The Power and Poverty of Mathematics in Physics

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Abstract. In this essay I shall defend the thesis that physics is about motion and that motion, as classical Greek philosophy has exhaustively demonstrated, is not tractable in any discrete domain of argument. For more than two millennia to come were Zeno's Paradoxes respected as the collegial advice: Do not open this box! Physics, as the science of motion, came of age only with Newton's fluxions and Leibniz' differentials, which rather than dissolving the paradox of motion, escaped it. Arguments will be presented in support of the thesis that infinitesimal calculus was the reason for the literally un-reason-able effectiveness of mathematics in physics and that the 'arithmetization of the continuum', i.e. the 'opening of the box', turned physics from the science of motion back to the affirmative logic of states in historical time.

I. **From Zeno**…to Newton…and back

Physics is deeply rooted in 'motion', which classical Greek philosophy had abandoned as a legitimate subject of philosophical enquiry. It was left to Newton to escape the paradoxical nature of motion through the use of a special kind of mathematics, and his escape was so convincing that his laws of motion were considered as a special category of physics by Kant, that is, as pure physics. Pure physics, in Kant's theory of science, contrasts with empirical physics which, rather than being a priori has been 'taken from the phenomena' and for this reason belongs to a different epistemological category, regardless how much mathematics it involves. Kant thus makes a significant distinction between natural (Newtonian) law and what today would be called (empirical) model, i.e. between intuition and representation. Let us retrace this historical development in some more detail with the aim to shed light on the power and poverty of mathematics in physics.

Zeno's Paradoxes are believed to be the response to criticism voiced against the idea of Parmenides that change in general and motion in particular is illusionary. Modern number (set) theory, while it formalistically dissolved the paradox of motion by way of discrete-definitional argument, failed to illuminate the underlying semantic dilemma that it was intuitively clear and warranted by experience (also to Zeno) that Achilles could outrun the tortoise despite its head start, whereas rational argument augmented by number theory proved that he could not. The Dichotomy Paradox, for example, holds that one must have arrived halfway before arriving at the goal, which is impossible, because it implies that one must have arrived quarter-way before arriving halfway and so forth ad infinitum – hence the goal can never be reached for it involves an infinite number of causal actions. For motion to become tractable at all it had to be broken into discrete phases, each representing a false (non-existing) phenomenon, e.g. the infamous *flying arrow at rest*. When, however, the *flying arrow at rest* was considered in isolation, there was no cause to be observed or argued connecting its successive states. With regard to the Arrow Paradox, Aristotle generally concurred with Zeno and tried to approach the problem logically in his *Physics*:

If everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless.

Finally, he tried to solve the problem of motion arithmetically – for several reasons without success, however. Also Aristotle's analyses of motion end up in a conclusion that is paradoxical and yet consistent with the knowledge of his era. Motion, understood as the temporal (historical) succession of phases, that is, of pseudo-states, simply vanished when inspected by causal, logically or arithmetical means. Successive states of the undeniably *flying arrow at rest* were therefore (and have remained to the current day) argumentatively unconnected. Zeno's Paradoxes and Aristotle's analyses were the capitulation of Greek antiquity as regards the problems of motion in particular and physical (or mathematical) time in general; motion in historical time had to remain a deep conundrum for principled reasons. And yet is Zeno's Flying Arrow Paradox – albeit involuntarily – anticipating classical physics, that is, the idea that inertial motion is causeless ($\Sigma F=0$) and logically groundless. However, in the absence of a proper concept of physical time the sole semantically sound conclusion that could be drawn was that motion (temporal change) is paradoxical, if not illusionary and factually inexistent. Classical Greek antiquity was well aware of the problem of infinite divisibility which, however, is different from the problem of infinitesimality, because the transition from the discrete to the infinitesimal involves a categorical change of argument in Hegel's sense, i.e. from quantity to quality.

II. From Zeno…**to Newton**…and back

In anticipating Hegel's dialectic view of nature by one century Newton finally escaped the paradox of motion by exiling causation and logic (i.e. verbal argument) as well as discrete proportions from the mathematical-philosophical part of physics and declaring absolute, explicit and infinitesimal time the domain in which the objects of physics exist. He thus separates the objects of physics from the phenomena by categorically exempting the latter from positive argumentative discourse. In the *Principia* (Scholium) he seems to foresee the danger of illegitimate, i.e. plain causal or logical, that is, positive reception of his system of motion *[my interpretation]*:

> *Wherefore relative quantities [time, place, space and motion] are not the quantities [objects] themselves, whose names they bear, but those sensible [phenomenal] measures of them (either accurate or inaccurate), which are commonly used instead of the measured quantities themselves. And if the meaning of words is to be determined [definiendae] by their use, then by the names time, space, place, and motion, their measures [mensurae sensibilies] are properly to be understood [namely phenomenally]; and [but] the expression will be unusual, and purely mathematical, if the measured quantities themselves [objects] are meant. On this account, those violate the accuracy of language, which ought to be kept precise, who interpret these words [time, place, space and motion] for the measured quantities [themselves]: nor do those less defile the purity of mathematical and philosophical truths, who confound real quantities [objects] with their relations and sensible measures [phenomena] [vulgaribus mensuris].*

With little exegesis Newton seems to specify two languages, *vulgar* and *mathematical-philosophical*, the relation between which he leaves undefined except that they must not be confounded. What Newton says here is that motion becomes tractable

in the presence of two non-confounded (pure?) languages. And it is mathematical-philosophical 'language', which allows him to escape the paradox of motion by introducing an argument (infinitesimality) incommensurable with discrete natural (vulgar) language. Infinitesimality – defined as a quantity less than any discrete quantity, yet not zero – is paradoxical in itself, i.e. not reachable by causal or logical argument. Intuitively, however, the continuum plays an important role in many of human conceptions, e.g. in how we think about lines and geometrical bodies in general. Newton thus constructs motion in a domain which is beyond verbal argument and therefore un-reason-able but not un-intuit-able. The crucial step Newton takes is the step from affirmation to negation (non-contradiction). If we consider the two 'languages' he posits incommensurable, they categorically escape the possibility of being contradictory. Then Newton's laws of motion, rather than affirmatively explaining anything, add a *not-false*, i.e. 'orthogonal' dimension to the world. In transcending the Zenonian paradox of motion (instead of solving it) his laws of motion are a priori in Kant's or a perfect synthesis in Hegel's sense. Newton's success was not in opening Zeno's 'box' but in 'putting wheels on it'.

III. From Zeno…to Newton…**and back**

Newton split physics in two incommensurable parts, i.e. into discrete (vulgar) phenomena and continuous (mathematical-philosophical) objects, the relation between which escapes not only causal and logical but any affirmative argument, for the two parts have no common measure. The continuum, which imaginatively we can slide our hands over, neither has nor needs verbal argument, that is, Ariadne's thread leads us through the most puzzling labyrinth without any instructions. Only Newton's 'dialectic' revolution should advance physics to the lead science for centuries to come and generations of physicists applied his system either from the phenomenal or from the mathematical philosophical end – with equal success. Physics became successful, because it could be pursued intuitively and intellectually, i.e. practically and theoretically. Then the experiment was not a blind search for something not yet known or the machinery from which knowledge was to be abstracted, but the intuitively targeted endeavor to expand physics' range of applicability, which in turn enabled theorists to find new solutions to the equations of motion, which in turn suggested novel, targeted experiments and so forth.

However, also physics' current perplexedness is already laid out in Newton's own interpretation of his system. The above quote from the Scholium leaves little doubt that of the two languages of physics, vulgar and mathematical-philosophical, only the latter refers to true knowledge, whereas the former is pre-scientific, i.e. vulgar at best. Accordingly became mathematics the decisive and, over time, the exclusive mark of physics and science in general. The problems began when differential equations became too complex to be solved in closed form and when already the formulation of differential equations became cumbersome, if not impossible (e.g. the N-body problem). Singularities and hence differentiability played a prominent role in the successive transition from infinitesimal calculus back to logically enhanced arithmetic (set theory). Weierstrass' functions (certain infinite Fourier sums), which are continuous yet not differentiable functions, were taken as the proof that infinitesimality is a mathematically untenable principle. It went unnoticed that in this proof the devil of infinitesimality was exorcized with the Beelzebub of infinity. Fourier series had been invented in order to approximate empirical sets of data by inevitably finite sums of Fourier coefficients, i.e.

in order to simplify the mathematical-philosophical side of physics. What we see in the 'proof' and its consequences is the all-too-human misconception to make the means into the purpose. Weierstrass' infinite Fourier sums are, in sharp contrast to the ideal triangle, not constructible in the mind, because they represent a complex process in historical time, i.e. they represent never ending change. In other words, the intuitively graspable continuum (geometry) had been discarded in favor of an infinite number of discrete operations in historical time. What happened in this process is that physics lost its original subject, i.e. the phenomena in general and motion in particular. The already by Newton belittled phenomena, by and by, got (literally and entirely) out of sight, which veiled the implication that physics was no longer speaking of motion, because in what precise sense galaxies and atoms 'move' is not manifest in language, i.e. in the phenomena. Already Coriolis force (due to its insignificant magnitude at 'human' velocities on earth) is not manifest in human language, which is why it is referred to as an apparent or pseudo force in physics. Had, however, humanity evolved on a sufficiently quickly rotating disk, there would not only be other verbs and nouns in our languages, i.e. other sentences and thus phenomena, but most likely would our anatomy be different from what it is today. Then Coriolis force would not be a pseudo-force but as natural as gravity is for us. It was due to the absence of linguistically embedded phenomena in modern physics that motion had to disappear from physics. Its position was taken by the state, which set theory so wonderfully supports at the expense that *the flying arrow at rest* returned to the physical debate – and the paradox should follow suit. In the absence of the foundational phenomena physics turned back to discrete causal, logical and mathematical argument. The dramatic confusion governing the return to discrete physics is captured in the call (Weierstrass, Cantor, Dedekind) to expel infinitesimals from mathematics for sole use in physics. How right they were for wrong reasons!

The return to the physical state was inevitably accompanied by the return of historical time as the domain in which physical objects exist. From above discussion it is only a minor step to interpret Newton's mathematical-philosophical time as Einstein's fourth dimension. In it nothing 'changes' because it is timeless (not eternal!). The disagreement about 'time' is not so much between Newton and Einstein, it is between Aristotle and the rest of the world. There are more than 20,000 verbs in the English language – plus the verb *to change*. What is change? Isn't the verb *to change* used by illiterates, i.e. by those not knowing what they talk about? In the absence of knowledge change became a problem, however, because causation names the grammatical subject and the predicate, whereas 'change' does not. Change had come about as the idea of grammarians that the verb indicates action or change of the subject. Change, however, which from a grammatical point of view appears to be the communality of more than 20,000 English verbs, is factually as meaningless as referring to white as the common characteristic of colour. Hence in his *time is the number of change* Aristotle elevated the impotent verb *to change* to a noun, which made his famous definition of time referring to everything and therefore to nothing. Also Leibniz belonged to the 'rest of the world' when he held that – against grammatical orthodoxy – the verb is *contained* in the subject, i.e. that *basic sentences* are by no means analyzable phenomena, but irreducible unities. Aristotle's dictum starts to make sense when unintelligible 'change' is replaced by a knowable phenomenon, e.g. by the motion of a clock. Then his dictum reads like Einstein's: *time is what the clock measures* and the ideal clock is nothing else but Newton's equably flowing mathematical-philosophical time, i.e. a purely hypothetical mathematical-philosophical) reference system bearing strictly periodical marks. Recalling that periodicity (from Latin: periodus) originally is a spatial property of

recurrence, there remains not so much of a controversy between Newton and Einstein as regards their ideas of time. It is, however, Aristotle's time which came to dominate not only physics, but all current sciences. Science in our time is science in historical time, i.e. the attempt to predict the future and sometimes even the past. 'Change', however, is unintelligible and hence not predictable, for it was intelligible, it would not need to be predicted. Newton's equably flowing time 'flows' only in our minds when we slide our imaginary hands over continual geometrical bodies – forth and back. There is no past or future to be found in Newton's laws of motion; temporality only emerges from the causal or logical succession of discrete states (Hume's causality), i.e. as the consequence of the need to argumentatively connect discrete states. Now let me come full circle in order to characterize the current state of affairs in physics by quoting Anaximander: the things *give justice and make reparation to one another for their injustice, according to the arrangement of time.*

IV. Conclusion

The 'arithmetization of the continuum' has opened Zeno's 'box' and thereby disregarded his advice that motion is illusionary, that is, not tractable by discrete argument for principled reasons. Newton's fluxions had kept the box closed by denying any positive argument as to the relationship between vulgar and mathematical-philosophical arguments. Today physics, after having replaced motion by animated states, has become an argumentative science, which made novel 'theories' the subject of argument rather than effectiveness. In the absence of phenomena there remains only an undecidable and unfruitful battle of words and mathematical symbols, because the proof of positive theories can only be 'demonstrated' in a domain which is inaccessible to humans, that is, the historical domain. The un-reason-able effectiveness of mathematics in physics, however, is restricted to the present. The historical path of physics – and its (dis)association with mathematics – sketched in this essay is the move from the enlightened, i.e. paradoxical positivism of the ancient Greeks through a brief period of effective anti-positivism (intuitionism) to current illusionary positivism, which is unreasonably complex historicism.