Indefinite causal structure can enhance predictability

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Abstract

I argue for the presence of indefinite spacetime causal structure in nature, and show how it removes predictability obstacles regarding 1) spacetime singularities, 2) ultra-high energy quantum field theory, and 3) quantum gravitational computations.

OLD SETH LOST HIS HORSE

There goes a millennium old legend[1]: "Old Seth lived near the border and was good at making predictions. His horse ran off to the foreign land with no reason. People comforted him, and he said: 'Why cannot this be a blessing?' Months later the horse returned, bringing another fine horse from the foreign land. People congratulated him, and he said: 'Why cannot this be a misgiving?' His son liked the fine horse, rode it, fell off and broke his leg. People comforted Seth, and he says: 'Why cannot this be a blessing?' One year later foreign soldiers invaded. Young and healthy men all took up their weapons to defend. Most died, but Old Seth and his son survived, for being old and going lame..."

INDEFINITENESS CAN ENHANCE PREDICTABILITY

To some people, it is unbelievable that indefiniteness can enhance predictability. To say there is indefiniteness in some physical quantity *is* to say there is unpredictability in that quantity. If a quantity can be predicted, it is not indefinite. Indefiniteness reduces predictability.

To some others, it is obvious that indefiniteness can enhance predictability. By incorporating uncertainties into mechanical laws, quantum mechanics is able to make far better predictions than deterministic classical mechanics on phenomena of all kinds, from running couplings in subatomic particle colliding, to tunnelings in superconductors, to fluctuations in the Cosmic Microwave Background.

Of course both views are correct and compatible, as they are making comparisons of different kinds. The former compares theories without reference to real world phenomena. Here one is free to imagine an idealistic world predicted perfectly by the theory with less indefiniteness. The latter compares theories in their description of real world phenomena. Here nothing prevents a theory with more indefiniteness to make predictions that agree better with certain actual phenomena.

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[1] This legend is from the *Renjian* chapter of *Huainanzi*, an ancient Chinese essay collection formed around 2nd century BC. The translation is mine.

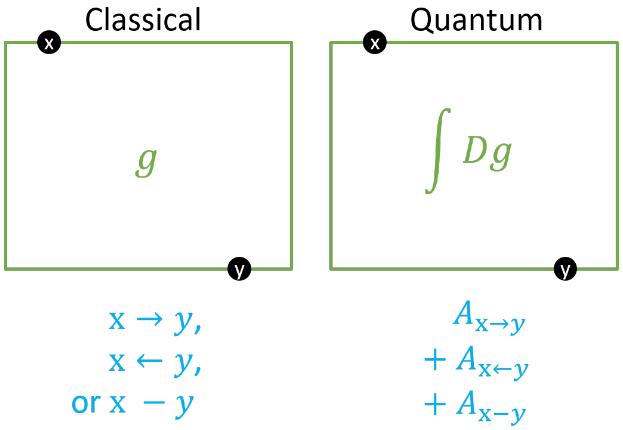

FIG. 1. On a classical spacetime configuration g, x precedes y, y precedes x, or x is disconnected with y. In a gravitational path integral amplitudes corresponding to the three cases are summed over.

In the latter sense indefiniteness can indeed enhance predictability, but this is hardly an exciting observation – hundreds of textbooks on quantum mechanics know it.

The point of this essay is that *indefiniteness in spacetime causal structure can enhance predictability regarding open problems of fundamental physics*. This is a topic of active research and certainly goes beyond textbooks quantum mechanics which presuppose a definite classical spacetime background. I discuss the prospects of having indefinite spacetime causal structures *resolving spacetime singularities, extending quantum field theories into the ultraviolet regime, and improving the computability of hardcore quantum gravity.*

AFFIRMING INDEFINITE CAUSAL STRUCTURE

The term “indefinite causal structure” is introduced by Lucien Hardy [1] in studies [2] of quantum gravity and quantum foundations:

> **Indefinite causal structure** is when there is, in general, no matter of fact as to whether the separation between two events is time-like or not.

There has recently been a surge of interest in indefinite causal structure, leading to inspiring works on both theoretical [3, 4] and experimental fronts [5–10]. While much remains to be clarified regarding the formalism of theoretical descriptions [11, 12] and the interpretation of experimental demonstrations [13], indefinite causal structure *is* expected to be in nature.

Indefinite causal structure is expected in quantum spacetime, since in General Relativity spacetime causal structure is dynamical, and in quantum theory dynamical quantities exhibit uncertainty [1, 2]. Concretely, indefinite causal structure is generically present in theories of quantum gravity that admit a path integral formulation (Figure 1). Since all spacetime configurations with different causal relations for events x and y (which can be identified physically meaningful e.g., by specifying the values certain scalar fields take at these events [14–16]) are quantum summed over in the region of interest, the causal structure is indefinite.

PRINCIPLE OF CAUSAL NEUTRALITY

Nature could surprise us and disappoint the expectation of indefinite causal structure. Nevertheless a practical strategy for a physicist is to take what general relativity and quantum theory suggest, work out the theoretical consequences, and test against experience.

In [17] I proposed the **principle of causal neutrality** to guide the theoretical investigations of indefinite causal structure:

> Fundamental concepts and laws of physics should be stated without assuming a definite spacetime causal structure.

This urges us to reexamine, generalize, and modify many fundamental concepts, results, and theories. For example, the concept of entanglement conceived only for spacelike systems needs, and can be generalized [18]. The axioms and derivations of quantum theory need, and can be generalized [19]. Other concepts and theories awaiting reconsideration are abundant, e.g., the microcausality axiom of quantum field theory (QFT), the notions of cosmological/black hole horizon, the "horizon problem" of the early universe, the notion of time's arrow and the second law of thermodynamics...

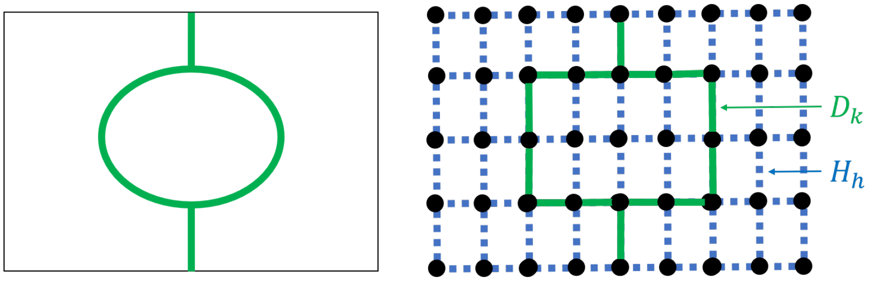

FIG. 2. Traditional Feynman diagram vs. correlation diagram for WQG

WORLD QUANTUM GRAVITY

Ultimately we want a theory of indefinite spacetime causal structure capable of making quantitative predictions. This would allow us to investigate the implications of indefinite causal structure on open problems of fundamental physics.

To meet this challenge I recently developed the World Quantum Gravity (WQG) formalism [20]. The key idea is to *express gravity in terms of the relational world function* $\sigma(x,y)$ [21], *instead of the metric field* $g_{ab}(x)$ *or other variables*. The **world function** is one half the squared geodesic distance, i.e., for $ds^2 = g_{ab}dx^a dx^b$, $\sigma(x,y) = \frac{1}{2}\int_x^y ds^2$, where the integral is along the geodesic from x to y. The metric g_{ab} can be extracted from σ through $g_{ab}(x) = -\lim_{y\to x} \frac{\partial}{\partial x^a}\frac{\partial}{\partial y^b}\sigma(x,y)$ [21].

The theory's configuration space lives on a skeleton graph[2] (e.g., illustrated on right of Figure 2). Each edge has two degrees of freedom, with $\sigma \in \mathbb{R}$ describing spacetime distance, and $\rho \in \mathbb{R}$ (originating from the second order derivative of σ) describing curvature. Importantly, σ **indicates causal structure**: $\sigma <,=,> 0$ manifestly correspond to time-, light-, and space-like separations.

The form of the probability amplitude for processes on quantum spacetime is analogous to Feynman diagram amplitudes of ordinary QFT (left of Figure 2):

$$A_M = \sum_\Gamma \prod_i \int dx_i \frac{V[\Gamma]}{N[\Gamma]} \prod_{k\in\Gamma} G_k, \tag{1}$$

where the sum is over Feynman diagrams Γ, the products are over vertices i and edges k of the diagrams, V is the vertex factor, and N is the symmetry factor. In WQG we have

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[2] See [22] for a discussion on the deeper reasons to locate degrees of freedom on a discrete graph, and why using a discrete skeleton in itself does not imply that spacetime is fundamentally discrete.

(right of Figure 2):

$$A_{QG} = \lim_{\text{sk.}} \sum_{g} \sum_{\Gamma} \frac{V[\gamma_\Gamma]}{N[\gamma_\Gamma, \sigma]} \prod_{h \notin \gamma_\Gamma} H_h(\sigma_h, \rho_h) \prod_{k \in \gamma_\Gamma} D_k(\sigma_k, \rho_k). \tag{2}$$

As illustrated in Figure 2, an old matter diagram Γ on the left is broken into smaller pieces to become the new graph γ_Γ in green with **matter-gravity amplitudes** D attached to its solid edges. Dashed edges are introduced to locate vacuum gravitational degrees of freedom and **pure-gravity amplitudes** H. The functional form of D and H will be presented below.

$V[\gamma_\Gamma]$ is the same vertex factor as in the old Feynman diagrams. $N[\gamma_\Gamma, \sigma]$ is a generalization of the Feynman diagram symmetry factor to count the number of skeleton graph relabelling that preserves the matter subgraph γ, the (σ, ρ)-configuration on the edges, and $V[\gamma_\Gamma]$ on the vertices. $\sum_\Gamma$ is the same sum over matter Feynman diagrams. $\lim_{\text{sk.}}$ indicates that a particular skeleton graph gives an approximation to the physical amplitude A_{QG}, the exact value of which is approached by going to ever larger skeleton graphs.

HARDY SUM

The path integral measure $\sum_g$ is a crucial object:

$$\sum_g = \prod_j \int_{-\infty}^{\infty} d\rho_j \int_{-a(\rho_j)^2/2}^{a(\rho_j)^2/2} d\sigma_j, \quad a(\rho) = \begin{cases} b\pi\sqrt{\frac{3}{\rho}}, & \rho > 0, \\ \infty, & \rho \le 0. \end{cases} \tag{3}$$

Different causal structures corresponding to different signs of σ_j are summed over on every edge j. This sum is so important that it deserves a special name. For an arbitrary function $f(\sigma)$,

$$\int_{-a^2/2}^{a^2/2} d\sigma f(\sigma) = \int_{-a^2/2}^{0} d\sigma f(\sigma) + \int_{0}^{a^2/2} d\sigma f(\sigma) \tag{4}$$

$$= \int_0^a ds\, s[f(\sigma = -s^2/2) + f(\sigma = s^2/2)], \tag{5}$$

where we changed variable from σ to $s = |2\sigma|^{1/2}$, the geodesic distance. Since σ is second order in s, we gather an additional factor of s in the integrand. We call

$$f_{\%}(s) := f(\sigma = -s^2/2) + f(\sigma = s^2/2) \tag{6}$$

the **Hardy sum** of $f(\sigma)$, in honor of Lucien Hardy who pioneered the study of indefinite causal structure as a central feature for quantum gravity [2]. The symbol "%" is a pictorial representation of combining timelike and spacelike contributions (two circles) separated by the lightcone (tilted line).

RESOLVING SPACETIME SINGULARITIES

General relativity fails to give physical predictions at what it describes as spacetime singularities, where spacetime curvature goes to infinity [23]. Intuitively, classical spacetime singularities originate from the gravitational focusing of non-spacelike curves. For example, in a Schwarzschild black hole spacetime curves which had entered the event horizon are strongly attracted so that they cannot escape and will terminate at the singularity in finite time...

Unless the curves have spacelike parts, which would allow the curves to escape. On a classical spacetime information carrying matter propagates on non-spacelike curves, so there is no escape. Yet on quantum spacetime with indefinite causal structure, no curve of information propagation is strictly non-spacelike. There is hope that this resolves singularities.

However, a gravitational path integral sums over all spacetime configurations, including singular ones. What does it mean to resolve singularities in this setting? Remarkably, in WQG singular configurations have zero amplitudes so do not contribute to the path integral. In this sense singularities are resolved.

To see this, we need the formula for the pure-gravity amplitude:

$$H_h(\sigma_h, \rho_h) = \frac{\sigma_h}{\sigma_h - 6i\alpha_h s_h \Delta_h^{-1} \ln \Delta_h}, \tag{7}$$

where $s_h = |2\sigma_h|^{1/2}$, $\Delta_h(s_h, \rho_h) = \mathrm{sinc}^{-3}\left(s_h\sqrt{\frac{\rho_h}{3}}\right)$, α_h is the neighboring edge coupling which for the present consideration can be regarded as a constant, and $0 < b < 1$ is a constant parameter of the theory.

Recall that ρ measures spacetime curvature. Singular spacetimes with divergent curvature has $\rho \to \infty$ on some gravitational edge(s). The constraint (3) implies $s \to 0$ there. If we plug in H_h for f in (5) and calculate the $s \to 0$ limit we find that the integrand goes to zero. That is, with a sum over causal relations, singular spacetime configurations contribute zero amplitudes to the gravitational path integral [24].

REGULARIZING ULTRAVIOLET DIVERGENCES

Matter QFT stops being predictive beyond Landau poles, which originate from ultraviolet (UV) divergences generically present for ordinary QFT. Since UV divergences appear in the limit of lightlike separation (short invariant distance) [25], it is intuitively reasonable that the gravitational path integral induces a smearing around the lightcone to cure the divergences [26–29]. WQG realizes this idea in a non-perturbative theory of quantum gravity.

The matter-gravity amplitude for a scalar matter field is:

$$D_k = \Delta_k^{1-3\xi} \int_0^\infty \frac{dl_k}{(4\pi i l_k)^2} \exp\{i[\sigma_k/2 - 3i\alpha_k s_k \Delta_k^{-1} \ln \Delta_k]/l_k - im^2 l_k\}, \tag{8}$$

where Δ_k and α_k are the same as in (7), ξ is the matter-gravity coupling constant, and m is the mass of the scalar field. The term proportional to α_k accounts for quantum gravitational correction to the matter propagator. Taking it away gives $D_k = \Delta_k^{1-3\xi} \int_0^\infty \frac{dl_k}{(4\pi i l_k)^2} \exp\{i\sigma_k/2l_k - im^2 l_k\}$, which up to a factor $\Delta_k^{1-3\xi}$ coincides with the Schwinger proper time representation [30] of the ordinary Feynman propagator. In the absence of quantum gravity, D_k inherits the standard ultraviolet divergences.

Quantum gravitational corrections show up at three places: a) the α-dependent term in (8), b) the Hardy sum $D_{\%}(s) = D(\sigma = -s^2/2) + D(\sigma = s^2/2)$ over causal relations as part of $\sum_g$, and c) the extra factor s in the integrand of the s-integral in (5) as part of $\sum_g$. The relevant quantity is then $sD_{\%}$. One can show that when $\rho < \infty$, $\lim_{s\to 0} s\overline{D}_{\%} = \frac{\rho\alpha}{\pi^2} < \infty$, and when $\rho \to \infty$, $\lim_{s\to 0} sD_{\%} = [24\pi^2\alpha \,\mathrm{sinc}^{6-9\xi}(x) \ln\left(\mathrm{sinc}^{-3}(x)\right)]^{-1} < \infty$ for some constant $0 < x < b\pi$. The three parts of quantum gravitational corrections work together to ensure UV-finiteness, and every part is indispensable [24].

COMPUTABILITY

To make quantitative predictions with any theory of non-perturbative quantum gravity one must face the challenge of computation. Here the three elements of Lorentzian signature, many degrees of freedom, and efficient computation are in tension.

By analytic continuation into the Euclidean signature, Monte Carlo simulation allows for efficient computation of the partition function with many degrees of freedom. Yet it is not known in general how to relate the result back to the Lorentzian case. By a reduction to few degrees of freedom, the computation problem can be greatly simplified and even analytic solutions are possible for special cases. Yet it is not known in general how the rest degrees of freedom affect the simplified background non-perturbatively. By dropping the requirement for efficient computation, we can still investigate certain qualitative properties of the theories we write down. Yet it is hard to extract quantitative predictions.

WQG offers a way out of the conundrum for pure gravity and gravity coupled with massless matter fields. Summing over causal relations turn complex numbers into non-negative real numbers to allow efficient Monte Carlo simulation for many degrees of freedom without needing to analytic continue to the Euclidean signature.

This follows from $H(\sigma = -s^2/2)$ and $H(\sigma = s^2/2)$, as well as massless $D(\sigma = -s^2/2)$ and $D(\sigma = s^2/2)$ forming complex conjugate pairs, respectively. Hence their Hardy sums are real. Furthermore, one can show that $H_{\%} \geq 0$ and $D_{\%} \geq, \leq 0$ for $\rho \geq, < 0$, respectively. Then Monte Carlo simulation can be applied efficiently without the problem of cancellation of complex amplitudes in opposing directions in the complex plane [24].

ON GENERALITY

The above results on enhanced predictability [24] are new and are based on a recently proposed still very speculative approach to quantum gravity [22]. We should leave some time to look for flaws, limitations, and mistakes in the works before drawing any firm conclusions.

Meanwhile we should consider if any general lesson can be gathered that is applicable beyond the particular approach of WQG for quantum gravity.

Perhaps the resolution of spacetime singularity and the regularization of UV divergence by indefinite causal structure are generic among different approaches to quantum gravity? A previous model-independent study adds some weight to this conjecture. Using an information-theoretic language [4], it was found that indefinite causal structure reduces quantum correlations generically, and the more indefinite the causal structure, the larger the reduction [31, 32]. In quantum gravity large causal structure fluctuations is expected at short distance scales. One cannot help but speculate that quantum gravitational indefinite causal structure reduces quantum correlations and amplitudes in the ultraviolet generically in theories of quantum gravity.

ON PREDICTABILITY AND INDEFINITENESS

There is a message to draw from Old Seth's story at the beginning: The matter of blessings and misgivings is subtle, and they sometimes give rise to each other.

I suppose the same can be said about predictability and indefiniteness.

Einstein believed that "God does not play dice". Many people share this belief, and set out to look for hidden-variables/ontological models of nature [33], for they think "reality" should be less indefinite than described by quantum theory. I think the lure to minimize indefiniteness for its own sake may lead to a trap that obscures the big picture.

I hope to have demonstrated with this essay that our overall understanding of natural phenomena – measured by our capability to make predictions – can be enhanced by incorporating indefiniteness. *More importantly, there are opportunities for indefiniteness to enhance predictability on yet open problems in fundamental physics.*

Einstein also believed that "God is subtle" [34]. The subtle interplay between predictability and indefiniteness may be the place to look for the deeper patterns of nature.

REFERENCES

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[1] Lucien Hardy. Quantum gravity computers: On the theory of computation with indefinite causal structure. In *Quantum Reality, Relativistic Causality, and Closing the Epistemic Circle*, pages 379–401. Springer, 2009.

[2] Lucien Hardy. Probability Theories with Dynamic Causal Structure: A New Framework for Quantum Gravity. *arXiv:gr-qc/0509120*.

Lucien Hardy. Towards quantum gravity: A framework for probabilistic theories with non-fixed causal structure. *Journal of Physics A: Mathematical and Theoretical*, 40(12):3081–3099, 2007.

[3] Giulio Chiribella, Giacomo Mauro D'Ariano, Paolo Perinotti, and Benoit Valiron. Quantum computations without definite causal structure. *Physical Review A*, 88(2):22318, 8 2013.

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron. Quantum computations without definite causal structure. *arXiv:0912.0195*.

[4] Ognyan Oreshkov, Fabio Costa, and Časlav Brukner. Quantum correlations with no causal order. *Nature Communications*, 3:1092, 10 2012.

[5] Lorenzo M. Procopio, Amir Moqanaki, Mateus Araújo, Fabio Costa, Irati Alonso Calafell, Emma G. Dowd, Deny R. Hamel, Lee A. Rozema, Časlav Brukner, and Philip Walther. Experimental superposition of orders of quantum gates. *Nature Communications*, 6:7913, 8 2015.

[6] Giulia Rubino, Lee A. Rozema, Adrien Feix, Mateus Araújo, Jonas M. Zeuner, Lorenzo M. Procopio, Časlav Brukner, and Philip Walther. Experimental verification of an indefinite causal order. *Science Advances*, 3(3):e1602589, 2017.

[7] K. Goswami, C. Giarmatzi, M. Kewming, F. Costa, C. Branciard, J. Romero, and A. G. White. Indefinite causal order in a quantum switch. *Physical Review Letters*, 121(9):90503, 8 2018.

[8] K. Goswami, J. Romero, and A. G. White. Communicating via ignorance. *arXiv:1807.07383*.

[9] Kejin Wei, Nora Tischler, Si-Ran Zhao, Yu-Huai Li, Juan Miguel Arrazola, Yang Liu, Weijun Zhang, Hao Li, Lixing You, Zhen Wang, Yu-Ao Chen, Barry C. Sanders, Qiang Zhang, Geoff J. Pryde, Feihu Xu, and Jian-Wei Pan. Experimental Quantum Switching for Exponentially Superior Quantum Communication Complexity. *Physical Review Letters*, 122(12):120504, 3 2019.

[10] Yu Guo, Xiao-Min Hu, Zhi-Bo Hou, Huan Cao, Jin-Ming Cui, Bi-Heng Liu, Yun-Feng Huang, Chuan-Feng Li, Guang-Can Guo, and Giulio Chiribella. Experimental Transmission of Quantum Information Using a Superposition of Causal Orders. *Physical Review Letters*, 124(3):030502, 1 2020.

[11] Ding Jia and Nitica Sakharwade. Tensor products of process matrices with indefinite causal structure. *Physical Review A*, 97(3):32110, 2018.

[12] Philippe Allard Guérin, Marius Krumm, Costantino Budroni, and Časlav Brukner. Composition rules for quantum processes: a no-go theorem. *New Journal of Physics*, 1 2019.

[13] Nikola Paunković and Marko Vojinović. Causal orders, quantum circuits and spacetime: distinguishing between definite and superposed causal orders. *arXiv: 1905.09682*.

[14] Hans Westman and Sebastiano Sonego. Events and observables in generally invariant spacetime theories. *Foundations of Physics*, 38(10):908–915, 10 2008.

[15] Hans Westman and Sebastiano Sonego. Coordinates, observables and symmetry in relativity. *Annals of Physics*, 324(8):1585–1611, 8 2009.

[16] Lucien Hardy. Operational General Relativity: Possibilistic, Probabilistic, and Quantum. *arxiv:1608.06940*.

[17] Ding Jia. Quantum indefinite spacetime: part II. *Master's thesis, University of Waterloo*, 2018.

[18] Ding Jia. Generalizing entanglement. *Physical Review A*, 96(6):62132, 2017.

[19] Ding Jia. Quantum theories from principles without assuming a definite causal structure. *Physical Review A*, 98(3):032112, 7 2018.

[20] Ding Jia. World quantum gravity: An approach based on Synge's world function. *arXiv: 1909.05322*.

[21] J. L. Synge. *Relativity: the General Theory*. North-Holland Publishing Company, Amsterdam, 1971.

[22] Ding Jia. World quantum gravity and the choices leading to it. *PIRSA: 19120028*.

[23] S. W. Hawking and G. F. R. Ellis. *The Large Scale Structure of Space–Time*. Cambridge University Press, 1973.

[24] Ding Jia. Ultraviolet regularity from quantum gravitational indefinite causal structure. *arXiv: 2003.00562*.

[25] John C. Collins. *Renormalization*. Cambridge University Press, Cambridge, 11 1984.

[26] S. Deser. General Relativity and the Divergence Problem in Quantum Field Theory. *Reviews of Modern Physics*, 29(3):417–423, 7 1957.

[27] L H Ford. Gravitons and light cone fluctuations. *Physical Review D*, 51(4):1692–1700, 1995.

[28] T Padmanabhan. Hypothesis of path integral duality. I. Quantum gravitational corrections to the propagator. *Physical Review D*, 57(10):6206–6215, 1998.

[29] Hans C. Ohanian. Smearing of propagators by gravitational fluctuations on the Planck scale. *Physical Review D*, 60(10):104051, 10 1999.

[30] Julian Schwinger. On Gauge Invariance and Vacuum Polarization. *Physical Review*, 82(5):664–679, 6 1951.

[31] Ding Jia. Quantum indefinite spacetime. *Bachelor's thesis, University of Waterloo*, 2017.

[32] Ding Jia. Reduction of correlations by quantum indefinite causal structure. *arXiv:1806.02331*.

[33] Nicholas Harrigan and Robert W. Spekkens. Einstein, incompleteness, and the epistemic view of quantum states. *Foundations of Physics*, 40:125–157, 2010.

[34] Abraham Pais. *Subtle is the Lord: the science and the life of Albert Einstein*. Oxford University Press, 2005.