Does quantum theory need space-time?

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Abstract

We argue that the notion of space-time has a physical meaning only for describing real classical bodies while for constructing fundamental quantum theories this notion is not needed at all. As an illustration, we describe our approaches to the cosmological constant problem and gravity.

The phenomenon of quantum field theory (QFT) has no analogs in the history of science. There is no branch of science where so impressive agreements between theory and experiment have been achieved. At the same time, the level of mathematical rigor in QFT is very poor and, as a result, QFT has several well-known difficulties and inconsistencies. The absolute majority of physicists believe that agreement with experiment is much more important than the lack of mathematical rigor, but not all of them think so. For example, such a famous physicist as Dirac who made a great contribution to QFT, wrote in Ref. [1]: *"The agreement with observation is presumably by coincidence, just like the original calculation of the hydrogen spectrum with Bohr orbits. Such coincidences are no reason for turning a blind eye to the faults of the theory. Quantum electrodynamics is rather like Klein-Gordon equation. It was built up from physical ideas that were not correctly incorporated into the theory and it has no sound mathematical foundation."* In addition, QFT fails in quantizing gravity since the gravitational constant has the dimension $(\text{length})^2$ (in units where $c = 1,\ \hbar = 1$), and as a result, quantum gravity is not renormalizable.

One of the key ingredients of QFT is the notion of space-time background. We will discuss this notion in view of the measurability principle, i.e. that a definition of a physical quantity is a description of how this quantity should be measured. Since physics is based on mathematics, intermediate stages of physical theories can involve abstract mathematical notions but any physical theory should formulate its final results only in terms of physical (i.e. measurable) quantities. Typically the theory does not say explicitly how the physical quantities in question should be measured (a well-known exclusions are special and general theories of relativity where the distances should be measured by using light signals) but it is assumed that in principle the measurements can be performed. In classical (i.e. nonquantum) theory it is assumed that any physical quantity can be measured with any desired accuracy. In quantum theory the measurability principle is partially implemented by requiring that any physical quantity can be discussed only in conjunction with the operator defining this quantity. However, quantum theory does not specify how the operator of a physical quantity is related to the measurement of this quantity. Probably this problem will be solved in the future quantum theory of measurements.

In classical physics, the space-time background is the four-dimensional space, the coordinates (t,x,y,z) of which are in the range $(-\infty,\infty)$ (e.g. the Galilei or Minkowski space). The set of all points of the space is treated as *a set of possible events for real particles in question* and the assumption is that at each moment of time t the spatial coordinates (x,y,z) of any particle can be measured with the absolute accuracy. Then a very important observation is that, from the point of view of the measurability principle, the space has a physical meaning only as a *space of events for real particles* while if particles are absent, the notion of empty space has no physical meaning. Indeed, there is no way to measure coordinates of a space which exists only in our imagination. In mathematics one can use different spaces regardless of whether they have a physical meaning or not. However, in physics spaces which have no physical meaning can be used only at intermediate stages. Since in classical physics the final results are formulated in terms of the Galilei or Minkowski space, this space should be physical. For example, the Maxwell equations make it possible to calculate the electric and magnetic fields, $\mathbf{E}(t,x,y,z)$ and $\mathbf{B}(t,x,y,z)$, at each point of Minkowski space. These fields can be measured by using test bodies at different moments of time and different positions. Hence in classical electrodynamics, Minkowski space can be physical only in the presence of test bodies but not as the empty space.

In General Relativity (GR) the range of the coordinates (t,x,y,z) and the geometry of the space-time are dynamical. They are defined by the Einstein equations

$$R_{\mu\nu}+\frac{1}{2}g_{\mu\nu}R_c+\Lambda g_{\mu\nu}=(8\pi G/c^4)T_{\mu\nu} \tag{1}$$

where $R_{\mu\nu}$ is the Ricci tensor, R_c is the scalar curvature, $T_{\mu\nu}$ is the stress-energy tensor of matter, $g_{\mu\nu}$ is the metric tensor, G is the gravitational constant and Λ is the cosmological constant (CC). In modern theory space-time in GR is treated as a description of quantum gravitational field in classical limit. The coordinates and the curvature of the space-time are the physical quantities which can be measured by using (macroscopic) test bodies. Since matter is treated as a source of the gravitational field, in the formal limit when matter disappears, the gravitational field should disappear too. Meanwhile, in this limit the solutions of Eq. (1) are Minkowski space when $\Lambda=0$, de Sitter (dS) space when $\Lambda>0$ and anti-de Sitter (AdS) space when $\Lambda<0$. Hence in GR, Minkowski, dS or AdS spaces can be only empty spaces, i.e. they are not physical. This shows that the formal limit of GR when matter disappears is nonphysical since in this limit the space-time background survives and has a curvature - zero curvature in the case of Minkowski space and a nonzero curvature in the case of dS or AdS spaces.

To avoid this problem one might try to treat the space-time background as a reference frame. In standard textbooks (see e.g., Ref. [2]) the reference frame in GR is defined as a collection of weightless bodies, each of which is characterized by three numbers (coordinates) and is supplied by a clock. Such a notion (which resembles ether) is not physical even on classical level and for sure it is meaningless on quantum level.

In approaches based on holographic principle it is stated that the space-time background is not fundamental but emergent. For example, as noted in Ref. [3], "*Space is in the first place a device introduced to describe the positions and movements of particles. Space is therefore literally just a storage space for information...*". This implies that the emergent space-time background is meaningful only if matter is present. The author of Ref. [3] states that in his approach one can recover Einstein equations where the coordinates and curvature refer to the emergent space-time. However, it is not clear how to treat the fact that the formal limit when matter disappears is possible and the space-time background formally remains although, if it is emergent, it cannot exist without matter.

As noted above, from the point of view of quantum theory, any physical quantity can be discussed only in conjunction with the operator defining this quantity. From this point of view, a problem arises how time should be defined on quantum level and whether it is possible to define an operator corresponding to time. For example, we cannot construct a state which is the eigenvector of the time operator with the eigenvalue -5000 years BC or 2013 years AD. The problem is very difficult and is discussed in a vast literature (see e.g., Refs. [4] and references therein).

In standard quantum mechanics the position operator of each particle is well defined but the quantity t is only a parameter defining evolution in classical limit. A problem arises how to define the position operator in relativistic quantum theory. Here a particle is described by an irreducible representation (IR) of the Poincare algebra implemented in the momentum space, i.e. in the space of functions $\psi(\mathbf{p})$ such that the momentum operator $\mathbf{P}$ is the operator of multiplication by $\mathbf{p}$. By analogy with nonrelativistic theory, one might try to define the position operator by using the Fourier transform of wave functions in momentum space. However, it has been well-known since the 1930s [5] that, when quantum mechanics is combined with relativity, there is no operator satisfying all the properties of the spatial position operator. In other words, the coordinates cannot be exactly measured even in situations when exact measurements are allowed by the non-relativistic uncertainty principle. For example, in the introductory section of the well-known textbook [6] the following arguments are given in favor of this statement. Suppose that we measure the coordinates of an electron with the mass m. When the uncertainty of coordinates is of the order of $\hbar/mc$, the uncertainty of momenta is of the order of mc, the uncertainty of the energy is of the order of mc^2 and hence creation of electron-positron pairs is allowed. As a consequence, it is not possible to localize the electron with the accuracy better than its Compton wave length $\hbar/mc$. Hence, for a particle with a nonzero mass the exact measurement is possible only either in the non-relativistic limit (when $c \to \infty$) or classical limit (when $\hbar \to 0$). If $m = 0$ is possible, the problem becomes even more complicated since the photon can create other photons with lesser energies.

A rather striking example demonstrating problems with space-time in relativistic quantum theory is as follows. Consider a photon emitted in the famous 21cm transition line between the hyperfine energy levels of the hydrogen atom. The phrase that the lifetime of this transition is of the order of $\tau = 10^7$ years implies that the width of the level is of the order of $\hbar/\tau$, i.e. experimentally the uncertainty of the photon energy is $\hbar/\tau$. Hence the uncertainty of the photon momentum is $\hbar/(c\tau)$ and with standard definition of the coordinate operators the uncertainty of the coordinate is $c\tau$, i.e. of the order of 10^7 light years. Then there is a nonzero probability that immediately after its creation at point A the photon can be detected at point B such that the distance between A and B is 10^7 light years.

A problem arises how this phenomenon should be interpreted. For example, one might say that the requirement that no signal can be transmitted with the speed greater than c has been obtained in Special Relativity which is a classical (i.e. nonquantum) theory which operates only with classical space-time coordinates. In quantum theory the existence of particles moving with the speed greater than c (tachyons) is not prohibited (see e.g. a discussion in Ref. [7]). On the other hand, a fully opposite explanation (pointed out to me by Alik Makarov) is as follows. We can know about the photon creation only if the photon is detected and when it was detected at point B at the moment of time $t = t_0$, this does not mean that the photon travelled from A to B with the speed greater than c since the time of creation has an uncertainty of the order of 10^7 years. Note also that in this situation a description of the system (atom + electric field) by the wave function (e.g. in the Fock space) depending on a continuous parameter t has no physical meaning (since roughly speaking the quantum of time in this process is of the order of 10^7 years). If we accept this explanation then we should acknowledge that in some situations a description of evolution by a continuous classical parameter t is not physical. This is in the spirit of the Heisenberg S-matrix program that in quantum theory one can describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$.

In QFT particles are described not only by IRs but also by local quantum fields. A quantum field $\psi(x) = \psi(t, \mathbf{x})$ combines together two IRs with positive and negative energies. The IR with the positive energy is associated with a particle and the IR with the negative energy is associated with the corresponding antiparticle. In that case there is no physical operator corresponding to x, i.e. x is not measurable. In addition, as it has been shown for the first time by Pauli, in the case of fields with an integer spin it is not possible to construct a positive definite charge operator and in the case of fields with a half-integer spin it is not possible to construct a positive definite energy operator.

Hence a problem arises why we need local fields at all. They are not needed if we consider only systems of noninteracting particles. Indeed, such systems are described by tensor products of IRs and all the operators of such tensor products are well defined. Local fields are used for constructing interacting Lagrangians which in turn, after quantization, define the representation operators of the Poincare algebra for a system of interacting particles under consideration. Hence local fields do not have a direct physical meaning but are only auxiliary notions.

It is well-known (see e.g. the textbook [8]) that quantum interacting local fields can be treated only as operatorial distributions. A well-known fact from the theory of distributions is that their products at the same point are poorly defined. Hence if $\psi_1(x)$ and $\psi_2(x)$ are two local operatorial fields then the product $\psi_1(x)\psi_2(x)$ is not well defined. This is known as the problem of constructing composite operators. A typical approach discussed in the literature is that the arguments of the field operators ψ_1 and ψ_2 should be slightly separated and the limit when the separation goes to zero should be taken only at the final stage of calculations. However, no universal way of separating the arguments is known and it is not clear whether any separation can resolve the problems of QFT. Physicists often ignore this problem and use such products to preserve locality (although the operator of the quantity x does not exist). As a consequence, the representation operators of interacting systems constructed in QFT are not well defined and the theory contains anomalies and infinities. Also, one of the well-known result in QFT is the Haag theorem and its generalizations (see e.g. Ref. [9]) that the interaction picture in QFT does not exist. We believe it is rather unethical that even in almost all textbooks on QFT this theorem is not mentioned at all.

In Loop Quantum Gravity (LQG), space-time is treated on quantum level as a special state of quantum gravitational field. This construction is rather complicated and one of its main goals is to have a quantum generalization of space-time such that GR should be recovered as a classical limit of quantum theory. However, so far LQG has not succeeded in proving that GR is a special case of LQG in classical limit.

We believe that in view of this discussion, it is unrealistic to expect that successful quantum theory of gravity will be based on quantization of GR or on emergent spacetime. The results of GR might follow from quantum theory of gravity only in situations when space-time coordinates of *real bodies* is a good approximation while in general the formulation of quantum theory should not involve the space-time background at all.

If the reader is still reading this note, he or she might say: "Well, suppose that I accept the above arguments. However, any criticism can be constructive only if something positive is proposed instead. Are there any ways to construct quantum theory without space-time?". In the remaining part of this note we argue that the answer is "yes" and, as an example, we describe our approach to the cosmological constant problem and gravity.

As noted above, space-time is not needed if we consider only systems of free particles and in standard theory space-time is used for constructing Lagrangians of interacting field. The interaction Lagrangians where the fields interact at the same points is the main source of difficulties and inconsistencies of QFT. So a problem arises whether the notion of interaction can be modified and moreover, whether this notion is needed at all.

Let us consider an isolated system of two particles and pose a question of whether they interact or not. In classical nonrelativistic and relativistic mechanics the criterion is clear and simple: if the relative acceleration of the particles is zero they do not interact, otherwise they interact. However, those theories are based on Galilei and Poincare symmetries, respectively, and there is no reason to believe that such symmetries are exact symmetries of nature. We will see below that in quantum theory based on de Sitter symmetry the relative acceleration of two particles is not zero even if no interaction is introduced (i.e. the particles are treated as free).

The usual approach to symmetry on quantum level is as follows. Since classical space-time (e.g. Minkowski or de Sitter space-time) is invariant under the action of a group (e.g. the Poincare or de Sitter group), the operators describing the symmetry should satisfy the commutation relations of the Lie algebra of the symmetry group. This approach is in the spirit of the well-known Klein's Erlangen program in mathematics.

However, as we argue in Refs. [7, 10], quantum theory should not be based on classical space-time background and the approach should be the opposite. Each system is described by a set of independent operators. By definition, the rules how these operators commute with each other define the symmetry algebra. So, a question is not what space-time background is "better" but what symmetry algebra is more pertinent for describing nature. From this point of view, the dS and AdS algebras are more pertinent than the Poincare algebra and detailed arguments are given e.g. in Refs. [7, 10]. In particular, the Poincare algebra is simply a special case of the de Sitter algebras when a parameter R, which can be called the radius of the Universe, goes to infinity. In the literature, instead of R the cosmological constant Λ is often used and the relation between those quantities is $\Lambda = 3/R^2$.

The calculation of the relative acceleration of two *free* particles in de Sitter invariant quantum theory involves the following steps.

At the starting point we have no space-time and no dimensionful parameters. The only information we have is how wave functions describing particles under consideration are constructed and how the operators of the algebra act on such wave functions. This is the maximum possible information in quantum theory.

The next step is that we introduce a parameter R with the dimension *length* and instead of the dS operators M^{40} and M^{4k} ($k = 1, 2, 3$) (see e.g. Refs. [7, 10]) work with the energy operator $E = M^{40}/R$ and the momentum operator $\mathbf{P}$ such that $P^k = M^{4k}/R$. Then we *define* classical time t as a parameter describing the evolution according to the Schroedinger equation and *define* the position operator $\mathbf{r}_j$ of particle j ($j = 1, 2$) such that it acts on wave functions $\psi(\mathbf{p}_j)$ of particle j in momentum representation as $i\hbar\partial/\partial\mathbf{p}_j$ (as in standard quantum mechanics).

A standard quantum-mechanical calculation, which is described in detail in our papers (see e.g. Ref. [10] and references therein) shows that in *classical* approximation to the dS quantum theory the relative acceleration $\mathbf{a}$ of two *free* particles is $\mathbf{a} = \Lambda c^2\mathbf{r}/3$ where $\mathbf{r}$ is the classical vector of the relative distance between the particles and $\Lambda = 3/R^2$. This result shows that the space-time description arises only in classical limit of quantum theory. From the formal point of view, the result is the same as in GR on dS space. However, our result has been obtained by using only standard quantum-mechanical notions while dS space, its metric, connection etc. have not been involved at all. The derivation clearly demonstrates that the phenomenon of the cosmological acceleration can be easily and naturally explained from first principles of quantum theory without involving dark energy, empty space-time background and other artificial notions (see Refs. [7, 11] for a more detailed discussion).

The above example shows that the choice of the symmetry algebra results in an effective interaction between the particles. Hence one might pose the following problem: for which symmetry algebra the relative acceleration of two *free* particles is the same as for the Newton gravitational law (with possible relativistic corrections)? In view of the above result one might think that the necessary alebra is not the dS one since the relative acceleration of two bodies in dS theory is repulsive and proportional to r, i.e. not attractive and proportional to $1/r^2$ for gravity as one would expect. In this connection we note the following. Since all the dS operators are conventional or hyperbolic rotations, the distances in dS theory should be given in terms of dimensionless angular variables. The angular distance φ and the standard distance r are related as $\varphi = r/R$ (see a discussion in Ref. [7]). It is well known that classical approximation to quantum mechanics cannot be applied for calculating quantities which are very small. If the distance between two bodies is large then the angular distance φ is not anomalously small and can be calculated in classical approximation. However, the distances between bodies in the Solar System are much less than R and therefore the angular distances between them are very small if R is very large.

In Ref. [7] it has been argued that standard classical approximation does not apply for macroscopic bodies in the Solar System and that the standard distance operator should be modified. We have given a modification, such that the distance operator has correct properties and classical approximation can be applied. As a result, the classical nonrelativistic Hamiltonian is

$$H(\mathbf{r},\mathbf{q}) = \frac{\mathbf{q}^2}{2m_{12}} - const\frac{m_1m_2R}{(m_1+m_2)r}(\frac{1}{\delta_1}+\frac{1}{\delta_2}) \qquad (2)$$

where $\mathbf{q}$ is the relative momentum, m_{12} is the reduced mass, *const* is of order unity and δ_i $(i = 1,2)$ is the width of the dS momentum distribution in the wave function of body i. The second term in the right-hand-side of this equation is the dS correction to the standard result in Galilei invariant theory. Therefore the Newton gravitational law can be recovered if $const \cdot R/\delta_i = Gm_i$ where G is a quantity which should be calculated (see below). It has also been shown that the proposed modification naturally gives a correct value for the precession of Mercury's perihelion. We have also discussed gravitational experiments with light but in view of the effect of the wave-packet spreading of the photon wave function (see Ref. [13]) this discussion needs to be revisited.

It is seen from Eq. (2) that the dS correction to standard Hamiltonian disappears if the width of the dS momentum distribution for each body becomes very large. In standard theory there is no strong limitation on the width of distribution; the only limitation in classical approximation is that the width of the dS momentum distribution should be much less than the mean value of this momentum. Therefore in standard theory the quantities δ_i can be very large and then the dS correction practically disappears. However, as shown in Ref. [7], in a quantum theory over a Galois field (GFQT) discussed in our previous publications, this is not the case. GFQT is an approach to quantum theory where wave functions and operators are considered in spaces not over complex numbers but over a Galois field. Each Galois field is defined by a prime number p and in GFQT this number can be treated such that no physical quantity can be greater than p. In other words, p is the greatest possible number in nature. Hence in GFQT there are no infinitely small and infinitely large numbers and divergences cannot exist in principle. In our papers (see e.g. Refs. [12] and references therein) we argue that GFQT is more physical than standard theory and sooner or later quantum theory will be discrete and finite.

As shown in Ref. [7], for the validity of the probabilistic interpretation of a wave function in GFQT, the width of the dS momentum distribution should be not only much less than p but even much less than lnp. Since p is expected to be a huge number, this should not be a serious restriction for elementary particles. However, when a macroscopic body consists of many smaller components and each of them is almost classical, a restriction on the width of the momentum distribution is stronger when the number of components is greater. This qualitatively explains that the width of the momentum distribution in the wave function describing a motion of a macroscopic body as a whole is inversely proportional to the mass of the body. As a consequence, Eq. (2) becomes the Newton law of gravity. In contrast to standard approach to gravity where the gravitational constant is taken from the outside, in GFQT it should be calculated. A very rough estimation [7] gives

$$G \approx \frac{R}{m_N lnp} \tag{3}$$

where m_N is the nucleon mass. If R is of the order of $10^{26}m$ then lnp is of the order of 10^{80} and therefore p is of the order of $exp(10^{80})$. In the formal limit $p \to \infty$ gravity disappears, i.e. in our approach gravity is a consequence of finiteness of nature.

In the mainstream approach, gravity is treated as the fourth interaction which should be combined with the electromagnetic, weak and strong interactions. However, in our approach gravity is not an interaction but simply a kinematical effect in dS theory over a Galois field when at least one body is macroscopic and can be considered in classical approximation to quantum theory. In particular, the notion of gravitational interaction between two elementary particles has no physical meaning.

The above examples with the cosmological constant problem and gravity give strong arguments that the space-time description has a physical meaning only for describing real classical bodies while the construction of fundamental quantum theories should not involve space-time at all.

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