3,1} \frac{\partial u}{\partial x_1} + a_{3,2} \frac{\partial u}{\partial x_2} + a_{3,3} \frac{\partial u}{\partial x_3} \right) = h \frac{\partial u}{\partial x} \end{aligned}$$

with quantity a denoting resulting conductivities, h denoting specific heat per unit volume, or the product of specific heat and the given density..."

⁴ *Et his principiis via sternitur ad majora*

⁵ *Et ignem regunt numeri. Nec meadona tibi studio disperta fidelis intellecta prius quam sint, contemta relinquo.*

⁶ *Nec meadona tibi studio disperta fidelis intellecta prius quam sint, contemta relinquo.* Modifications mine.

⁷ He expresses both atomic inner space and intuitive observational space through a “function P of locations such that

$$\frac{1}{4\pi} \int \frac{\partial P}{\partial p} dS$$

expresses the ponderable mass of the inner state of a closed surface S.” He speculatively assumes “a space-filling substance, a homogeneous incompressible fluid without inertia,” which I would identify as predecessor for the concept of “ether.”