The price for mathematics

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Abstract

In this short essay I argue that the development of modern physics, in which the use of mathematical techniques plays a central role, comes with a price. Some important questions become meaningless and cannot be asked in the framework of the current scientific paradigm adopted in physics. Using mathematical methods we can understand and describe only idealized processes and systems.

The emergence of modern science, along with the establishment of the modern philosophy of politics, was, arguably, the most important result of the great intellectual breakthrough of the 17th century. Not infrequent, even in the second half of that century, were the voices claiming that in order to understand the world and our place in it, it is sufficient to study ancient thought; in the middle of the 18th century it was clear that the era of this method of investigations had come to an end. The ancient writings, regarded previously as the primary source of knowledge, were replaced by natural philosophy, which became the kernel of the new physics and started to employ mathematical techniques in an increasingly successful way.

As in the case of all great revolutions, the emergence of the modern scientific paradigm, in which mathematical techniques play such an important role, cannot be associated with a single incident; instead it was a slow, decades long process of questioning of dogmas, creation of pioneering methods, and identification of new challenges. But it also came with a price: the use of the quantitative methods and emergence of the modern scientific paradigm shed light on enormous unknown areas of inquiry, but at the same time started hiding others in the shadows. It seems that only in recent years have we become aware of the existence of these shadowy areas, and especially in the field of cosmology we have been forced to ask questions that we have found very hard to answer in the framework of the existing paradigm.

In order to understand the power and limitations of mathematical methods, one should go back in time and take a look at the development of physics (or natural philosophy) in the 17th century, before it emerged in the shaped form in the great Newton's *Philosophiae Naturalis Principia Mathematica*.

From the 13th century and the scholastic synthesis of Thomas Aquinas to the beginnings of the 17th century, the Aristotelian tradition was widely accepted as a starting point for philosophical and theological inquiries. Rejecting the Aristotelian view and breaking free of the hindering straightjacket of the scholastics was a big challenge, and it took decades, if not centuries, to cope with it.

According to Aristotle, there are three types of knowledge: metaphysics, physics, and mathematics, all three primarily characterized by their fields of interest. Metaphysics deals with entities that do not change and have an independent existence (that is, in fact, God). Physics, or natural philosophy, deals with entities that change and have an independent existence, that is – in modern words – natural phenomena. Finally, the objects of interest of mathematics are entities that do not change and do not have an independent existence. Aristotelian methodology is based on this fundamental division. According to Aristotle, the purpose of scientific research concerning natural phenomena should be the revealing of the changing and independent essences, underlying the phenomena. In order to describe the fundamental properties of objects, phenomena, or processes, as understood by Aristotle, one has to employ the principles consistent with the interest of physics – and only those. This means that only the physical principles should be applied in the study of physical systems, and, by the same token, mathematical principles can be applied exclusively to investigate the properties of mathematical entities. They cannot be mixed with each other, because physical and mathematical principles concern completely different areas of knowledge (a more detailed discussion can be found in [Gaukroger 2006, Chapter 11]). In other words, it is equally wrong to use mathematical explanations to obtain physical knowledge, as it is to apply physical insight to get mathematical knowledge.

It is worth asking what, according to this paradigm, was valid and informative explanation. Gaukroger writes:

> Aristotle, and the whole ancient and medieval tradition after him, thought that to explain a physical phenomenon one needed to distinguish between accidental features of a body and its essential properties, and any behaviour that could be said to be due to the body itself was due to the essential properties it had. These essential properties explained the behaviour. Such properties were material and Aristotle argued that they could not be captured by employing mathematical or empirical concepts. [...]
> The physical account of something – such as why the celestial bodies are spherical – is an explanation that works in terms of the fundamental principles of the subject matter of physics, that is, it captures the phenomena in terms of what is changing and has an independent existence, whereas a mathematical account of something – such as the relation between the surface and the volume of a sphere – requires a wholly different kind of explanation, one that involves principles commensurate with the kind of things that the mathematics entities are. [Gaukroger 2006, p. 401]

In modern words, physical processes, being usually the time-evolution processes, cannot be explained by mathematical structures that are, by definition, timeless. In modern physics we apparently avoid this problem by saying that although the physical theories, expressed in the language of mathematics, indeed describe processes, are themselves timeless. Indeed, this timelessness of physical theories is a part of the Copernican principle, which assures us that there is nothing special about our particular time and place, and that the two identical experiments performed here now and there tomorrow will give identical result[1]. This makes it possible to write the theories in the mathematical language, as it is done in physics. But the clash between the timelessness necessity of the laws and the contingency of real process is still there: How is it possible that things know how to behave?

The first great physical theory being at odds with the Aristotelian methodology was the Galilean kinematics expounded in his famous *Discorsi e dimonstrazioni matematiche intorno a due nuove scienze* of 1637. In this work Galileo presented his theory of motion using mathematical techniques5: in his approach, motion is nothing but a change of the position of an object in time and space which can be described with the help of geometrical and algebraic methods. The significance of this work lies not only in the fact that it became the first step towards the formulation of the theory of dynamics of Huygens, Hooke and Newton in the second half of the seventeenth century. Galileo realized that in order to create an adequate mathematical model of a physical phenomenon – and in order for this model to be

[1] Of course there were many proposals, contemplating, for example, variability of the physical constant. But then the rule governing this variability turns out to be timeless again.

effectively used – one has to limit oneself to far-reaching idealizations. For example, to describe the trajectory of a bullet, in the first approximation one has to neglect air resistance, curvature of the surface of the Earth, etc., effectively treating the bullet as a point particle moving in a vacuum over flat surface. Such a simplistic approach can be criticized by claiming that it describes a situation that never takes place in reality. As a consequence, one could argue that such a perfect mathematical description has nothing to do with the real phenomena in nature. Galileo was able to fend off the criticism by analyzing, using thought experiments and referring to the observation, how effects such as friction or buoyancy influence motion. This made it possible for him to formulate the idealized laws of motion that hold when these effects are eliminated. It can be concluded, therefore, that although the Galilean laws of motion describe ideal situations, they often could describe real motion with quite good accuracy. For example, the motion of a small metal ball thrown at a certain angle with small velocity will be described by Galilean laws pretty accurately, but in the case of a soccer ball, the deviations from the idealized motion could be significant (as anyone who has ever seen a football match knows perfectly well).

It is worth remarking at this point that although the criticism of the Galilean method could have been rebutted, and the method of idealizations, being the first step of the successful application of the mathematical techniques, works very efficiently in physics, it fails miserably in the case of other fields of inquiry. For example the world economic crisis of the last decade has proved without doubts that one cannot shape social sciences in this way; it is especially clear in the case of economics, which in spite of using sophisticated mathematical technics found itself rather helpless in predicting the crisis, and proposing the ways out of it.

Galileo formulated the rule of thumb for the construction of physical models: examine simple situations only, simple enough to be depicted in a mathematical form, so that with the help of mathematical operations one can describe the process or the state of a system of interest. This is the first price that one has to pay for the use of mathematics in physics: using mathematics we can describe only idealized processes, simple enough that their mathematical model can be effectively used to get predictions concerning this process. It also means that in order to verify predictions of physical theories we have to create experiments in such a way that external effects disturbing the investigated process are minimized. Building physical theories that make use of the mathematical formalism is therefore the art of fishing out the relevant information about a system from a sea of ubiquitous disturbances.

Nevertheless, the question remains, how is it possible that physical phenomena can be described using mathematical methods? To answer this question, it should be firstly noted that, contrary to the views of Aristotle, the theories of modern physics do not aim at the discovery of the substance or essence of the phenomena. Physical theory should describe the type of input-output situation: given the system in a specified initial state we are usually interested to see how its evolution will look like, or in what final state the system will be after a certain time has passed. In order to describe the initial and final states of the system (and, in practice, only some of its properties), we use metering devices. These measures gives us some numbers characterizing the system under investigation: the distance from the measuring point, speed, temperature, etc. Therefore what we measure in (most of) the experiments are relations between these numbers for the initial and final states. Physical theories must explain the observed correlations between these numbers, the quantitative characteristics of the processes and systems and it is for this reason that we find it convenient to have our theories expressed in the mathematical terms. In practice, we express our physical theories in terms of the mathematical functions, depending on some kind of the evolution parameters, such that, for one particular value of these parameters, the numerical values of the functions form the input, and for some other – the output set of numbers. We can also ask ourselves the questions what are all the possible outputs, given the input, or even under what conditions some functions describe all the possible input-output relations. These conditions are usually expressed in terms of some equations the functions are satisfied. For example, if we want to know, where a body in the Solar System is going to be tomorrow, if it was at some given place and had given velocity yesterday, we solve the Newton's equations of motion, or the geodesic equation.

The aim of a physical theory is to link a set of numbers specifying the initial state (the input) with a different set of numbers with the status of the final (output). From this perspective the use of quantitative methods in physics seems very natural. Such an approach, however, has, of course, very little to do with the Aristotelian quest for the essence of the phenomenon or physical processes. This is another price which has to be paid for the use of mathematics in physics.

Physicists are, of course, interested in the process that transforms the input data into the output. Imagine that one day an advanced alien civilization provides us one day with a device that takes as an input the characteristics of the initial state of a system and then tell us what will be a result of any measurement performed on that system. Will we consider it a major advance in our understanding of nature? Certainly not. What we really want to understand is `what happens', what is the relevant process. This process, however, will necessarily be described in abstract mathematical terms and its interpretation, especially in the case of sophisticated contemporary physical theories, is not easy and often leads to deep conceptual problems and puzzles. The debate concerning the interpretation of quantum mechanics, lasting for almost a century, serves as a perfect example.

The fact that the main focus of physics is description of the properties of processes transforming the initial conditions into some final state determines also which branches of mathematics are most useful and interesting from the physicist's point of view. Starting from the times of Newton and Leibniz, the main tool of physics are the differential equations that can be used to describe how the system changes over time. Most, if not all, of the fundamental equations of physics, from Newton's equations describing the motion of the planets to Einstein's equations of general relativity, or Schrödinger's equation of quantum mechanics – are (partial) differential equations. The basic feature of differential equations is the fact that, in order to solve them, the initial conditions which determine the future (and past) of the evolution of the system should be given. For example, specifying the position and velocity of a planet at a given moment makes it possible to find (using Newton's or Einstein's equations) the position and movement of the planet at any time, in the future and in the past. This feature of mathematical formalism naturally coexists with the experimental scheme described above, where one prepares a system in a particular state and then observes its evolution.

There is, however, the other side of the coin. We do not have any theory which makes it possible for us to decide which initial conditions should be chosen. For example, the theory of gravity allows you to specify how a planet moves, but you cannot use it to answer the question why the movement of the planet matches the specified initial conditions. Moreover, mathematical formalism makes it necessary to introduce the parameter (or parameters) characterizing the evolution (for example, time) that must be measured by an observer placed outside the system. This was not a problem in most of the situations that physics encountered in the last 300 years of its development: the physical systems of interest could have been observed from the "bird's eye view", from the perspective of an external observer.

The situation changes radically when we start thinking about the evolution of the universe as a whole. In this case, the view from the outside is not available; moreover, one of the most important questions of cosmology is why is the universe as it is? Why are other self-consistent models of the universe not realized in nature? Or aren't they? These questions refer essentially to the problem of the initial conditions for the universe, which within the framework of the existing mathematical methods of physics simply cannot be addressed. It should be stressed that this is not due to the limitations of this or that theory, this is a direct consequence of the paradigm of physics as shaped in the seventeenth century.

The great scientific revolution launched by Galileo, at the heart of which was the use of numerical and mathematical methods, has resulted in theoretical and technological advances unimaginable by those involved in the 17th-century intellectual breakthrough. Like every revolution, this one also had its price, however: powerful methods, further refined over the centuries, are not universal and today more and more clearly we realize the existence of problems in which they are helpless to solve. To tackle these problems, we will probably have to transcend the current scientific paradigm.

"The more the universe seems comprehensible, the more it also seems pointless" Steven Weinberg proudly proclaims. He simply fails to notice that in the framework of modern physics it is meaningless to ask about meaning. More than 300 years ago, facing the dawn of modern science Blaise Pascal famously bemoaned "The eternal silence of these infinite spaces frightens me." For many of his great contemporaries, Descartes, Spinoza, Newton, Leibnitz, to name just a few, understanding nature was an important part of the synthesis they seek, the synthesis that would tell us, what is the meaning of human life. We long abandoned such grandiose projects. Science is now just about numbers and formulas. This is the major price that we had to pay for mathematics.

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References

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