

Philosophical Foundations of Mathematical Universe Hypothesis Using Immanuel Kant

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1 Introduction

"Since in every doctrine of nature only so much science proper is to be found as there is a priori cognition in it, a doctrine of nature will contain only so much science proper as there is mathematics in it" - Immanuel Kant (Metaphysical Foundations of Natural Science, 1786)

All our understanding about the world originates in our reason characterized by the processes in our brain. Therefore a theory of everything which is currently sought in physics and which would explain fundamental forces, particles, space and time cannot leave questions of other fundamental sciences unanswered. Theory of Everything should synthesize philosophy, cognitive science, natural language, information theory, mathematics and physics into a single framework or system of our reason itself. In this essay, however, we will briefly investigate what the framework of our reason has to teach us about the nature of mathematics, how mathematics relates to the physical world and why we find ourselves in the world described by mathematics.

Human reason seems to be superior being able to pierce through the objects of sense experience and understand how they are constituted and what laws govern them. Moreover, it can express those laws in equations of mathematics which seems to relate to the objects *a priori*. What our reason has to teach us about the nature of a priori propositions of mathematics? Where is this realm of mathematical objects – in human mind (human brain) or do they exist in the Universe in general independent of us? How our reason gets to understand the objects of experience?

Some theories in philosophy of science such as ontic structural realism (OSR) argue that the world might have an underlying invariant mathematical structure which is retained under theory change and paradigm shift. Physicist Max Tegmark argues that the Universe is mathematical structure which supports self-aware substructures who perceive themselves as existing in physically "real" world. Noam Chomsky argues that all our natural languages share the same underlying structure – universal grammar, while Steven Pinker argues that language is the window into the human mind. Philosophers such as Kant, Hegel and Wittgenstein held logic to be fundamental.

What they all have in common? They seem to impose that human mind operates on some kind of invariant structure (matrix, network or framework) on which all our cognition is based. This invariant cognitive framework lies at the foundation of our thinking, logic, mathematics, natural language and organization of sense-data (experience). The idea about such fundamental and universal framework can be traced back to Leibniz' *characteristica universalis* and *calculus ratiocinator*.

How this cognitive framework might look like? Let's assume that human mind works as a quantum computer or cellular automaton. The phenomenal world in space and time is presented to our mind like computers output video games to the screen so we perceive ourselves in phenomenal empirically real world. Then we have to look for a grid or framework of cells on which our mind operates. In a search for such cognitive framework we would make our efforts much easier if we first knew the

answers to these questions: is it possible for our reason to know everything about the physical world or just a part of it and why? How science is possible? How we discover and understand things about the physical world? That is, we must understand how we understand things in general. We must understand how our reason works when we think, know things or do mathematics.

These epistemological questions are not new and some of them were investigated by Immanuel Kant in his work "Critique of Pure Reason" (1781) (CPR). David Hume had shown the importance of experience through the senses but he could not explain the nature of *a priori* cognition (e.g. mathematics) and establish it on the firm basis. Kant defends a priori knowledge of the Universe from Hume's skepticism. Kant asks *how are synthetic a priori judgments possible?* According to Kant, pure mathematics, pure natural science (theoretical physics) and metaphysics (scientific metaphysics) are based on *synthetic a priori* propositions. Kant investigates how synthetic a priori judgments are possible and what are the limits of our knowledge. In order to do so he argues about the necessary *a priori* conditions in our mind to make experience in general possible. Kant calls those immanent necessary conditions of experience *transcendental*, not to confuse with *transcendent* which means beyond experience and unknowable.

Kant takes the transcendental idealist position that it is our mind which structures and organizes our perceived reality (Kant's Copernican Revolution in philosophy). He argues that the phenomenal world in space and time is the product of an underlying framework which makes our experience possible – the original synthetic unity of apperception (synthetic unity of self-consciousness). In some sense it is similar to Platonic realm of forms or Wilczek's quantum grid. The constitution of our senses and the way that they provide data makes nature possible materially as a totality of appearances in space and time. The senses give the content of our knowledge. The constitution of our understanding makes nature possible formally, as a totality of rules that regulate appearances in order for them to be thought of as connected in one consciousness. According to Kant, the necessary laws of nature (such as causality) that we seem to discover in perceived objects in space and time have actually been derived from our own understanding. That is, the understanding prescribes laws to nature. Nature and the possibility of experience in general are the same.

Kant's starting point is self-conscious subject investigating the nature of his experience. He argues that all our experience must be such so it can become self-conscious experience: "it must be possible for the 'I think' to accompany all my representations; for otherwise something would be represented in me which could not be thought at all, and that is equivalent to saying that the representation would be impossible, or at least would be nothing to me." (B131).

This is where we can use Kant's transcendental philosophy and his architectonic system of reason to model our cognitive framework. This framework is the original synthetic unity of apperception – the logico-mathematical framework which makes synthetic a priori judgments possible. Similarly to Tegmark's Mathematical Universe Hypothesis, this framework is such so the transcendental subject (the 'I' - the self) is self-aware and perceives itself in empirically real phenomenal world. Kant had explained quite well why the Universe is described by mathematics and how we find ourselves in such a Universe 200 years before Tegmark has even raised this idea. The problem is that Kant's philosophy is quite hard to understand and requires a lot of time and effort. Nevertheless, it has a lot to teach us.

2 The conditions of experience

Let's give a very brief summary of the necessary conditions of experience which Kant gives in CPR. For those who are not familiar with Kant's construction of corporeal nature, we found an essay "The Unity of Kant's Thought in His Philosophy of Corporeal Nature" by James W. Ellington very useful and clarifying. It can be found in his translation of "Metaphysical Foundations of Natural Science" (Indianapolis: Bobbs-Merrill, 1970) and "Philosophy of Material Nature" (Indianapolis: Hackett Publishing Co., 1985).

We need to look for a structure made of cells which would be characterized by the necessary conditions of experience given by Kant in CPR:

The faculty of sensibility:

"If the *receptivity* of our mind, its power of receiving representations in so far as it is in any wise affected, is to be entitled sensibility, then the mind's power of producing representations from itself, the *spontaneity* of knowledge, should be called the understanding. [...] Without sensibility no object would be given to us, without understanding no object would be thought." (A51)

Through the faculty of sensibility we receive intuitions (the sense-data). Space and time are pure forms of sensible intuition. Space is the form of all appearances of outer sense (A24, A26). Time is the form of inner sense (A33) and is the form of all appearances whatsoever (A34). Therefore a unit cell of our framework must have two forms of receptivity – inner (time) and outer (space):

- Unit cell must be a unit sensor characterized as pure receptivity for sensation. Time as form of inner sense must be present inside every cell in the framework since time is a form of all appearances whatsoever.
- Space as form of outer sense must be outer relations of a cell to neighbouring cells and through these relations (channels) sensory information must move (flow) within the framework.

The faculty of understanding:

"We can reduce all acts of the understanding to judgments, and the *understanding* may therefore be represented as a *faculty of judgment*. For, as stated above, the understanding is a *faculty of thought*. [...] The functions of the understanding can, therefore, be discovered if we can give an exhaustive statement of the functions of unity in judgments." (B94)

"The synthetic unity of apperception is therefore that highest point, to which we must ascribe all employment of the understanding, even the whole of logic, and conformably therewith, transcendental philosophy. Indeed this faculty of apperception is the understanding itself." (B134)

"*The principle of contradiction* must therefore be recognised as being the universal and completely sufficient *principle of all analytic knowledge*" (B191)

"The highest principle of all synthetic judgments is therefore this: every object stands under the necessary conditions of synthetic unity of the manifold of intuition in a possible experience." (B197)

The synthetic unity of apperception is the objective condition of all knowledge (B138). The understanding is the *faculty of thought* and the *faculty of knowledge*. Categories are the synthetic functions of the understanding (forms of thought) which synthesize (combine) cells together.

- The framework must be the model of the original synthetic unity of apperception where the cells must be connected with each other (synthesized) by the logical functions of judgment (forms of thought), i.e. by transcendental logic. Unit cell must be binary (with two contradictory states) expressing the principle of analytic knowledge.

The imagination and schematism:

“Inasmuch as its synthesis is an expression of spontaneity, which is determinative and not, like sense, determinable merely, and which is therefore able to determine sense *a priori* in respect of its form in accordance with the unity of apperception, imagination is to that extent a faculty which determines the sensibility *a priori*; and its synthesis of intuitions, conforming as it does to the *categories*, must be the transcendental synthesis of *imagination*. This synthesis is an action of the understanding on the sensibility; and is its first application – and thereby the ground of all its other applications – to the objects of our possible intuition.” (B152)

”An application of the category to appearances becomes possible by means of the *transcendental determination of time*, which, as the *schema* of the concepts of understanding, mediates the subsumption of the appearances under the category.” (B178)

“Whatever the origin of our representations, whether they are due to the influence of outer things, or are produced through inner causes, whether they arise *a priori*, or being appearances have an empirical origin, they must all, as modifications of the mind, belong to inner sense. All our knowledge is thus finally subject to time, the formal condition of inner sense. In it they must all be ordered, connected, and brought into relation.” (A99)

“The most fundamental and original act of imagination is the production of time-consciousness.” (Kang 83)

According to Kant, the schematization and synthesis of the spatio-temporal manifold of intuition (the framework) is performed by the productive imagination. Synthesis is the work of spontaneity. Therefore the productive imagination must synthesize (combine, integrate) the framework and present phenomena to the mind. In other words, it must “read” the framework and output phenomena (experience).

- Unit cell must be the transcendental determination of time - a *transcendental clock* with the time parameter. Unit cell must be the purest *schema* of imagination homogeneous with pure receptivity for sensation, with the pure act of spontaneity (the “I think” – pure apperception) and with the pure transcendental imagination. Every cell must be characterized by spontaneity.

The faculty of reason:

“If we consider in its whole range the knowledge obtained for us by the understanding, we find that what is peculiarly distinctive of reason in its attitude to this body of knowledge, is that it prescribes and seeks to achieve its *systematisation*” (B673)

- Kant argues that the ideas of reason provide us with the greatest possible systematic unity. Therefore the framework (the understanding) must have greatest possible systematic unity and must be seen as a unitary architectonic system.

3 The model of the original synthetic unity of apperception

The only framework which meet the characteristics given above and by Kant in CPR is the structure made of unit Eulerian circles where the center of each circle is on the circumference of six surrounding circles (this structure is also known as flower of life):

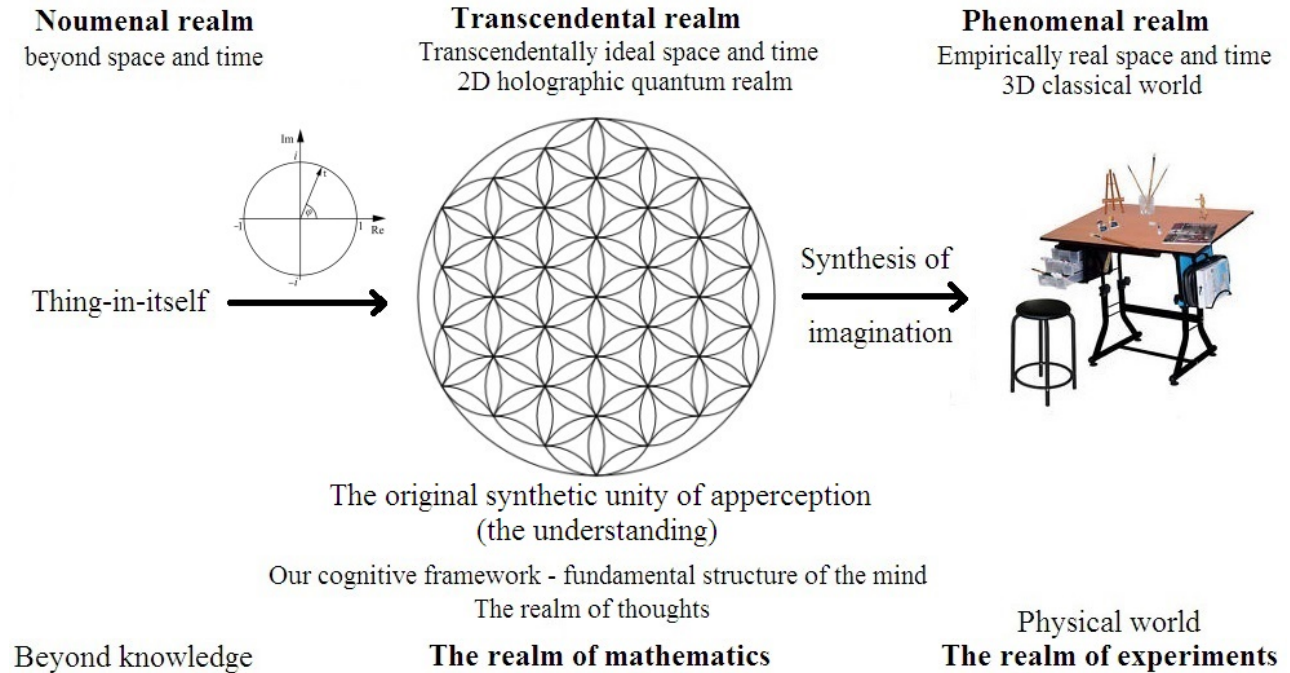


Figure 1: The model

3.1 Why unit circle is the fundamental element of the universe?

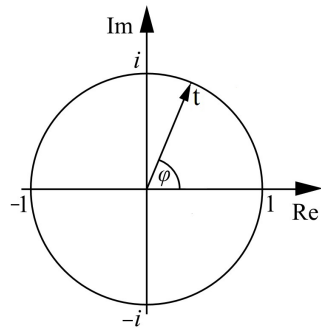


Figure 2: Unit cell

The idea that unit circle is fundamental element of the Universe is not new. It can be found in pythagoreanism and neoplatonism. Plato in "Timaeus" argues that the Universe started as a spinning circle. Leibniz' monad was referred to as *logical sphere*. Why unit Eulerian circle should be fundamental to our Universe?

Kant argued that reason is characterized by the three ideas which provide the *greatest possible systematic unity*. Hegel who developed Kant's philosophy further equates Reason with "the Idea" (equivalent to pure consciousness). Unit circle is the expression of pure consciousness (pure Reason or "the Idea") in our imagination:

- Unit cell expresses *pure systematic unity*: unit cell is pure passive receptivity for sense perception (a unit sensor), pure active spontaneity generating the "I think" (pure apperception) and pure productive imagination (i). *Spontaneity* is the circular motion of the time parameter (arrow) around the boundary of the circle which creates simple harmonic vibration of the cell and generates the representation "I think". Unit cell is the transcendental determination of time (*transcendental clock*) – the purest *schema* of imagination homogeneous with the pure receptivity for sensation and the pure spontaneity of thought (pure apperception). $e^{i\pi} + 1 = 0$ (Euler's identity) expresses the idea of pure systematic unity mathematically.

- Imaginary unit (i) is related to the faculty of transcendental imagination.
- We consider natural number 1 (unit cell) as the expression of reason’s idea of the whole and unity (the Universe). It is pure identity.
- The cell is binary and contains two contradictory (opposite) states (1, -1) which is the purest expression of the dialectic nature of reason. Cell is the primordial case of the identity of the opposites.
- Frege defined 0 as the number of the concept not being self-identical, and that 0 thereby is identified with the extension of all concepts which fail to be exemplified. That is, 0 is derived from the two opposite (not identical) states in the cell.
- We equate Absolute Infinite with “the Idea”. We consider unit cell to be the expression of the Absolute Infinite. The motion of the time parameter around the circle is the expression of infinity. It is related to point at infinity in mathematics. “So Kant links the concept of infinity with that of a thing in itself. Leibniz once said: ‘The true infinite exists, strictly speaking, only in the absolute, which is anterior to all composition, and is not formed by the addition of parts’; and Kant seems to echo this idea that the province of ‘the true infinite’ is the realm of unempirical monads or things in themselves.” (Bennett 135) “The infinite in its simple notion can, in the first place, be regarded as a fresh definition of the Absolute” (Hegel, Science of Logic, §270)

3.2 The framework is logical structure

Unit cell is pure apperception (pure consciousness). The framework (the flower of life) shown in Fig. 1 is *the original synthetic unity of apperception*. The logical forms of thought *synthesize* (connect) cells together into a unitary framework forming the *synthetic* unity of consciousness. Various forms of intersection (relation) of overlapping cells express logical forms (using Venn diagrams): the logical functions of judgment, syllogisms, Boolean algebra and set operations. How are the forms of syllogisms, Boolean and set operations represented using overlapping circles (Venn diagrams) is obvious and requires no further discussion. The framework is the logical universe or logical space. Cells can be considered as sets and the framework as a model or the universe of set theory. For example, it would be interesting to consider this framework as a Von Neumann universe under ZFC, etc.

The framework is one quantum entangled network which forms *the unity of consciousness*. The framework is holographic because of the analytic unity of consciousness – the “I” in all its parts and acts throughout the framework is the same. The analytic unity of consciousness simply means that all my representations are my representations. However, “the *analytic* unity of apperception is possible only under the presupposition of a certain *synthetic* unity” (B133).

We acknowledge the works of logic after Kant and do not stick to Kant’s version of logic. We must study how modern logic (such as Frege and Russell) can be modelled on this framework. Here we lay a fundamental claim that the framework is completely logical structure which lies at the foundation of logic. The Universe is pan-logical. Metaphysics is ultimately logic. As Hegel puts it: “metaphysics is nothing else but the entire range of the universal determinations of thought, as it were, the *diamond net* into which everything is brought and thereby first made intelligible.” (Hegel, Philosophy of Nature).

3.3 The framework within which our thoughts are formed

This structure (the flower of life) is the framework of the faculty of understanding which is the *faculty of thought*. It is the framework within which our thoughts are formed. The understanding is the ability (performed by active spontaneity) to synthesize (combine) cells using the forms of thought which are the synthetic functions of unity. Our thinking is governed by Hegelian dialectic – unity of the opposites. Synthesis is performed when the time parameter moves around the circle. Different

rates of time parameters of cells create different rates of vibrations of cells. Our thoughts are waves in this structure. Unit cell is the basic building block – logical atom. Logic, mathematics and natural language are based on this structure but in this essay we briefly discuss only the nature of mathematics.

3.4 How the 3D physical world appears after the synthesis of the 2D framework

This framework is transcendently ideal, i.e. it is not an object of experience but is a necessary condition of experience. We see that *transcendentally ideal space* is 2D framework where each cell has 6 outer relations – 6 ‘curled-up’ transcendental dimensions which we do not experience (like in string theory). These are energy (or information) transfer channels. This six-dimensionality of transcendently ideal space and the spatial relations by an angle of 60° arise from the logical forms of the understanding. That is, *transcendentally ideal space* is defined by logic and is invariant and Euclidean.

Empirically real space which we experience has 3 dimensions which appear to us after the manifold of intuition (the framework) is synthesized by the transcendental productive imagination. Transcendental synthesis of productive imagination subsumes (synthesizes or integrates) the framework. The cell (unit circle – qubit) encodes information on its boundary because the time parameter moves around it. If we add unit circles in 1D we get a cylinder whose surface is 2D. If we do that in 2D framework we get a 3D space. Curvature of empirically real space depends on the rate of time parameter – the rate of synthesis (or information-processing) which defines the energy (‘mass’) of sensation.

Unit cell is a unit sensor (like a unit pixel on the computer screen). The framework (the screen) forms the *unity of consciousness*. The framework at each moment represents the *state of affairs*, a ‘now’. Time is not fundamental and time flow appears as states of cells change. Our soul (pure reason or pure consciousness) is outside space and time and stares at this screen. The synthesis of the framework (the manifold of sensible intuition) is performed by the transcendental productive imagination which produces experience (phenomena). In this process apperception makes cells acquire definite states (wavefunction collapse) and our experience (empirical consciousness) in empirically real space and time emerges. This way we perceive ourselves in empirically real 3D world.

3.5 All synthetic a priori judgments originate within the invariant framework of the original synthetic unity of apperception

Synthetic a priori judgments describe nature a priori. According to Kant, mathematics, pure natural science (theoretical physics) and metaphysics (scientific metaphysics) are grounded on synthetic a priori judgments. Kant gives the famous example of a synthetic a priori proposition: $7 + 5 = 12$.

A particular pattern (schema) of cells is an algebraic structure (a set) corresponding to a concept. Analytic propositions are those where the predicate (a set of cells) is a subset of the subject (a larger set of cells). This produces no new knowledge. Synthetic a priori propositions are those where the predicate is outside the set of the subject and requires a [synthetic] connection. This produces new knowledge. This way the original synthetic unity of apperception makes synthetic a priori propositions possible and is the invariant framework of such knowledge. All mathematical objects are constructed within this framework.

3.6 The framework is logico-mathematical structure where all mathematical objects are constructed

”Mathematical knowledge is the knowledge gained by reason from the *construction* of concepts. To *construct* a concept means to exhibit *a priori* the intuition which corresponds to the concept. For the construction of a concept we therefore need a *non-empirical* intuition.” (B741)

”The only intuition that is given *a priori* is that of the mere form of appearances, space and time.” (B748)

”Time and space are ... two sources of knowledge, from which bodies of *a priori* synthetic knowledge can be derived. (Pure mathematics is a brilliant example of such knowledge ...)” (A38-39)

”[Mathematics] being able to realise all its concepts in intuitions, which it can provide *a priori*, and by which it becomes, so to speak, master of nature” (B753)

According to Kant, mathematics constructs all its objects in pure intuition of space and time. Pure intuition means absent of sensation and it contains only the form under which something is intuited. That is, mathematics studies this framework of cells (our cognitive framework) in general – it studies the structure in which the phenomenal world appears. For example, $7 + 5 = 12$ is true since it follows from the synthesis of cells in pure intuition of space and time, that is it follows from our cognitive framework which is invariant logico-mathematical structure. All true mathematical statements state something true about this framework and this way mathematics describes the world *a priori*. This way mathematics is the master of nature. Kant claims that construction of numbers happens in time (pure form of inner sense), while of objects of geometry in space (pure form of outer sense). It is interesting in the context of cognitive neuroscience, that is how various mathematical cognition is represented by our brain?

Wittgenstein stated: ”The logic of the world, which is shown in tautologies by the propositions of logic, is shown in equations by mathematics.” (Tractatus Logico-Philosophicus, 6.22) Contrary to Kant’s intuitionism, such thinkers as Dedekind, Frege and Russell belong to logicist school which holds mathematics to be an extension of logic. It is necessary to study how mathematical logic and the schools of formalism and logicism in philosophy of mathematics can be modeled on this cognitive framework (the fundamental structure of our mind). This framework is logical structure where cells are connected by the logical forms (laws of thought). As mentioned earlier, cells can be considered as sets. It is logic which defines the structure of space and time, while space and time are pure forms of our intuition. In ”The Nature and Meaning of Numbers” (1887) Dedekind states:

”In speaking of arithmetic (algebra, analysis) as a part of logic I mean to imply that I consider the number-concept an immediate result from the laws of thought ... numbers are free creations of the human mind ... [and] only through the purely logical process of building up the science of numbers ... are we prepared accurately to investigate our notions of space and time by bringing them into relation with this number-domain created in our mind.”

3.7 How does mathematics relate to the world?

”The synthesis of spaces and times, being a synthesis of the essential forms of all intuition, is what makes possible the apprehension of appearance, and consequently every outer experience and all knowledge of the objects of such experience. Whatever pure mathematics establishes in regard to the synthesis of the form of apprehension is also necessarily valid of the objects apprehended.” (A166)

Mathematics applies not directly to phenomena but to the pure forms of space and time (pure forms of intuition) in which all phenomena appear. For example, there is no perfect circle in the phenomenal world. This is the nature of the split between “The Unreasonable Effectiveness of Math-

ematics” and “The Unreasonable Ineffectiveness of Mathematics”. Pure natural science (theoretical physics) is also grounded on synthetic a priori judgments. This is why physics uses mathematics. We perform the experiments in the phenomenal realm, while we reason about them and express the laws of physics in mathematical equations in our thoughts, that is in the realm of the mind (Platonic/transcendental/quantum realm). Phenomenal world is an “output” of this logico-mathematical realm of the mind.

The uncertainty principle, for example, is observed in the physical world, however it has mathematical basis – the uncertainty pairs are Fourier transform pairs. Quantum field theory and string theory are very mathematical because they study this framework of cells (which is in quantum realm).

Unit cell (unit circle) is the prototype for natural units. Cells are separated by Planck length. Unit cell acts as analog (infinite, noumena) to digital (finite, phenomena) converter and performs quantization. There are some suggestions to derive physical constants from mathematical constants using geometric relationships (Makinson, 2011). This way we could understand the connections between the key equations of physics and the nature of them.

To sum up, we do live in the universe where all phenomena in space and time are subject to mathematics. The connection between physics and mathematics is not mysterious – mathematics provides the framework for the physical world (the world of appearances) where the forces of physics move things (i.e. energy) around. However, it is not mathematics which is fundamental. It is logic. There are some suggestions to derive the laws of physics from the laws of logic (e.g. Zizzi). Kant again has here a lot to teach us. In his “Transition to Physics” programme he already tried to derive the forces of physics from logic. However, this would require a separate essay to discuss.

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