

# Quantum Vacuum Sustaining Force Model

Russell Jurgensen

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## Abstract

When elementary particles first emanated from the quantum vacuum, there may have remained a connection still propelling their internal motion. A possible model for this connection is explored through a quantum vacuum energy potential producing a particle's motion through a series of perpendicular forces. In the process of producing internal particle motion, a secondary energy potential develops to produce acceleration on other particles in the form of a reinterpreted Coulomb force. Equations are introduced describing the combination of electromagnetic force, strong interaction, and gravity, with units naturally working out from first principles. While this exploratory model is in a preliminary stage of development, it offers a refreshingly different view of reality.

## Introduction

In response to FQXi's invitation for essays on "Is Reality Digital or Analog?" and with encouragement from associates, I have attempted to describe research done so far into a model of a quantum vacuum sustaining force. I hope you find it thought provoking.

In considering whether reality is digital or analog we are looking for the mechanism that produced particles in the first place and that may still propel their motion. Perhaps analyzing particle motion in extra dimensions while considering a hidden force will offer a new way to look at the problem and lead to additional clues to the elusive originating mechanism.

In physics a hidden sustaining force is not readily visible, so particles are considered to have properties of self-contained energy as described through the first law of thermodynamics where energy is neither created nor destroyed. Two fundamental theories in science, general relativity and the standard model of quantum physics, stand in agreement with the laws of thermodynamics. As useful as these theories are, many scientists are not satisfied with them because they do not predict each other and they do not provide answers to some other questions remaining in science [8]. Particularly interesting to me is their lack of explanation for the first law of thermodynamics. Perhaps an accurate description of internal particle motion, whether digital or analog, would provide a more cohesive whole and explain why the energy of particles is constant.

If nature has a hidden sustaining mechanism, can science reproducibly test it? Could a model predicting such a system lead to new important predictions of physical behavior?

The quantum vacuum sustaining force model I am proposing is based on a conjecture that a constant energy potential in the quantum vacuum applies force to elementary particles propelling their internal motion. With this basic idea in place, the model analyzes a possible way that it might work. Perhaps it could lead to answers for the preceding questions.

The quantum vacuum, as defined in the proposed model, is the invisible originating source of the universe and particles. The model carefully defines a measurable potential in the quantum vacuum and its dimensions of operation. While invisible, the quantum vacuum potential still connects with every point in space – even at our fingertips. The model describes through continuous equations how the potential operates, but cannot say of what it is made or why it works that way.

To get a visual picture of how the sustaining force propels the internal motion of elementary particles, consider an unplugged box fan sitting in a window. When a wind gust comes through the window, and through the fan, it sets the fan blades to turning like a pinwheel. This seems a common enough occurrence, but let's look more closely at what is happening. In the fan, the blades rotate in a two-dimensional plane that also includes the plane of the window and wall. The wind comes from a third dimension that is perpendicular (at right angles) to the fan's plane of rotation. No matter which position a blade is located in its rotation; the wind develops an equally constant force on the blade in a direction perpendicular to the wind direction.

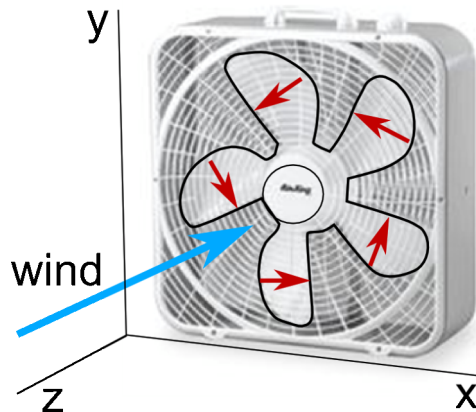


Figure 1. Wind can produce force equally in any direction of a perpendicular plane.

In the box fan analogy we see an example of energy potential, in this case the wind, operating in one dimension but causing equal levels of force in all possible directions within a two-dimensional plane that is perpendicular to the wind's dimension. See Figure 1.

The sustaining force model describes a mechanism where something like the wind can produce steady force equally in three dimensions of free space, not just two dimensions like in the box fan. While the wind of a box fan can produce steady force equally in a two-dimensional plane, it cannot produce equal force in three dimensions. For example, if the box fan were turned to be edgewise to the wind (and the shroud is removed) each fan blade would not receive the same force. There does, however, exist a method that works for three dimensions of a particle.

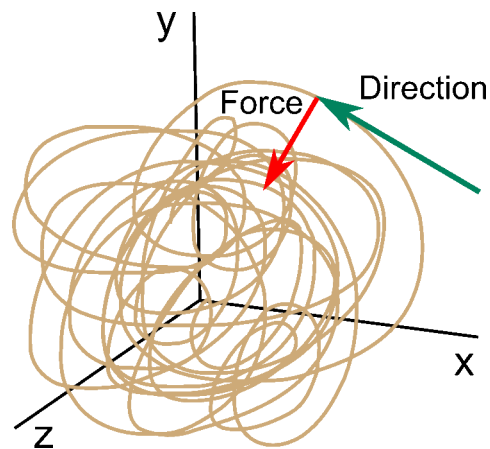


Figure 2. Equations are analyzed that show how a single-dimensional potential can apply force to a particle equally in three dimensions sustaining its motion.

Particles can move in three dimensions sometimes producing an apparently spherical, although fuzzy, shape such as the electrons around a Neon atom [6]. For now it does not matter if the particle is a photon, electron, or proton; we will just call it a particle. Something propelling the motion of the particle would need to act equally in every direction of the particle's fuzzy motion. It would need

a potential like the wind that is perpendicular to each of the three-dimensions. The wind was easy to visualize for the fan, and it is almost as easy to visualize logically for a particle. To do this, we thoughtfully add one more dimension, naturally giving the desired feature of being perpendicular to each of the three dimensions of motion.

We can't see a fourth dimension or know exactly its nature. This may be the biggest hurdle in picturing a quantum vacuum sustaining force. The key is remembering it is perpendicular to each of the other dimensions. The next steps involve analyzing how the quantum vacuum potential relates to internal motion and external forces between particles. But first, let's step through a slightly more formal definition.

### Directions of Measurement for a Quantum Vacuum Force

To analyze a hidden sustaining force, let's first identify its location in a dimensional measurement framework. Starting with three measurement dimensions of visible space  $x$ ,  $y$ , and  $z$ , we specify that each dimension is perpendicular to each other. The  $x$ ,  $y$ ,  $z$  measurement dimensions may be rotated to line up with objects being measured. Remembering that each new measurement dimension is perpendicular to every other dimension will help when thinking about them. It may also help to realize that while the model treats extra dimensions as spatial dimensions with length for calculation purposes, they could be interpreted differently. The reason the model treats them as full spatial dimensions is it allows a logical system of forces and accelerations to describe motion within these dimensions.

It also may help to note that extra-dimensional motion is microscopic. If the motion was large and a person could flex a special muscle to move in an extra dimension, then there would be no trouble visualizing it because it would be normal [9]. In actuality, the extra-dimensional motion is tiny and automatic so we don't notice it.

The first extra dimension added to the model is a measurement dimension named "s" that is perpendicular to each of the existing three dimensions. The s dimension plays an important role as it provides the direction of quantum vacuum sustaining potential that is analogous to the wind of the box fan.

The next two added dimensions are the electromagnetic dimensions, "e" and "m". The model treats e and m as spatial dimensions with measurements of length.

The model holds that from the perspective of an elementary particle, it has a six-dimensional center position in  $x$ ,  $y$ ,  $z$  visible space, the s dimension, and the dimensions of the e-m plane. The position in e-m is spatial and is different than electro-magnetic fields that are treated separately later. The model pinpoints the center position leaving the size and shape undefined.

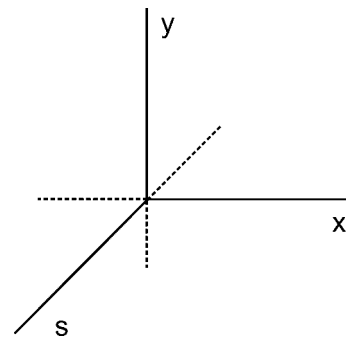


Figure 3. s is shown perpendicular to  $x$  and  $y$ .  $z$  is not shown for clarity.

If extra spatial dimensions actually exist in nature, there may be more than the six dimensions of  $x$ ,  $y$ ,  $z$ ,  $s$ ,  $e$ , and  $m$ , but this is all the model needs to illustrate the operation of a quantum vacuum sustaining force.

The model has a concept of time that is not a dimension. Time is defined by motion and is produced by the magnitude of the quantum vacuum potential. The potential cannot be seen, but the motion of objects can be seen. By observing the motion of one object relative to the motion of another object, time can be measured. Through motion, the magnitude of quantum vacuum sustaining potential also can be measured (more on this later).

Measuring time is much like the zeroth law of thermodynamics [10]. There is not an absolute measurement but a relative one that works well for calculations. The zeroth law of time could state: *An event occurs when an object changes position relative to another object. If event A has the same duration as event B (a clock), and if event B has the same duration as event C, then event C has the same duration as event A.* For example, event A could be the Earth making one rotation; event B, the number of ticks on a clock; and event C, the time for a chemical reaction to take place. Essentially, this says we can use a clock to measure the time of events relative to other events. Of course, the model accounts for quantum and relativistic effects according to the observing reference frame. With a set of dimensions and time, we are ready to look at the quantum vacuum potential.

## Force of a Quantum Vacuum Sustaining Potential

Earlier, the quantum vacuum potential was described as perpendicular to three-dimensional visible space in order for the sustaining force, analogous to the wind, to act evenly in all directions. The model, however, requires the sustaining force to first act in the electromagnetic  $e$ - $m$  plane.

Imagine the surface of a pond as representing the  $e$ - $m$  plane. Let's say north is the  $e$  direction and east is the  $m$  direction. Now place a water hose in the pond with the end of the hose pointing straight up just under the surface of the pond. Turn the water hose on just enough to make a small hill of water that flows out evenly in all directions of the pond.

At the top of the water hill, place a ping-pong ball. At first, it rapidly accelerates to one side, and as it gets farther away the acceleration slows as the water moves slower. This is a very loose analogy of how an elementary particle is accelerated into the  $e$ - $m$  plane by the quantum vacuum sustaining force. Since the pond is a limited analogy, let's leave it behind and look at an elementary particle at a much smaller scale. At first, the force into the  $e$ - $m$  plane is very great but reduces inversely proportional to the distance into the plane. It matters little which direction into the  $e$ - $m$  plane. The force acts in the radial direction outward from the zero  $e$ - $m$  position.

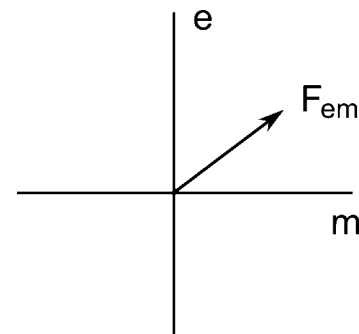


Figure 4. The quantum vacuum sustaining force first presses a particle into the  $e$ - $m$  plane.

The elementary particle would continue accelerating (although gradually less acceleration as it goes) into the e-m plane except two things happen. First, the quantum vacuum potential producing the acceleration is counteracted by the kinetic potential of the particle. In other words, as the particle picks up speed into the e-m plane, the difference between the quantum vacuum potential and the kinetic potential goes to zero, and the force goes to zero. The particle is still moving like a baseball hit by a bat. The ball is still moving, but the force has stopped.

Second, the particle still has a large speed heading outward into the e-m plane, which is where a new interaction happens. The kinetic potential of the particle produces a force perpendicular to its current direction towards other directions in  $x$ ,  $y$ ,  $z$ ,  $s$ ,  $e$ , and  $m$ . Why the force is perpendicular to the motion is a mystery but it is the same mechanism as a known and measured force in physics. That mechanism is seen in the Lorentz force [6] where force is produced perpendicular to both electron motion and a magnetic field. The direction of force in the model can be any direction in six dimensions, but it follows specific rules similar to the right-handed rule in the Lorentz force. Technically this is a cross product [4] of the existing motion and the quantum vacuum potential. In other words, a force is developed at right angles to the existing motion. The direction of this force constantly changes as the particle changes internal direction and speed. Depending on the kind of particle, it may end up in a fuzzy sphere for a proton or in a straight-line motion with oscillation for a photon.

You may find the preceding description to be a lot to absorb with this limited description, especially in visualizing with the extra dimensions. Optionally, for those who want to analyze equations, see (1) and (2) in the technical endnotes.

In all particles, an oscillation is set up in the e-m plane that appears to have a self-regulated average radial magnitude. As forces (introduced later) press the particle towards zero in the e-m plane or slow the particle, the sustaining force gets that much stronger to press it back to its stable magnitude. (Remember the force is inversely proportional to the distance into the e-m plane.) As the position moves towards zero the force grows to press it out. This feature regulates the average radial distance into the e-m plane like a cruise control regulates the speed of a car.

Now that the model has a rough description for the transfer of sustaining force to the particle motion through the e-m plane and into visible  $x$ ,  $y$ , and  $z$  dimensions, several interesting features arise. One feature is that motion also may develop in the  $s$  dimension in the shape of an oscillation. Later, this  $s$  dimension motion becomes important in the quantum interactions between elementary particles.

Another important feature of the model is that the position of the particle in the e-m plane produces energy potential in surrounding space depending on the distance and direction into the e-m plane.

## **Induced E-M Directional Potential**

Up until now the model describes the internal motion of a particle with no other particles around. When other particles are added to the system, a new interaction is introduced. A particle's distance and direction in the e-m plane causes an energy potential that extends into space affecting other particles. If the particle lies purely in the  $e$  direction, it produces an electric

potential as an electric field. If it lies purely in the  $m$  direction, it produces a magnetic potential and field but no electric potential. If it lies somewhere between  $e$  and  $m$ , it produces both an electric potential and a magnetic potential, although it is not really two different potentials. It is one potential and field with a direction in the  $e$ - $m$  plane. When the position changes in the  $e$ - $m$  plane, which it constantly does, the potential direction and field change with it. Optionally, see equation (3) in the technical endnotes.

Particles such as electrons and protons have an average position in the  $e$ - $m$  plane that is held in the negative  $e$  direction or positive  $e$  direction, giving them a permanent average potential or charge. These particles still move around in the  $e$ - $m$  plane, but the average position is held in one direction by the nature of the way the particles interact with the combination of cross product (perpendicular) forces for the type of particle.

While the model treats all directions in the  $e$ - $m$  plane as receiving sustaining force, there is one direction that seems to have a natural property making all electrons or protons, for example, have the same average direction rather than randomly distributed  $e$ - $m$  directions. The model essentially rotates the  $e$ - $m$  measurement dimensions so that  $e$  lies on the natural average direction of this charge. For a photon there is not an average direction in the  $e$ - $m$  plane, where it just rotates. Instead, the photon receives a constant straight-line force in a visible direction.

The induced energy potential, produced by an elementary particle's position in the  $e$ - $m$  plane, extends outward into the  $x$ ,  $y$ , and  $z$  dimensions as an ever-reducing energy potential known as a field. The energy potential drops off inversely proportional to the distance from the particle. Here we turn to standard electromagnetic theory with an interesting twist. The sustaining force model does not use charge in the equations. Instead, it is the average distance into the  $e$ - $m$  plane that produces our concept of electric charge. Incidentally, the equation used to calculate force between two charged particles, Coulomb's force equation [6], may be revised with simpler units with this approach. Optionally, see equation (4).

With a revised equation for Coulomb force, the model shows that the force between two elementary particles is dependent on each of their instantaneous positions and directions within the  $e$ - $m$  plane and their distance from each other.

### The Strong Interaction

In the preceding section the quantum vacuum sustaining force model describes the Coulomb force as being dependent on an elementary particle's distance and direction within the  $e$ - $m$  plane. When particles are far away from each other, the  $e$ - $m$  directional potentials average out, and it is only the average positions and directions that matter in the form of electric charge.

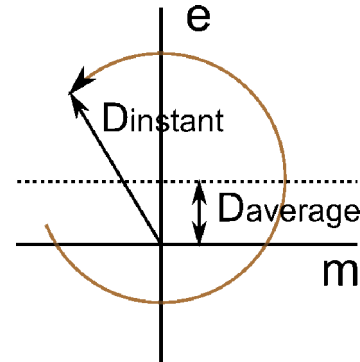


Figure 5. A particle induces an  $e$ - $m$  directional potential dependent on its distance and angle. Protons and electrons have an average distance in  $e$ . Later the instantaneous direction is important for the strong interaction.

Bring the particles closer than several widths of a proton, however, and Coulomb's force gains an important aspect dependent on the particles' instantaneous distance and direction within the e-m plane. When two particles have opposite directions in the e-m plane, they attract, while particles with the same direction repel. When the frequencies of the particles match and are in opposite phase, particles suddenly see a nearly continuous attractive force. Two protons still have an average repulsion but may suddenly have much less repulsion and need only one more ingredient to stick together.

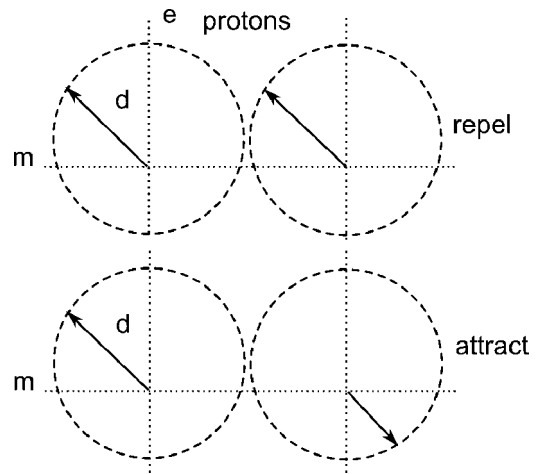


Figure 6. The direction of particles in e-m causes them to repel or attract because of their induced e-m directional potentials producing the strong interaction. The average force for protons is repulsion.

Let's examine the interaction between an electron and a photon. An electron may absorb a photon of sufficient energy/frequency. In the sustaining force model it is the e-m oscillating Coulomb force at close distance that keeps the particles together in their synchronized motion. The photon has the effect of increasing the electron's internal velocity while the self-regulating features of the sustaining force keep a constant average distance into the e-m plane maintaining a constant charge. In essence, the photon's energy is added to the electron, which is important for the next interaction.

When an electron gains sufficient energy/frequency and comes into close proximity with a proton, the directional Coulomb force can synchronize to cause the proton to absorb the electron producing a neutron. Normally, an electron not having the right frequency cannot be completely absorbed because their phases periodically line up to repel, getting repeatedly kicked out and appearing to do a random dance. In the quantum vacuum sustaining force model, a neutron is composed of a proton, an electron, and sufficient photon energy to provide the necessary energy/frequency/phase match.

When a proton and a neutron of sufficient energy/frequency come together, the e-m directional potentials can synchronize to form a net attractive force. The sustaining force model takes a proton and a neutron, such as in a deuterium atom [5], and sums up these synchronized directional Coulomb forces to calculate a net attractive force explaining why they stick together.

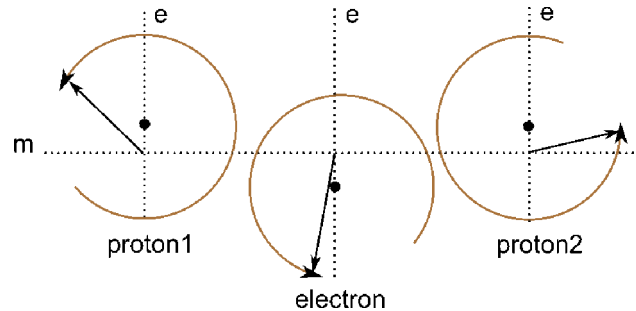


Figure 7. A deuterium atom nucleus where particles attract through synchronization of their rotating e-m potentials.

In other words, two protons have an average repulsive force, although parts

of their e-m oscillation may temporarily attract while the other parts of the oscillation repel. By adding an electron, the ingredients are provided for attractive forces to synchronize, like gears, and be greater than the repulsive forces. The two protons apparently share the electron instead of it being bound to just one proton. If for some reason a free neutron is produced, experiment shows it has a half-life of about 12 minutes [4]. A neutron needs at least one other proton to be stable. Helium, the next common atom up from Hydrogen, has two protons and two neutrons indicating a greater number of particles in the nucleus offer greater stability. Interestingly, a Helium atom in the sustaining force model consists of four protons, two electrons, and some photons. It is expected that computer simulations using the model's directional e-m potentials could provide accurate modeling and interesting predictions for atomic nuclei. A computer model, however, needs to take into account six dimensions of motion.

## **Electromagnetic Potential and Gravity**

Let's take a look again at the quantum vacuum potential and examine the nature of gravity. In the model, a constant quantum vacuum potential exists throughout space defined as  $S$ , and it applies force into the e-m plane. In addition, a new thing happens not previously discussed. A particle existing in space reduces the maximum electromagnetic potential available to other particles. The amount of potential reduction depends on the first particle's energy and inversely on the distance between particles. Optionally, see equation (5) in the technical endnotes.

It is the gradient of the e-m potential reduction caused by other particles that produces the force or acceleration of gravity on a given particle. Taking the differential of this potential reduction equation produces the equation for the acceleration of gravity.

Essentially, the model holds that gravity comes through the same mechanism as electromagnetic force. Gravity, however, does not use a directional e-m plane potential but instead applies to the overall e-m potential magnitude. Even particles with no net charge have an oscillation in e-m and come under the influence of a gradient in the e-m potential magnitude, and receive the force of gravity.

The model also attempts to define particle energy by its velocity and distance within the e-m plane. Those who ventured a look at equation (6) may have been surprised by the terms used. Instead of mass and a gravitational constant as used by Newton's gravitational equation, it uses the particle velocity and the average radial distance into the e-m plane. The units come out very clean with no special constant to supply the missing units. It would be very difficult, however, to measure the velocity and distance in the e-m plane, so it still is more practical to use Newton's gravitational equation. Optionally, see equation (6).

## **Relativity of Time and the Value of S**

When considering relativity, the model treats time with respect to an inertial reference frame with no other particles around. When things start moving and other particles are introduced, the energy potential within the e-m plane changes. A clock within this new reference frame will measure time differently.

The model introduces relativity through the magnitude of the quantum vacuum sustaining potential,  $S$ , with units of  $m^2/s^2$ . The model multiplies  $S$  by the particle mass to get units of



energy. Then, the well-known energy-mass equation is associated to get  $E = mc^2 = mS$ . The mass, algebraically, drops out to produce  $c^2 = S$ . The speed of light is on the left hand side to show that this speed is dependent on the available quantum vacuum sustaining potential. In other words, as the available potential changes, the speed of light changes. Interestingly, a particle cannot tell that the speed of light has changed because time is relative to  $S$ . Optionally, see equation (5).

This approach of viewing relativity through a changing speed of light is not the primary focus of the model at this point. It simply represents an interesting idea that may be explored further as the model develops.

## Conclusion

The quantum vacuum sustaining force model explores how a single dimensional potential could propel the internal motion of particles. Using observed (but otherwise unexplained) behavior from nature such as the Lorentz force we can see how perpendicular forces can drive the motion of particles like a motor. The internal motion, then, generates a secondary potential with a gradient that affects other particles. The force of this potential gradient produces electromagnetism, the strong interaction, and gravity. Optionally, see summarizing equation (7). At this introductory level, only the tip of the iceberg is imperfectly described through simplified descriptions and equations.

The model helps explain the first law of thermodynamics where energy cannot be created or destroyed. With self-regulating forces keeping particles from collapsing, it becomes understandable how particles can seem to have their own self-contained energy with incredibly long lives. The motion in extra dimensions, especially in  $s$ , helps explain some quantum data such as spin, anti-particles, and effect-before-cause such as when an electron emits a photon before a photon is absorbed [2]. These topics, not analyzed in this essay, deserve whole papers on their own. For, now it is the motion in the  $s$  dimension that may help explain Heisenberg measurement uncertainty and offer clues to other discrete quantum effects.

I find it very interesting that using a hidden force model allows existing fundamental forces to be reinterpreted with units naturally falling into place. I also find it fascinating that we seem to have no knowledge on which to base the quantum vacuum potential and of what it is made. Can anything explain the root cause of the Lorentz force equation? Perhaps it could be explained as a digital program running in a true reality. Space as a quantum computer makes a good logical device for analyzing the concepts of our own reality and the limits of exploring a deeper reality. The sustaining force model, however, describes motion in analog terms and leaves the quantum vacuum potential as undefined beyond the point where we can measure it.

While more of an exploration than an authoritative report on a sustaining force, this essay describes ideas I have been developing and hope to test further against more data. I hope that you find this introduction to the quantum vacuum sustaining force model interesting as well as relevant to the theme: "Is Reality Digital or Analog?"

## Technical Endnotes

The following equations are provided for illustration only to help define the general ideas discussed in the essay and are not intended as final forms.

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Sustaining force into the e-m plane:

$$F_{em} = \frac{m}{r_{em}} (S - V^2) \quad \text{Equation (1)}$$

S is the sustaining potential in  $m^2/s^2$  with a value of approximately  $c^2$ ; V is the particle velocity in all dimensions; m is the particle mass; and  $r_{em}$  is the distance into e-m. The units for force readily work out to  $kg\ m / s^2$ .

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Multi-dimensional cross product producing force at right angles to the motion, ultimately developing a spinning motion in its many dimensions (not exact; will depend on particle type):

$$F = \frac{m}{r_{em}} (V_s \times V) \quad \text{Equation (2)}$$

The resulting motion resembles the wave observations of Schrödinger, de Broglie, and others.

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Potential produced by a particle's position in the e-m plane:

$$P = \frac{kSD}{r} \quad \text{Equation (3)}$$

D is the particle distance and direction in e-m, k is a constant in  $kg/m$ , and r is the distance away from the particle. To get the force on particle2 caused by particle1, we take the differential of the potential in (3), multiply by the distance in e-m of particle 2, and account for e-m directionality of each particle with a cosine function.

$$F = \frac{kSD_1D_2}{r^2} \cos(\theta_2 - \theta_1) \quad \text{Equation (4)}$$

When the particles are farther apart than the distance of a proton, it is the average e-m distance and angle that becomes the controlling factor. At distances the size of a proton, the

instantaneous angle becomes very important for the strong nuclear force between particles as the particles continuously rotate in e-m. With only k in kg/m providing adjustment to units, the resulting units of force are readily seen from the equation as kg m / s<sup>2</sup>.

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The absolute electromagnetic potential reduction for other particles is modeled by an adjustment to the sustaining potential S. The full equation is complicated by multiple particle interactions. For a single particle, the simplified equation:

$$S \cong S_0 - \frac{V_{em}^2 d}{r} \quad \text{Equation (5)}$$

V<sub>em</sub> is the particle velocity in the e-m plane; d is the average radial distance into the e-m plane; and r is the distance away from the particle. It is the gradient in the potential that produces gravitational acceleration on other particles. The gravitational force on a second particle, then, is the mass of the second particle multiplied by acceleration induced by potential gradient from the first particle.

$$\begin{aligned} F_2 &= m_2 a = m_2 \frac{dS}{dr} \cong m_2 \frac{V_{em1}^2 d_1}{r^2} \\ &\approx \frac{Gm_1 m_2}{r^2} \end{aligned} \quad \text{Equation (6)}$$

It is interesting to note that the acceleration of a particle is independent of its own energy just as Galileo noted that gravity is independent of a falling object's mass. Equation (6) can be related to Newton's gravitational equation by assuming V<sup>2</sup>d is equal to Gm<sub>1</sub>. Note that using V<sup>2</sup>d allows the units for force to readily work out to kg m / s<sup>2</sup> without a special constant.

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From (4) and (6) the combined force on a particle due to another particle demonstrating e-m directional Coulomb force, the strong interaction, and gravity:

$$F_2 = \frac{kSD_1 D_2 \cos(\theta_2 - \theta_1) - m_2 V_{em1}^2 d_1}{r^2} \quad \text{Equation (7)}$$

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