Planck units and wave-particle duality

A simple Planck unit theory by deduction

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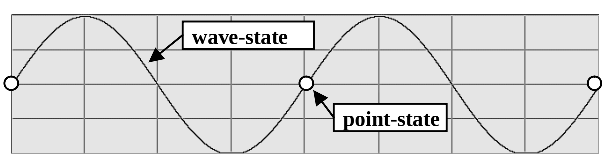
The essay topic asks if reality is digital or analog. It may be that reality oscillates between both.

There is no inductive method that could lead to the fundamental concepts of physics. Failure to understand this fact constituted the basic philosophical error of so many investigators of the nineteenth century.
-Albert Einstein[1]

Wave-particle duality[2] implies a wave-state and a non wave-state. If we may re-define these 2 states in terms of a simple oscillation instead of a duality, then it becomes possible to deductively construct the parameters for a hypothetical Planck unit[3] universe theory whereby the frequency of discrete (digital) Planck events is regulated by the presence of (analog) particles.

Definition:
For the particular purposes of this essay, wave-particle duality is depicted as an oscillation between an analog wave-state and a digital point-state. At the completion of 1 period (1 particle wavelength), it is presumed that the wave-state contracts (collapses) into the point-state before once again expanding into the wave-state… and so on, ad infinitum.

A simple particle (x-axis = time, y-axis = amplitude) ;



If the wave-state curve can be drawn as a series of theoretical dots (joined together over time), then no individual dot will give us information about the particle; we need to make a measurement of that particle over time… in other words, time is 1 of the dimensions of the particle wave-state and so must be included in our experiments and our measurements. We can infer the wave-nature of a particle, but we cannot physically describe that wave at or during any discrete moment (unit) of time.

The point-state however has no time interval; or rather we could define the point-state as occupying the smallest possible unit of measurable time. The point-state is analogous to a single frame of a movie with the wave-state as the interval between frames. If this single unit of time can be defined as a unit of Planck time[4]; then the point-state becomes a Planck state.

1. The wave-state is continuous.
2. Planck time is defined as the duration of the point-state.

The total energy of the particle system does not change; the point-state is therefore analogous to a particle potential energy PE state, the wave-state to the oscillation between PE and KE (kinetic energy), i.e. a form of particle pendulum motion.

Gravity:
If we may presume that gravity is an interaction between particles that are simultaneously (the same unit of Planck time) in the point-state and that the electric force occurs between particles that are simultaneously in the wave-state, then the probability of gravity (the point-state) occurring between any 2 electrons chosen at random for any discrete moment of (Planck) time equals $(m_P/m_e) \times (m_P/m_e) = (m_P^2/m_e^2)$. As this is also the ratio between the strong force and gravity, we see that the magnitude of the strong force is equivalent to the magnitude of our gravitational force[5].

Gravity appears weak in our Planck universe only because it seldom occurs, to put it in perspective, for every equivalent unit of time an electron spends in the gravity 'point-state', it spends about m_P/m_e = 23 900 000 000 000 000 000 000 units of time in the electric 'wave-state'.

1. Gravity is a simultaneous point-state to point-state interaction
2. The electric force is a wave-state to wave-state interaction

Mass:
At present, there is no good theory explaining why the masses of particles should be what they are – whereas 'mass' is a concept intimately bound up with the concept of gravity. Indeed, mass acts uniquely as the 'source' of gravity.
-'In search of Quantum gravity', Roger Penrose[6]

If gravity occurs between point-states, then mass is a function of the point-state. Furthermore, if the point-state is a Planck state and it has mass, then that mass is also a Planck unit; by definition it would be a Planck mass[7] (m_P). The point-state now becomes a micro (Planck) black hole and gravity has become, perhaps not surprisingly, an interaction between black holes.

In summary, for every particle;
1. During the wave-state; particle mass = 0
2. During the point-state; particle mass = Planck mass m_p

A particle has mass or it doesn't have mass, there is no ½ mass. Our Planck universe therefore has only 1 unit of mass, that unit is Planck mass. There are no ½ units of Planck mass. If mass is a function of Planck time (the point-state), then what we define as mass is the average number of units of Planck mass per second.

For example, if a particle wavelength = 12 (11 units of time in the wave-state followed by 1 unit of time in the point-state), then we will measure the mass of that particle as;
$m_P/12 = 2.1767\ 10^{-8}kg/12 = 1.8139\ 10^{-8}kg$.

That particle has mass (i.e. that particle is mass) for only $1/12^{th}$ of a second. In our Planck universe, there is no need for a Higgs boson.

3. An object/particle mass = units Planck mass / units Planck time

Because the wavelength of a proton is less than the wavelength of an electron, the proton point-state will occur more frequently than the point-state of the electron, and the proton will appear to have more mass.

If we add momentum to an electron (increase its velocity) then the wave amplitude will increase and the period will decrease, the point-state will occur more often and the electron will seem to have acquired more mass.

In our Planck universe, relativity becomes relatively simple, for Planck time joins light speed *c* as a universal constant… just as velocity is relative to *c* and mass to Planck mass, so also local time is relative to Planck time.

The dimensions of our universe fade away, leaving us with particles as dimensionless formulas that dictate the frequency of our Planck events. Without particles, our Planck universe would be a pure black-hole, akin to a gigantic computer hard-disk that contains no information. Particles are how this Planck universe would store and manipulate its data.

E=hv:
This equation is correct, but in a Planck universe it is misleading for it implies that the energy of a photon depends on its wavelength. In fact, what it states is that energy is related to the frequency of occurrence of Planck constant (per second). If every photon point-state expresses 1 unit of Planck energy (instead of Planck mass) then a high frequency photon will have more occurrences of Planck energy per second that a low frequency photon. The same principle applies as with Planck mass, there are no ½ units of Planck energy ($E_p = m_P.c^2$);

In summary, for every photon;

1. During the wave-state; photon energy = 0
2. During the point-state; photon energy = Planck energy E_p

As gravity is a point-state interaction, gravity can act on photons whilst they are in the point-state.

This may seem simplistic or even counter-intuitive, that the complexity of our universe may be reduced to a few Planck units… but reality may be even more simplistic, for it may be that there is only 1 fundamental Planck unit; Planck momentum.

An analogy: water in a perfectly spherical container. A drop falls in the centre of the container… a ripple (wave) expands outward; reaches the container wall, then reverses its direction inwards… as that ripple returns to the precise center, the wave (for the briefest of moments) overlaps to form a single motionless point… before continuing its journey outward again.

I conjecture that a particle wave-state 'ripple' is simply momentum; at the 'precise center' it forms a unit of Planck momentum. If units of this Planck momentum interact with each other, then we will interpret (measure) this interaction as gravitational mass. There is no actual requirement for a physical unit of Planck mass.

The formula $E=mc^2$ then becomes a description of different aspects of this momentum. A (Planck) black hole is simply (a unit of Planck) momentum.

Planck force:
When we calculate the gravitational force between the earth and the moon, we need to know the mass (not the size) of each planet and the distance between the centers of each planet. As

we are measuring force, we could propose a Planck force unit ($F_p = E_p/l_p$) to replace G; after all, this is a Planck unit theory. We can then measure force relative to this Planck force.

$$F = \frac{M_{earth} M_{moon} G}{R^2}$$

$$G = \frac{E_p l_p}{m_P^2}$$

$$F_p = \frac{E_p}{l_p}$$

If gravity is an interaction between black holes, then we can replace mass with the black hole Schwarzschild radius[8] (SR), l_p = Planck length…

$$M_{earth} = \frac{1}{2} \frac{SR_{earth} m_P}{l_p}$$

$$M_{moon} = \frac{1}{2} \frac{SR_{moon} m_P}{l_p}$$

$$F = \frac{1}{4} \frac{SR_{earth} SR_{moon} F_p}{R^2}$$

Even in a hypothetical Planck universe, clearly our earth and moon are not black holes, despite the equation above. However if we could see the earth at the Planck unit level, where a Planck universe would operate, it would not resemble a solid dependable planet; instead it would presumably be a mixture of waves and points. We could not even recognize it as a planet (at the Planck level, as mentioned in the introduction, we do not even have particles). In our Planck universe therefore, the above equation could be a valid interpretation.

Bohr and gravity:

In the Hydrogen atom, there are 2 general orbital models. The Schrödinger wave equation, which assumes the electron and proton are waves and the Bohr model[9], which assumes the electron and proton are points (the electron orbits the proton as a planet orbits the sun).

The Bohr model can accurately solve some orbits, but it cannot explain some phenomena and, as it appears to break certain rules, physics tells us that it cannot be correct, and therefore it is not correct. Why it can be used to solve some orbits still remains a mystery.

However, if we have a 'wave-state' and a 'point-state', then we might anticipate a wave model and a point model… and as particles are predominately in the 'wave-state', and as the point model cannot describe the 'wave-state', the point model would naturally be an incomplete model, but it would not necessarily be an incorrect model. There is a distinction.

The Bohr model is a planetary orbit model; it should therefore also apply to planetary orbits. If we replace wavelength with Schwarzschild radius, then λ_{earth} = .00887m and λ_{sun} = 2953.25m. In our Hydrogen atom there are 4 integer 'quantum numbers', the most interesting for us here is *n*, the principal quantum number (for *n* orbits are the simplest circular orbits).

Another interesting number is α, ‘$\alpha \approx 137.036$’ (Sommerfeld fine structure constant[8]). We now need only 2 more numbers (constants); λ and *c*.

Radius:
The radius R of the n-orbits, gravity or electric; R = Bohr radius:

$$R = \alpha\, n^2\, \lambda$$

For a simple 2 body system such as the earth and the moon, or an electron and a proton, we just add the wavelengths (note: this addition of wavelengths is otherwise referred to as the reduced mass of the electron):

$$R = \alpha\, n^2\, (\lambda_1 + \lambda_2)$$

A Hydrogen atom (electron + proton);

$$R_{hydrogen} = \alpha\, n^2\, (\lambda_{electron} + \lambda_{proton})$$

Gravity, λ = Schwarzschild radius.

$$R_{sun\ earth} = \alpha\, n^2\, (\lambda_{sun} + \lambda_{earth})$$

Velocity: (the velocities of the electric and gravity orbits):

$$v_{electric} = \frac{c}{\alpha\, n}$$

$$v_{gravity}{}^2 = \frac{c^2}{2\, \alpha\, n^2}$$

Acceleration:

$$a_{electric} = \frac{c^2}{\alpha^3\, n^4\, \lambda}$$

$$a_{gravity} = \frac{c^2}{2\, \alpha^2\, n^4\, \lambda}$$

Period:

$$T_{electric} = \frac{2\, \pi\, \alpha^2\, n^3\, \lambda}{c}$$

$$T_{gravity}{}^2 = \frac{8\, \pi^2\, \alpha^3\, n^6\, \lambda^2}{c^2}$$

Example; satellite orbiting the earth’s surface:
$\lambda_{earth} \approx .00887$m, $\lambda_{satellite} \approx 0$m, so $\lambda_{earth + satellite} = .00887 + 0$

At n = 2290, R_g = 6374km, a_g = 9.81m/s^2, T_g = 5064.8s and v_g = 7907m/s.
At n = 2291, R_g = 6380km, a_g = 9.79m/s^2, T_g = 5071.4s and v_g = 7904m/s.

Semi-major axis:
For elliptical orbits (between circular *n* orbits), the semi-major axis = R_a can be calculated using the following approximation (with Bohr formulas);

$$R_v = \frac{1}{2}\frac{T\, v_g}{\pi}$$

$$R_a = \frac{2}{3}R_v + \frac{1}{3}R_g$$

Earth's geocentric gravitational constant[10]; μ = **398600441.8**
Schwarzschild radius (μ = SR.c^2 /2); SR = **.00887005608** m
Acceleration of gravity; g = **9.80665**… m/s^2
a^2 = μ / 9.80665; a = **6 375 416.3** m (Kepler[11])
T = 2.π.sqrt(a/9.80665) T = **5066.10055** s (84.44mins)

Using n = 2290, we find a solution for R_a…
$\mathbf{R_g}$ = α.n^2.SR = **6 374 292.7** m
$\mathbf{R_v}$ = (2.π.sqrt($\mathbf{R_a}$ /9.80665))*sqrt(c^2/(2.α.n^2))/(2.π)
$\mathbf{R_a}$ = 2.R_v /3 + R_g /3 $\mathbf{R_a}$ = **6 375 416.3** m (Bohr)

For the planets[12]:

Mercury:	T = 87.9691days * 86400
	a = 57 909 100km (observed)
	k = 57 909 066 (Kepler)
n = 378	$\mathbf{R_a}$ = 57 909 096 (Bohr)
Venus:	T = 224.65
	a = 108 208 930
	k = 108 192 563
n = 517	$\mathbf{R_a}$ = 108 192 563
Earth:	T = 365.25636
	a = 149 597 887
	k = 149 597 724
n = 607	$\mathbf{R_a}$ = 149 597 722

F = ma:

Physics has both inertial mass and gravitational mass, theoretically they should be different, but this difference has never been detected.

If $\lambda_1 = SR_{earth}$, $\lambda_2 = SR_{satellite}$:

$$F = \frac{1}{4}\frac{SR_{earth}\, SR_{satellite}\, F_p}{R^2}$$

$$R = \alpha\, n^2\, (SR_{earth} + SR_{satellite})$$

$$a_{gravity} = \frac{1}{2}\frac{c^2}{\alpha^2\, n^4\, (SR_1 + SR_2)}$$

$$F_{gravity} = \frac{1}{2}\frac{m_P\, SR_{earth}\, SR_{satellite}\, a_{gravity}}{(SR_{earth} + SR_{satellite})\, l_p}$$

If the mass of the satellite is insignificant;

$$F_{gravity} = \frac{1}{2}\frac{m_P\, SR_{satellite}\, a_{gravity}}{l_p}$$

$$F_{gravity} = m_{satellite}\ a_{gravity}$$

F = ma when M » m.
When M = m:

$$F = \frac{1}{4} \frac{SR_M\ SR_m\ F_p}{\alpha^2\ n^4\ (SR_M + SR_m)^2}$$

$$F = \frac{1}{16} \frac{F_p}{\alpha^2\ n^4}$$

M & m may represent 2 stars or 2 grains of sand, nevertheless, if M = m then the force between them depends on the principal quantum number *n*.

The actual mass is does not appear in the equations. If *n* is the same, then the force is the same. Whilst for us, in terms of km or miles, $R = \alpha.n^2.(\lambda_{star} + \lambda_{star})$ » $R = \alpha.n^2.(\lambda_{sand} + \lambda_{sand})$, a Planck universe does not seem to make the same clear distinction.

As n = 1 is the lowest possible orbit, 2 objects (or particles) under the influence only of gravity (or the electric force), cannot collide; i.e. the minimum attainable distance in the absence of an external force would be; $R_{min} = \alpha.1^2.(\lambda_1 + \lambda_2)$.

We could also note that neither gravity nor the electric force can change an orbit, from n=2290 to n=2291 for example. This requires an (i.e. external source) addition/reduction of total system momentum. I will maintain my present orbit around the earth (sitting at my desk) until a graviton (i.e. a tennis ball) hits me, thereby changing my total gravitational momentum and so my orbit. At the Planck level, my desk and I are orbiting the earth. There is no attractive gravitational force pulling me to the earth.

Euler's digital pi[13]:
If *n* represents the number of units of Planck time since the big bang, then the accuracy of the universe π could be increasing as I write this article;

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

Wave-state:
I have defined the point-state as a Planck unit state; whose units; mass, energy, time, length are discrete units with dimensions. The wave-state however is particle dependent; an electron wave-state is clearly different to a proton wave-state, yet they both theoretically comprise that same single unit of Planck momentum. The particle wave-state also has no apparent dimensions; instead it dictates the frequency of those Planck dimensions.
For example; the electron;
mass: '$m_e = m_p$ x electron wave formula'
wavelength: '$\lambda_e = 2.\pi.l_p$ / electron wave formula'

In the submission to the previous fqxi essay contest, I suggested the following as an example of an electron wave formula[14];
Elementary charge: **e = 1.602 176 487 × 10^{-19}** C
Planck time: **t_p = 5.391 24 × 10^{-44}** s
Speed of light: **c = 299 792 458** m/s

Using dimensionless $\mathbf{t_x}$, $\mathbf{e_x}$, $\mathbf{c_x}$ to convert from Planck to SI units;

$$\frac{t_p}{t_x} = \frac{5.3912...e^{-44}s}{5.3912...e^{-44}} = 1s$$

$$\frac{e}{e_x} = \frac{1.6021764...e^{-19}C}{1.6021764...e^{-19}} = 1C$$

$$\frac{c}{c_x} = \frac{299792458m/s}{299792458} = 1m/s$$

The electron wave formula becomes…

$$g_e = \frac{2}{3}\frac{\pi^2}{\alpha^2 e_x c_x}$$

$$electron = 2\, t_x\, g_e^{\;3}$$

This wave formula is constructed from dimensionless Planck level units (Planck time, elementary charge and speed c), where g_e is a dimensionless magnetic monopole (‘$A \times l_p = e \times c$’)[15]. Encoded into this mathematical (geometrical) formula are the attributes and magnitude of the electron. Furthermore, this formula can be used to solve the Rydberg constant; with a 12-13 digit precision it is the most accurate of the fundamental constants.

Using (Q_p = Planck momentum);

α	= 137.035 999 679 (CODATA 2006)
Q_p	= 6.525 666 874 7820
$l_p = 5^7.\alpha.Q_p^{\,4} / 2^4.\,\pi^3.c^5$	= .1616 036 580 1567e-34
$e = \sqrt{(2^{11}.\,\pi^3.l_p^{\,2}.c^4 / \alpha^2.Q_p^{\,3})}$	= .1602 176 494 4601e-18
$h = Q_p.2.\pi.l_p$	= .6626 069 020 9762e-33
$\mu_0 = \alpha.\,Q_p^{\,4} / 2^9.\,\pi^2.l_p.c^5$	= $4.\pi.10^{-7}$ (exact)
$R_\infty = m_e.e^4.\mu_0^{\,2}.c^3 / 8.h^3$	= 10 973 731.568 526
m_e	= .9109 382 228 8078e-30

Using 2, 3, 5^7, π, Q_p, α and c, we have;
R, h, e, m_e, μ_0 with CODATA 2006 precision, and
l_p (G) with CODATA 2002 precision

If we can use Euler’s integer solution for pi, and given that *c* has a (fixed) integer value, then there are only 2 non-integer numbers used in the above formulas; α and Q_p. As Q_p is unit dependent, we could, for example, set M=L=T=1, thereby Q_p=1.

CODATA 2006 values:
R = 10 973 731.568 527(73) [16]
h = 6.626 068 96(33) e-34 [17]
α = 137.035 999 679(94) [18]
l_p = 1.616 252(81) e-35 [19]
e = 1.602 176 487(40) e-19 [20]
m_e = 9.109 382 15(45) e-31 [21]
$\mu 0 = 4\pi/10000000$ [22]

Note: If $Q_p = 2.\pi.Q^2$ then we can define elementary charge (the wave-state) in terms of a charged (square root) Planck momentum[14].

$$e = \frac{16\ l_p\ c^2}{\alpha\ Q^3}$$

In terms of momentum Q (or Planck momentum Q_p), α and c…

$$\alpha = \frac{2h}{\mu_0 e^2 c} = 2\ 2\pi Q^2 2\pi l_p\ \frac{32 l_p c^5}{\pi^2 \alpha Q^8}\ \frac{\alpha^2 Q^6}{256 l_p^2 c^4}\ \frac{1}{c}\ = \alpha$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \mu_0 \epsilon_0 = \frac{\pi^2 \alpha Q^8}{32 l_p c^5}\ \frac{32 l_p c^3}{\pi^2 \alpha Q^8}\ = \frac{1}{c^2}$$

$$q_p = \sqrt{4\pi\epsilon_0 \hbar c} = \sqrt{4\pi\ \frac{32 l_p c^3}{\pi^2 \alpha Q^8}\ 2\pi Q^2 l_p\ c}\ = \sqrt{\alpha} e$$

$$r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} = \frac{256 l_p^2 c^4}{\alpha^2 Q^6}\ \frac{1}{4\pi}\ \frac{\pi^2 \alpha Q^8}{32 l_p c^3}\ \frac{1}{m_e c^2}\ = \frac{l_p m_P}{\alpha m_e}$$

$$\frac{4\pi\epsilon_0 G m_e m_p}{e^2}\ = 4\pi\ \frac{32 l_p c^3}{\pi^2 \alpha Q^8}\ \frac{l_p c^3}{2\pi Q^2}\ m_e m_p\ \frac{\alpha^2 Q^6}{256 l_p^2 c^4}\ = \frac{\alpha m_e m_p}{m_P^2}$$

$$R_\infty = \frac{m_e e^4 \mu_0^2 c^3}{8h^3} = m_e\ \frac{65536 l_p^4 c^8}{\alpha^4 Q^{12}}\ \frac{\pi^4 \alpha^2 Q^{16}}{1024 l_p^2 c^{10}}\ c^3\ \frac{1}{8}\ \frac{1}{8\pi^3 Q^6 8\pi^3 l_p^3}\ = \frac{m_e}{4\pi l_p \alpha^2 m_P}$$

$$E_n = -\frac{2\pi^2 k_e^2 m_e e^4}{h^2 n^2} = 2\pi^2\ \frac{\pi^2 \alpha^2 Q^{16}}{16384 l_p^2 c^6}\ m_e\ \frac{65536 l_p^4 c^8}{\alpha^4 Q^{12}}\ \frac{1}{4\pi^2 Q^4 4\pi^2 l_p^2}\ = -\frac{m_e c^2}{2\alpha^2 n^2}$$

Conclusion:
In our simple Planck universe, the visible world is a point-state world, the atomic world is a wave-state world and the Planck level world is a time-less mixture of analog waves and digital points.

The fundamental components of this Planck universe are; momentum, an expanding black-hole universe, a contracting white-hole twin and alpha. The white hole transfers momentum (dark energy) to the black hole, this forces the expansion of the black hole (dark matter) universe and so gives us time (and the arrow of time). If this transfer occurs at the speed of light then we have c. Alpha determines the magnitude of the forces between particles. Particles define the intervals between Planck events; information inside a black hole is not lost.

Complexity arises with the addition of the time dimension, itself a consequence of the expansion of the universe.

I have outlined a few of the main points, clearly there is much more. The question of why the wave-state (expansion and contraction) behaves as it does is quite interesting for this also seems to involve a duality. Light waves, for example, reflect an oscillation between electric and magnetic fields. Consequently, the philosophical implications of an analog-digital oscillation may also be worthy of study[23].

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