“Experimental Mathematics” and The Digital Nature of Reality

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Albert Einstein said that he couldn’t believe that God would “play dice” with the universe. It has continued to be a source of considerable frustration that there is no equation that could predict the detailed timing of emission from an excited state, whether from atomic or molecular events such as fluorescence or from nuclear events such as radioactive decay. The best prediction as to the timing of these events has been to assume randomness at the molecular or atomic level, and settle for predicting the average rate of many such identical events over a period of time. While these predictions are acceptable on the larger scale, there is still a need to explain why these events do not scale down to the scale of a small number of events.

True randomness requires that there be no relationship between one event and the next event. Pseudorandomness, however, simply implies that we have not (yet) been able to establish the cause-effect relationship of one event to the next. For example, “random” numbers that are generated by computer algorithms are actually pseudorandom rather than random. That is to say, an observer who is equipped with the algorithm can, in principle if not in practice, start at a given point and continue the series.

Fluorescence and radioactive decay are examples of a property that is highly predictable at one level of scale but apparently becomes random at a small scale. While a continuous reality might not change properties as the scale gets smaller, the question arises whether a digital reality might model the scaling properties of such events. To provide a satisfactory model, the mathematical function must

(1) exhibit a causal relationship between events that is independent of scale,

(2) equal the continuous version of the function at large scale,

(3) diverge from the continuous version at a smaller scale,

(4) behave pseudorandomly at smaller scale and

(5) derive a size “quantum”, where smaller scale becomes meaningless.

While it is difficult to conceive of a continuous function having these properties, it is not at all difficult to conceive a discontinuous function where the transition from smooth to chaotic behavior depends on the value of a parameter. If the iterative formula is not known, an observer will find the chaotic output to be pseudorandom and indistinguishable from random.

A familiar function with these properties is the population function widely used in population biology. In this context the number of a individuals in a population depends directly on the number the prior year, but is confined by the maximum carrying capacity of their environment. If the fraction of the maximum population in year n is Xn and that of the prior year Xn-1, they can be related by a parameter R , by the equation:

$$An = R * Xn\text{-}1 * (1 - Xn\text{-}1) .$$

(Note that for our purposes it doesn't matter whether the population actually follows this equation; we are concerned here only with the mathematical behavior of the model.)

In the population biology case, the independent variable is time. Metaphorically, we will let time represent ANY dimension of our reality. If this function were continuous rather than discontinuous in time, it would produce a stable value for A (between 0.0 and 1.0) when Xn = Xn-1 and the parameter R is between 1.0 and 4.0 . That value is:

$$Ac = (R - 1) / R ,$$

Where the subscript c refers to the continuous function. We can eliminate the parameter R by solving this equation for R:

$$R = 1 / (1 - Xc) .$$

Substituting back in the original equation:

$$An = Xn\text{-}1 * (1 - Xn\text{-}1) / (1 - Xc) .$$

It is, however, not continuous and we are interested in the behavior of this function as Xc goes from 0 to 1 . The easiest way to model the behavior of this function is with a spreadsheet that iterates the value of Xn with each fixed value of Xc . (Changing the value of a spreadsheet parameter to evaluate the behavior of a function is what I am calling "experimental mathematics".) Appendix 1 gives instructions for constructing such as spreadsheet. Inputting values of Xc between zero and one, the spreadsheet shows the behavior of the Xn 's . Each row gives sixteen consecutive values of X , the next gives sixteen more, etc.

For values of Xc between 0 and 2/3 the values of Xn approach a behavior that is independent of Xo , the initial value (endnote A). This difference equation gives the same result as the continuous version (ie: Xn = Xc). Figure 1 shows a portion of the spreadsheet for Xc = 0.60 . When Xc increases from 2/3 to about 0.72 the value of Xn starts oscillating, first between two values, then more as Xc increases within the range. Figure 2 shows a portion of the spreadsheet for Xc = 0.70 . One is tempted to expect the average value of Xn over its oscillation period to equal Xc , but surprisingly it doesn't; instead the average value of Xn drops from its maximum value (at Xc = 2/3) to a lower value of about 0.65 as Xc approaches 0.72 .

When Xc increases further, to the range 0.72 to 3/4 , Xn becomes chaotic. Figure 3 shows a portion of the spreadsheet for Xc = 0.74 . Moreover, the average value of Xn starts misbehaving. The chaos in this range hides the fact that Xn remains completely causal; that is to say that each value is entirely predictable from the previous value.

Finally, when Xc increases to between 3/4 and 1 , the spreadsheet value of Xn becomes negatively infinite. Figure 4 shows a portion of the spreadsheet for Xc = 0.76 . That is to say, there is no meaning in reality to this range of parameter.

While it is easier to discuss the mathematical variation of Xn with varying Xc , comparison with our reality criteria (above) is made simpler by using a parameter we will call Q = (1 - Xc) . The range of Q is, of course, between 0 and 1 . Comparing Q with our criteria:

(1) Xn exhibits a causal relationship between events that is independent of Q ,

(2) Xn = Xc , its continuous version, when Q is greater than 1/3 ,

(3) the average value of Xn diverges from the continuous version when Q drops below 1/3 to about 0.28 ,

(4) Xn behaves pseudorandomly (chaotically) when Q drops from 0.28 to 1/4 and

(5) there is no meaningful value for Xn when Q is less than 1/4,
Q therefore defining a size "quantum", where smaller scale becomes meaningless.

In review, where did the continuous function go wrong? Within one range of the parameter, it didn't go wrong at all. But over a different range the discontinuous nature of the function induced chaotic behavior that could not be distinguished from randomness. The continuous version predicted meaningful Xn in a range where reality had Xn meaningless.

In population biology, the time interval is one generation or one year and it makes no sense to subdivide the time variable any further. Similarly, I believe that the so-called randomness of molecular or atomic events, is best and most-simply explained by a division of time into small parts ("time quanta" if you like) beyond which it is meaningless to split time further. Consequentially time (and dimensions in general and reality in general) is best explained by being discreet (digital, if you prefer) and not continuous.

The fundamental equations of physics are differential equations such as the Schroedinger equation. They are traditionally evaluated by integration over time and space, and give very acceptable results over a broad range of parameters. This essay suggests that the predictability of small-scale or small-number events, however, would be amenable to better analysis if these equations treated time and space as finite, rather than a differential, variables.

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Figure 1: Normal Behavior -- Xc = 0.60 .

Xo = 0.5 Xn = Xn-1 * (1 - Xn-1)/(1-Xc) R = 2.5
Xc = 0.6

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Average
0	0.625	0.58594	0.60654	0.59662	0.60166	0.59916	0.60042	0.59979	0.6001	0.59995	0.60003	0.59999	0.60001	0.6	0.6	0.6	0.60095
1	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
2	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
3	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
4	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
5	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6

Figure 2: Oscillating Behavior -- Xc = 0.70

Xo = 0.5 Xn = Xn-1 * (1 - Xn-1)/(1-Xc) R = 3.33333
Xc = 0.7

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Average
0	0.83333	0.46296	0.82876	0.47305	0.83091	0.46832	0.82999	0.47036	0.8304	0.46944	0.83022	0.46985	0.8303	0.46967	0.83027	0.46975	0.64985
1	0.83028	0.46971	0.83028	0.46973	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.65
2	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.65
3	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.65
4	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.65
5	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.65
6	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.83028	0.46972	0.65

Figure 3: Chaotic Behavior -- Xc = 0.74

Xo = 0.5 Xn = Xn-1 * (1 - Xn-1)/(1-Xc) R = 3.84615

Xc = 0.74

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Average
0	0.96154	0.14224	0.46926	0.9579	0.15509	0.504	0.96148	0.14246	0.46986	0.95804	0.1546	0.50268	0.96151	0.14234	0.46953	0.95797	0.55691
1	0.15487	0.5034	0.96149	0.1424	0.46969	0.95801	0.15473	0.50304	0.9615	0.14237	0.46961	0.95799	0.1548	0.50323	0.9615	0.14238	0.50881
2	0.46965	0.958	0.15477	0.50313	0.9615	0.14237	0.46963	0.95799	0.15479	0.50318	0.9615	0.14238	0.46964	0.95799	0.15478	0.50316	0.52903
3	0.9615	0.14238	0.46963	0.95799	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50316	0.9615	0.14238	0.46964	0.95799	0.5569
4	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.5088
5	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.52903
6	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.5569
7	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.5088
8	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.9615	0.14238	0.46964	0.95799	0.15478	0.50317	0.52903

Figure 4: Divergent Behavior -- Xc = 0.76

Xo = 0.5 Xn = Xn-1 * (1 - Xn-1)/(1-Xc) R = 4.16667

Xc = 0.76

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Average
0	1.04167	-0.1808	-0.8898	-7.0063	-233.73	-228596	-2E+11	-2E+23	-2E+47	-1E+95	-5E+190	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!
1	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!
2	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!

Endnote 1 -- Resonance:

The independence of the stable value of X_n on the initial value, X_o, is not quite complete. When $X_o = X_c$ the spreadsheet shows X_n to stay at this value, even when X_n is in the chaotic regime. This “resonance” disappears with even very small variations from equality in X_o and X_c. These singularities are therefore unstable. We will not speculate here whether these singularities imply something like the selection rules for fluorescence.

Endnote 2 -- Precision:

After a large number of iterations, one must wonder whether how much of the behavior of the function might be due to round-off error in the iterations. While in the application of population biology the function is known to behave chaotically (see for example "Chaos" by James Gleick) the best check of the relative importance of precision here is to make a small change in the value of X_o and see how much change in the function's behavior occurs before stabilization or before chaos. For $X_c = 0.6$ and $X_o = 0.5$ for example, adding $(1.0 * 10^{-10})$ causes no change; the value of X stabilizes at 0.6 (within the precision of the computer) in sixteen iterations. The same incremental change in X_o with $X_c = 0.66$ again causes a stable value of X. Again, for $X_c = 0.67$, where the X now oscillates between two values, the increment in X_o has no effect.

Since the computer is calculating to the sixteenth decimal place, and an increment of one in the tenth decimal place produces no apparent change, it is safe to conclude that the precision of the calculation is sufficient, and the behavior of the function is truly chaotic in this range.

Appendix 1 -- Construction of the spreadsheet

Parenthetical notes are instructions. All other entries below are text except those that begin with an equal sign.

Row 1

cell	A	Xo =
	B	(Enter arbitary value for Xo. 0.50 is convenient)
	D	Xn = Xn-1 * (1 - Xn-1)/(1-Xc)
	S	R =
	T	=1/(1-B2)

Row 2

cell	A	Xc =
	B	(Enter value for Xc .)

Row 5

cells	C - R	(Enter numerals 1 through 16.)
	T	Average

Column A

Rows 7 - 27	(Enter numerals 1 through 20.)

Row 7

cell	C	=B1*(1-B1)/(1-B2)
	D	=C7*(1-C7)/(1-B2)
	E - R	(Copy right cell D7)
	T	= AVERAGE (C7 ... R7)

Row 8

cell	C	=R7*(1-R7)/(1-B2)
	D - R	(Copy down cells D7 - T7)

Rows 9 - 27

cells	C - R	(Copy down cells C8 - T8)