# A well-founded formulation for quantum chromo- and electro-dynamics

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Although we often speak of fundamental particles, those in the standard model lack a formal mathematical foundation. That is, the standard model of particle theory is empirically-founded but could benefit from a formal, causal basis for consistency with cosmology. This essay will explore the notion that such a mathematical foundation exists and discuss its ability to address known problems in fundamental physics.

**Fundamental Space-time Filling Objects**

The existence of a fundamental basis for particle physics is strengthened by formal mathematical arguments about causal cosmology which form a basis for the No-Boundary Wave Function (NBWF)[1]. A mathematical foundation for Quantum Chromo-Dynamics (QCD) means to base the construction of the theory upon a formal theorem, one which provides a causal quantum[2] basis. It is anticipated to be a space-time geometric basis with a formulation equivalent to NBWF instantons, for consistency.

Recall that QCD has been shown to be based on a simpler, 'more fundamental' algebra explored in preon models. The tantalizing minimalist aspect of preon algebra suggests that is very likely to have a well-founded formulation, based on a *theorem* without *any* assumptions about particle properties. One could argue whether simplicity itself makes preon algebra 'more fundamental', or whether it economically affords a single fundamental theorem?

A key characteristic of a foundational theory is that it be consistent across all physical scales. Thus a causal particle's formulation must be consistent with the No-Boundary Wave Function, which is consistent with cosmological evolution. Takeuti further proved[3] that a self-consistent mathematical system must be *finitary*. This means that fundamental particles are finite in extent (whether observable or not).

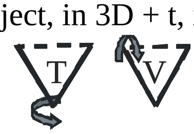
Consider the form of the NBWF: $\Psi(b,\chi) \approx \exp\{[-I_R(b,\chi) + iS(b,\chi)]/\hbar\}$ (1)

and observe that any finite, minimal, particle will have area and curvature metrics. Such fundamental geometric metrics can be assigned to instantons (so that they are no longer 'fuzzy' but quantized) and thus equated to the terms $-I_R(b,\chi)$ and $iS(b,\chi)$.

Our search for a fundamental theorem must be limited to only those which preserve causality and can replicate QCD (and QED) faithfully. Keep in mind that preons use a combinatorial algebra comprised of only two elements, which taken in groups of three produce the eight fundamental particles of one family of quarks. Research toward a foundational representation geometry requires a space-filling object that has an intrinsic temporal vector.

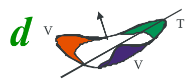
Consider the 'first theorem of particle physics':

The simplest space-time filling object, in 3D + t, is a spinning triangle.



**Fundamental Basis for Quantum Chromo- & Electo-Dynamics**

There are of course two topologically distinct versions, one which spins along an edge and one which spins orthogonal to an edge. Identify these with T & V to construct the representation geometry of quarks, leptons and neutrinos. The T & V (~Rishon[4]) geometric basis doesn't include curvature, which is constructed when three are taken as a group, e.g. quark. Geometric intuition suggests that they are 'fused', as partitions of a closed string,[5] band, etc. Construction of all QC/ED particles and interactions has been modeled,[6,7] including its equivalence with Dirac notation, and shown to be 1-1 with this dual algebra-geometry. Although the algebra is quite interesting, understanding how fundamental quanta and properties are a geometric construct requires interpretation of:




This finite representation geometry provides a consistent, causal formulation of QCD particles. QCD color is determined by the orientation of the intrinsic spin centerline with respect to the colored partitions, Up or Down quanta by the number of T (or V) partition/preons, and Electro-dynamic charge is assigned as usual. Mass and energy are represented by nano-geometric area and curvature. Thus ALL fundamental particle quanta, mass and energy quantities are attributed to a geometric basis having a dual algebra, Я, with **no** geometric properties left over. The well-founded representation geometry and Langland-dual algebra Я are thus *one-to-one and onto* with QC/ED!

With both geometric and dual algebraic insight, a first answer to this essays relevant question is posed, without assumptions:

Qx: What is "Fundamental" (for particle theory)?
Ans: A foundational theorem which defines geometric-algebraic space-time objects.

The first theorem of particle physics provides the basis for constructing a consistent (i.e. finitary) representation geometry-algebra dual formulation for QC/ED. The dual formulation is intrinsically causal, has a mass quantum, and is thus consistent with cosmology. Further demonstration that Я replicates all QC/ED particles and interactions is not a fundamental concern within the scope of this short essay. The representative noncommutative matrix algebra, Я - a subgroup of the cross-product of two wreath products,[8] is a "closed" group; it does not include any *new* fundamental particles.

Understanding the foundation of a complex theory such as QCD raises many fundamental questions. Three very broad questions will be considered further:

(a) How does a well-founded particle physics differ in *an observable way* from a tightly-constrained empirical theory?

(b) How is a causal particle's formulation consistent with cyclic cosmology?

(c) Do the mathematical structures constructed upon this fundamental basis answer any current questions about fundamental physics?

**A Well-Founded Formulation Explains Neutrino Oscillations**

The proposed approach to particle theory bears a striking geometric similarity to string theory.[5] However, to replicate particulate ground states, the string must be tripartite.[6] In particular, the representation geometry for a neutrino is deduced to be a trecoil – not a knot. A trecoil is the natural ground state of a Band, which is a *closed string with intrinsic stiffness*.[9] A trecoil has three topologically distinct states:

Note trecoil neutrino states oscillate between three distinct winding numbers, and that the highest mass (area) state can only be reached via the intermediate (muon neutrino) state. This is fundamentally consistent with observations[11] that ν mixing angle $\theta_{13}$ is very low, and depends on propagation distance.[11] Qualitative agreement with neutrino observations is a strong testament of support for the fundamental construction of representation geometry used in Band Theory. The Band as a fundamental representation geometry will soon be tested by DUNE particle experimental observations.

The Planck scale, α', is a natural quantum metric of mass which is consistent with gravitation.[12] Defining a quantum metric for energy as *i*ç, a unit of intrinsic curvature, allows one to write the explicit neutrino state function:

$$|H_{\nu\mu}\rangle = (\alpha' + i\text{ç})|e^{-\lambda(\nu\mu+\text{gluon})}B^2(\bar{P}O P\bar{O})^2(\bar{O}GO\bar{G})^2B^2\rangle\ , \qquad (2)$$

where λ is a Randall-Sundrum scaling exponent defined as a functional ב (Beth) having *six extra intrinsic dimensions*:

$$\lambda = ב(r_R, r_Y, r_B, \text{Ø}_R, \text{Ø}_Y, \text{Ø}_B). \qquad (3)$$

This formulation is both causal and an explicit QC/ED ground state of band theory, and thus resolves coupled fundamental problems of particle theory.

## A Fundamental Formula, Consistent with Cyclic Cosmology

The standard model particle theory uses a space-time average of allowable particle interactions over weak interaction scales accessible by high-energy colliders. The scale used in equation 2 is near the Planck scale to construct a color-state-specific particle formula. We now assert that, since the *universe is simply the sum of all its particles*, for consistency the form of the respective equations must be the same. Consider a cosmological space-time average of the causal particle formulation, in which individual particle quantum states are rendered moot, but the underlying causal formulation remains.

A cosmological space-time equivalent to equation 2 is the ***fundamental formula***:

$$e^{(i\theta_{mt} - \lambda)}|\text{Я}\rangle, \qquad (4)$$

where temporal evolution is included as in the *original* cyclic cosmology[13] imaginary time variable, $\theta_{mt}$. The QC/ED group Я allows the fundamental formula to be evaluated, "$|_{weak}$", at the usual weak interaction scale to produce particle interaction calculations.

First observe that the well-founded cosmology is cyclic in *mass* and time. Formula 4 can also be evaluated "$|_{now}$"at a fixed (current) $\theta_{mt}$ 'time' to yield the well-known equation of General Relativity, but with an imaginary temporal term. The generalized QC/ED state algebra, Я, averages to unity at macroscopic scales, as usual.

The result is set equal to one universal 'size' to produce equation 5:

$$\frac{G^{C}_{\mu\nu}}{\Lambda g_{\mu\nu}} + \frac{8\pi i}{\Lambda g_{\mu\nu}} T_{\mu\nu} = 1 \tag{5}$$

where the Randall-Sundrum scale factor, λ, has been evaluated to reveal $G_{\mu\nu}$ and $T_{\mu\nu}$. Here the cosmological constant simply appears in the denominator, although it can be multiplied through and terms re-arranged to easily recognize General Relativity. Temporal curvature is naturally an imaginary quantity, like any curvature out of the coordinate plane in physics.

Cyclic cosmology thus requires a flat Riemannian geometric space-time. It also eliminates the cosmological coincidence problem, since the legacy GR is orthogonal to the cyclic formulation when evaluated at the present time.[14]

**Current Questions about Fundamental Physics**

String theory has one very important property which is shared by Band Theory – it has a minimum distance. The minimal Planck scale is thus taken to be the scale of the *physical singularity* of a black hole. Note that a finitary geometry is again required to describe a black hole, which naturally takes the form of a tetrahedron. This geometric insight fundamentally explains the factor of ¼ in the Hawking-Bekenstein Area-Entropy Law… it is the area of a single tetrahedral surface that determines black hole entropy!

The complexities of the mathematical description of a black hole are well beyond the scope of this paper. Nonetheless, note that a tetrahedron is bi-laterally asymmetric, and that (string theoretic) minimum distance law, $r \leftrightarrow \alpha'/R$ , are fundamental properties of the forgoing theory which have very important impacts on black hole theory and astrophysical observations.

Another important aspect of a well-founded theory is that is should provide insights into answers for well-known fundamental problems. The foregoing representation geometry and associated algebra provide qualitative answers to a few of the "Top Ten Problems in Fundamental Physics" from the 2000 International Superstrings Conference:

Q1. Are all the (measurable) dimensionless parameters that characterize the physical universe calculable in principle or are some merely determined by historical or quantum mechanical accident and uncalculable? [D.J. Gross]
A1. At least one is empirical. The cosmological constant is a measure of the mass-time scale of the universe, and thus 'merely determined'.

Q4. Is Nature supersymmetric; if so, how is supersymmetry broken? [S. Ferrara, G. Kane]
A4. Finitary particles can have an extra symmetry, when characterized as Majorana particles. SUSY exists only as an intrinsic property in $\pi^{o}$ and a few other bosons. Since these particles cannot have a super-partner, then no super-partner particles exist.

Q5. Why does the universe appear to have one time and three space dimensions? [S. Kachru, S. Mukhi, H. Ooguri]
A5. The six extra dimensions needed to achieve consistency are intrinsic dimensions of a finitary particle. So there are only 3 space dimensions, and time.

Q6. Why does the cosmological constant have the value that it has, is it zero and is it really constant? [A. Chamblin, R. Kallosh]
A6. The cosmological constant is the most constant of all, as it represents the scale of the universe's total mass and time.

## References


[1] J.B. Hartle, S.W. Hawking and T. Hertog, "The Classical Universes of the No-Boundary Quantum State" hep-th/0803.1663v1 March 2008.

[2] N. Seiberg, L. Susskind and N. Toumbas, "Space/Time Non-Commutivity and Causality", hep-th/0005015v3, May 2000.

[3] G. Takeuti, *Proof Theory*, Dover Publications, 1975.

[4] H. Harari and N. Seiberg, "The Rishon Model", Nucl. Phys. B, Vol 204 #1, p 141-167, September 1982.

[5] M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory*, Cambridge University Press, p 21-22, 1987.

[6] W.R. Lundberg, "Topological Combinatorics of a Quantized String Gravitational Metric", Proc. 7th APS/DPF Conf., p 1589-1591, physics/9712042, November 1992.

[7] W.R. Lundberg, "Causal Particle Theory", Proc. DPF `09 http://indico.cern.ch/event/41044/session/68/contribution/38, July 2009.

[8] D. Joyner, *Adventures in Group Theory*, Johns Hopkins Univ Press, p 177, 2002.

[9] W.R. Lundberg, "Neutrino Oscillation: A Natural Effect of Band Theory", Proc. DPF `15, http://vixra.org/pdf/1510.0382v2.pdf , October 2015.

[10] N. Haba1, H. Ishida, K. Shimada and Y. Yamaguchi, "What triggers $\theta_{13}$ discrepancy between Daya Bay and T2K?" /abs/1508.05238v1, Aug. 2015.

[11] O. Mena and S. Parke, "Unified Graphical Summary of Neutrino Mixing Parameters", hep-ph/0312131v1, 2003.

[12] C.W. Misner, K.S. Thorne and J.H. Wheeler, *Gravitation*, W.H. Freeman and Co., p 536, 1973.

[13] W.R. Lundberg, "A Cyclic Universe without Missing Mass: Implications of R↔α'/R", Proc DPF `96, astro-ph/0007100

[14] W.R. Lundberg, "No Cosmological Coincidence!" presentation at Eastern Gravity Meeting, April, 2015.