

Condensed matter lessons about the origin of time

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It is widely hoped that quantum gravity will shed a profound light on the origin of time in physics. The currently dominant approaches to a candidate quantum theory of gravity have quite naturally evolved from general relativity, on the one hand, and from particle physics, on the other hand. In this essay, I will argue that a third important branch of 20th century ‘fundamental’ physics, namely condensed-matter physics, also offers an interesting perspective on quantum gravity, and thereby on the problem of time. The bottomline might sound disappointing to those who have become used to claims that quantum gravity or a ‘Theory of Everything’ will solve most of the conceptual problems of fundamental physics: To understand the origin of time, experimental input is needed at much higher energies than what is available today. Moreover, it is far from obvious that we will ever discover the true origin of physical time, even if we become able to directly probe physics at the Planck scale. But we might learn plenty of interesting lessons about time and the structure of our universe in the process.

I. INTRODUCTION

The pursuit of a quantum theory of gravity is usually presented as a quest for the unification of the two great 20th-century revolutions in physics: quantum mechanics and general relativity. When looking at the currently most popular approaches to quantum gravity, string theory and canonical (loop or spin foam) quantum gravity, we see that these indeed faithfully reflect those two starting points in many respects: string theory is a natural extension of quantum field theory and the associated particle concept, whereas canonical quantum gravity is an attempt to formulate a quantum theory of gravity which is diffeomorphism invariant by construction. But part of the success of both approaches is also sociological. Ironically, shortly after particle physics knew its greatest moment of glory by the experimental confirmation of the standard model of particles, it entered into a deep crisis: all predicted particles had been detected (with the notable exception of the Higgs boson) and nothing fundamentally new was expected for the foreseeable future. So many particle physicists engaged in a more esoteric pursuit: a quantum theory of gravity, which until then had mainly been the privilege of relativists. Indeed, general relativity has seemed incomplete from its very conception, since it is a classical theory which contains its own seeds of destruction by predicting singularities. So the pursuit of a quantum version of general relativity has always been a quite natural goal for its practitioners.

When looking back at the revolution brought about by quantum mechanics, however, particle physics represents only part of the package. While particle physics was entering into the deep crisis just mentioned, condensed-matter physicists were still happily playing around and pursuing Nobel prizes with superconductors, superfluids and the like. So it is perhaps not surprising that little attention has been paid by people working in condensed matter to quantum gravity and vice versa. However, lately, this has started to change.

Indeed, an increasingly popular approach to quantum gravity rests on the idea that gravity (and maybe electromagnetism and the other gauge fields) might be an ‘emergent phenomenon’, in the sense of representing a macroscopic

collective behaviour resulting from a very different microscopic physics. A prominent example of this emergent approach to gravity is precisely the condensed-matter scheme for quantum gravity, which considers the possibility that gravity emerges as an effective low-energy phenomenon from the quantum vacuum, in a way similar to the emergence of collective excitations in condensed-matter systems. A basic observation to this effect is the following. The fundamental ‘quantum gravity’ theory is generally assumed to have the Planck level as its characteristic scale. Expressed as a temperature, this Planck level lies at approximately 10^{32} K. On the other hand, most of the observable universe has temperatures that barely exceed the cosmic background radiation temperature of a few Kelvins. Even the interior of a star such as the sun is more than 20 orders of magnitude colder than the Planck temperature. So the degrees of freedom of the quantum vacuum of our universe, whatever their fundamental structure, are probably effectively frozen out in most of our universe, just like in a laboratory condensed-matter system. The physics that we observe might then well be due to collective excitations that result from the—comparatively tiny—thermal (or other) perturbations of this vacuum. From this point of view, the term ‘high-energy physics’ to describe, say, the energy level at which the Higgs boson is hoped to be detected at the LHC, is slightly ironic.

A more technical observation that supports the condensed-matter approach to quantum gravity is the following. Even in relatively simple, weakly interacting condensed-matter systems, such as Bose-Einstein condensates in atomic gases, the kinematics of the low-energy excitations or phonons can be described by a relativistic field theory. The curved background spacetime is provided by the collective behaviour of the condensed part of the constituent atoms. In more complicated fermionic systems, in particular $^3\text{He-A}$, gravitational and gauge fields emerge as the low-energy bosonic degrees of freedom together with fermionic quasi-matter in a similar way. All these emergent components share surprisingly many characteristics with their counterparts in ‘high-energy’ physics: Einstein gravity and the standard model of particles. This leads one to speculate that these latter theories might really just be effective theories emerging in the low-energy corner of a fundamental theory of quantum gravity describing the ‘atoms’ composing the quantum vacuum.

In what follows, I will try to convey the basic ideas behind this condensed-matter approach to gravity in slightly more detail, and discuss some of the possible lessons that it offers us with respect to the physical origin of time in our universe.

II. CONDENSED MATTER AND EMERGENT GRAVITY

An intriguing theorem of mathematical physics [1] shows that the equation of motion of acoustic perturbations in a perfect (irrotational, inviscid and barotropic) fluid is described by a d’Alembertian equation in curved spacetime:

$$\square\phi \equiv \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu\phi = 0, \quad (1)$$

so that these acoustic perturbations travel along the null geodesics of the effective metric $g_{\mu\nu}$, with g its determinant. This formula might seem a bit scary to the non-specialist, but it is actually well known from relativistic field theory as the equation of motion for a massless scalar field ϕ propagating in a curved spacetime. Moreover, the only input needed to derive it is the Euler equation, the continuity equation and some basic properties of perfect fluids such as their irrotationality. For an effective Minkowski metric, the previous equation simply becomes

$$(-\partial_t^2 + c^2\partial_x^2 + c^2\partial_y^2 + c^2\partial_z^2)\phi = 0, \quad (2)$$

which is the familiar wave equation in the flat spacetime of special relativity, with the speed of sound c playing exactly the same role as the speed of light in relativity. So the acoustic perturbations or phonons in perfect fluid systems behave as if they were moving in a relativistic spacetime, whose metric $g_{\mu\nu}$ is determined by the bulk of the fluid, i.e., by the collective behaviour of its constituent atoms, and in general will be curved as in (1). In particular, the physical time that these perturbations experience is relativistic in exactly the same sense as the one we know from special and general relativity, including all the observer-dependence of simultaneity and associated paradoxes. This is the more curious because the background system in which this relativistic spacetime emerges can simply be described in Newtonian terms: it consists of a fluid in a laboratory, where all velocities are extremely low compared to the speed of light and thus relativistic corrections are irrelevant, and where there is therefore a clear preferred time, imperfectly indicated by the clocks on the wall of the laboratory.

Building on these observations, the idea developed to study certain aspects of general relativity and quantum field theory by analogy with such perfect fluid systems [2]. To take maximal advantage of the analogy, the microscopic physics of the fluid system should be well understood, theoretically and experimentally, even in regimes where the relativistic description breaks down. Then, full calculations based on firmly verified and controlled physics are (at least in principle) possible, even beyond the relativistic regime. Additionally, laboratory experiments might even become feasible that could shed light on issues of high-energy physics. The paradigmatic example is that of Bose-Einstein condensates (BECs). BECs fulfil all the listed conditions, and are for example considered a good candidate for a possible future experimental detection of Hawking radiation [3].

The simple model considered up to now is indeed a useful analogy or toy model, but not a full-fledged model for quantum gravity. A first reason is that the metric $g_{\mu\nu}$ in (1) does not reproduce all possible general relativistic metrics. However, this might just reflect the possibility that not all mathematical solutions of general relativity represent physically realistic spacetimes. A second and more important problem regards the dynamics of the system. In a BEC, the ‘analogue gravitational dynamics’ turns out to be (semi-)classical, i.e., can be encoded in a modified Poisson equation but does not lead to the Einstein equations [4]. More abstract toy models have been studied, but apparently none of those studied so far are sufficiently complex to reproduce the Einstein dynamics of general relativity. Nevertheless, the idea that gravity might emerge from an underlying microscopic ‘condensed-matter-like’ quantum system has at least two additional trumps to play.

First, in more complicated fermionic systems with a Fermi-point topology¹, and in particular ³He-A, fermionic quasi-matter emerges at low energy together with effective bosonic gauge and gravitational fields from the quantum vacuum [5]. The Fermi-point scenario can essentially be understood as follows. The Fermi point is the point in momentum space where the quasi-particle energy is zero. Spatial and temporal perturbations do not destroy the Fermi point, because of its topological stability. They only lead to a general deformation of the energy spectrum near the Fermi point. This deformation can be split into two components. The dynamical change of *slope* in the energy spectrum near the Fermi point leads to the emergence of an effective *gravitational* field, whereas an effective *electromagnetic* field reflects changes in the *position* of the Fermi point. So the quasi-particles move along the geodesics

¹ Remember that in normal metals and semiconductors, the fermionic excitations are defined by the *Fermi surface*, the highest occupied level in momentum space. In more complex structures, this Fermi surface can be replaced by a Fermi line, or even a Fermi point, as in ³He-A.

of an effective metric $g_{\mu\nu}$, which arises as a consequence of perturbations of the quantum vacuum, whereas the emergent quasi-particles and gauge fields show striking similarities with the ones known from the standard model of particles, including chiral or Weyl fermions and effective quantum electrodynamics. The essential point with regard to the metric $g_{\mu\nu}$, as in the case of BECs, is that $g_{\mu\nu}$ is Lorentzian even if the underlying system is plainly Newtonian. The additional emergence of effective quantum electrodynamics has led to suggestions that the condensed-matter analogy might not be limited to the gravitational sector, but that by carefully studying the topological properties of quantum vacua, this might also provide a hint for a ‘theory of everything’ that gives a unified description of gravity and matter [5].

Second, apart from the ‘esthetic’ issue of unifying quantum mechanics and general relativity, there is arguably at least one empirical motivation for a quantum theory of gravity: the accelerated expansion of the universe, which seems to imply some form of repulsive ‘dark energy’ [6]. The first intuition from quantum field theory to explain this mysterious repulsive force was that dark energy is simply the energy of the quantum vacuum, which makes its entry in the Einstein field equations in the guise of the cosmological constant [7]. Infamously, the experimentally obtained value of the cosmological constant turned out to disagree with theoretical estimates of the quantum vacuum energy by more than a hundred orders of magnitude. However, if one takes the condensed-matter analogy seriously, then this intuition might prove to be right after all [8]. Indeed, the quantum vacuum energy E_{vac} relevant for the cosmological constant problem in a condensed-matter system in equilibrium is regulated by macroscopic thermodynamic principles. It is obtained from the expectation value $E_{\text{vac}} = \langle \mathcal{H} - \mu\mathcal{N} \rangle_{\text{vac}}$, with \mathcal{H} the many-body Hamiltonian, μ the chemical potential and \mathcal{N} the number operator. The equation of state relating the energy density and the pressure of the vacuum of a quantum many-body system is then simply $\epsilon_{\text{vac}} = -p_{\text{vac}}$. Liquid-like systems can be in a self-sustained equilibrium without external pressure at $T = 0$. So the natural value for ϵ_{vac} at $T = 0$ in such a system is $\epsilon_{\text{vac}} = 0$. At $T \neq 0$, the thermal fluctuations and quasi-particle excitations lead to a matter pressure p_M , which is compensated by a non-zero vacuum pressure such that $p_{\text{vac}} + p_M = 0$. The vacuum energy therefore automatically evolves towards the value $\epsilon_{\text{vac}} = p_M$ in equilibrium [9]. The cosmological constant mystery then becomes a lot less insurmountable: What remains to be explained is why the cosmological constant is slightly bigger than the equilibrium value which would exactly cancel the matter contribution: $\Omega_\Lambda \approx 0.7$ versus $\Omega_M \approx 0.3$.

So far, I have exposed the conceptual idea that the quantum vacuum of our universe might be similar to the condensed state of a condensed-matter system, from which a relativistic spacetime emerges in the form of collective excitations on top of the quantum vacuum background. As a sidenote, in spite of this background dependence, the physics describing the excitations is not only Lorentz invariant at low energies, but also diffeomorphism invariant. Indeed, as long as the excitations are confined to their low-energy effective world, they have no way to detect this background, at least not in a geometric way,² and a natural description of their ‘internal’ physics is fully relativistic. So this condensed-matter framework provides the basis for an approach to understanding gravity that promises to shed an original light on some aspects of quantum gravity, such as the problem of dark energy, without the need to postulate extra dimensions, parallel universes or other exotic concepts of which we have no experimental indication whatsoever.

² If the excitations are advanced enough to detect that their universe is expanding, they might infer the existence of a non-zero energy provided by the background vacuum, which they could for example call ‘dark energy’.

III. EMERGENT SPACETIMES AND LORENTZ INVARIANCE

In this essay, I am following the rather conservative assumption that the essence of time as we experience it lies in the Lorentzian character of our universe, i.e., a spacetime with a metric of signature $(-+++)$. As in the previous condensed-matter examples, the Lorentzian character of a spacetime can arise even when the fundamental structure itself is not Lorentzian, but can simply be Newtonian, with an absolute time defined by the ‘laboratory’ setting in which the atoms composing the microscopic condensed-matter system lie.³ There are also other possibilities. Mathematically speaking, it is not so difficult to obtain an effective low-energy Lorentzian structure from a global ‘timeless’ one. The following example illustrates this [10].

Consider the following equation:

$$a(\partial_t^4 + c^4\partial_x^4 + c^4\partial_y^4 + c^4\partial_z^4)\phi + (-\partial_t^2 + c^2\partial_x^2 + c^2\partial_y^2 + c^2\partial_z^2)\phi = 0, \quad (3)$$

where a is a dimensionless parameter, for example E/E_{Planck} , and compare this with the wave equation of (2). Globally, (3) is an *elliptic* partial differential equation. t is a coordinate that behaves exactly as x , y and z do. In particular, if x , y and z are spatial coordinates, then so is t , whereas c is just a dimensional constant without any possibility to interpret it as a velocity. However, when the energy E becomes much lower than the Planck energy E_{Planck} , a approximately vanishes, and the second, *hyperbolic*, part of the equation becomes dominant. A Lorentzian structure is then obtained, and so it is perfectly legitimate to interpret t as a time coordinate and c as a velocity at low energies.

This is of course just a crude mathematical example. However, it has become nearly common to claim that quantum gravity is or should be timeless, with ‘time’ just a property arising in the low-energy limit. The previous example illustrates that this particular feature of quantum gravity is perhaps not as hard to obtain as one might expect, at least in a mathematical sense. Also, since we could have replaced the part of the equation between the first pair of brackets by nearly anything and still obtain the same low-energy limit, it also illustrates that a variety of microscopic theories could lead to an effective Lorentzian spacetime in the low-energy limit. However, it is not necessarily as easy to think of a physically realistic microscopic or high-energy structure that is not Lorentzian in itself, but still leads to a Lorentz invariant spacetime in the low-energy limit. Moreover, in the past few years, intense experimental attention has been paid to the possibility that Lorentz invariance might be an effective low-energy phenomenon, broken at high energies [11]. At the moment, no indication of such Lorentz violation has been found, and actually there exist stringent bounds on possible Lorentz violations at the Planck scale. So maybe quantum gravity should after all include Lorentz invariance from the start? Perhaps curiously, it is not clear how the dominant approaches to quantum gravity are assumed to recover our classical spacetime, and in particular whether Lorentz violations are to be expected at high energies or not. In scenarios of emergent gravity based on condensed-matter analogies, on the contrary, the situation is very clear: Lorentz invariance is a low-energy effective symmetry, and so it is expected to break at some scale, although not necessarily related to (and therefore possibly much higher than) the Planck scale. This poses an interesting question: How does the transition between our low-energy relativistic spacetime and the

³ I will resist the temptation to describe the universe as ‘God’s laboratory’, although I would not be surprised if some cartoonist has already found inspiration in the idea of God in a white laboratory coat, watching the clock on the wall indicating universal time.

fundamental microstructure take place? Condensed matter models again provide useful clues.

Let us return to the two condensed-matter models that we discussed in the first section. Consider massless particles and write c for the invariant speed of the theory (the speed of light or the speed of sound). Phenomenologically speaking, Lorentz breaking can be described simply by the following power law for the dispersion relation between the energy E and the momentum p :

$$E^2 = c^2 p^2 + \alpha c^4 p^4 / E_{\text{LV}}^2 \quad (+ \text{ higher-order terms}) , \quad (4)$$

where E_{LV} indicates the Lorentz violation energy scale, and $\alpha = \pm 1$ (we assume that uneven powers of p are ruled out to lowest order, since they would lead to strong parity violations). Let me again emphasise that there is no a priori reason to expect the Lorentz violation scale to be equal to (or even of the same order of magnitude as) the Planck scale, and, for the sake of clarity, point out that the relativistic dispersion relation $E^2 = c^2 p^2$ is obtained from (4) at low energies.

1. Bose-Einstein condensates

For Bose-Einstein condensates, the following dispersion relation is obtained from the microscopic theory in terms of the frequency ω and the wave number k :

$$\omega^2 = c^2 k^2 + \frac{1}{4} c^2 \xi^2 k^4 , \quad (5)$$

where ξ is the healing length of the condensate (roughly speaking, the distance needed for the condensate to smoothen out a sharp inhomogeneity in the atomic density). Comparison with (4) identifies $E_{\text{LV}} = 2\hbar c/\xi$. So how does this compare with the ‘Planck scale’ of the theory? Various characteristic scales can be constructed from the fundamental parameters of the microscopic theory. A sensible candidate for the analogue of the Planck scale in a BEC is the characteristic scale $E_{\text{ch}} = \hbar c/a_0$, with c the speed of sound and a_0 the interatomic distance. E_{ch} can be interpreted as the energy scale at which the granularity of the vacuum due to the separation between the condensate atoms becomes significant. In BEC gases, in general, $E_{\text{LV}} \ll E_{\text{ch}}$, indicating that Lorentz violations are expected at much lower energies than the energy at which the discreteness of the vacuum becomes apparent. Thus the BEC approach for gravity seems to be ruled out experimentally. However, this should not worry us too much, since I already indicated that the BEC model should be regarded mainly as a useful toy model for the kinematic aspects of gravity.

The main lesson to be drawn from this example is then simply that naive dimensional estimates indicating that quantum gravity effects should be expected around ‘*the* Planck scale’ are indeed naive. Different types of quantum gravity phenomenology might be characterised by different, mutually independent energy scales, which are composed from the fundamental constants of the microscopic theory and not necessarily accessible to an internal observer who is limited to the effective low-energy physics.

2. Fermionic vacua

In the Fermi-point scenario based on $^3\text{He-A}$, the gravitational and gauge bosons are composites made from the fundamental fermionic degrees of freedom of the microscopic theory. In such a scenario, the Planck scale can be

understood as the energy scale above which the bosonic content of the low-energy theory starts to dissolve into its fundamental fermionic components. Theoretical estimates based on such a scenario show that, for the quantum vacuum of our universe, $E_{\text{Planck}}/E_{\text{LV}} < 10^{-8}$ and probably even several orders of magnitude smaller [12], which is in agreement with current astrophysical constraints. Incidentally, $E_{\text{Planck}} \ll E_{\text{LV}}$ is also required for the Einstein equations to be obtainable from a hydrodynamic model in general, since fermions with energies above the Lorentz violation scale would contaminate the effective action for bosonic fields with non-covariant (hydrodynamic) terms. Note that this condition is not fulfilled in ${}^3\text{He-A}$, which is precisely why the Einstein equations do not emerge there.

In any case, the additional lesson with respect to the case of BECs might well be that the various characteristic scales in our universe ‘conspire’ to protect the effective low-energy symmetries such as Lorentz invariance, and hide the microscopic physics from low-energy observers like ourselves. Indeed, if the compositeness scale of the bosons provides a gravitational cut-off, while the Lorentz violation scale lies at much higher energies, then the latter might be strongly suppressed from direct observation. This would then mean that a relativistic spacetime subsists well above the Planck scale, even if the fundamental ‘quantum gravity’ theory itself does not obey Lorentz invariance.

Intriguingly, this suggests the possibility that the Lorentzian character of spacetime would survive at energies E for which $E_{\text{Planck}} < E < E_{\text{LV}}$, while gravity would be modified and might even vanish completely before E_{LV} is reached. One would then be left with a relativistic spacetime in the pure sense of special relativity [13]: a non-gravitating, and hence flat (Minkowski), but still perfectly defined and Lorentzian spacetime, where the invariant speed c , which at low energy was the signalling velocity of the massless composite bosons, is now the limiting velocity of the fermions. Curiously, the lesson from condensed matter shows that the order of fundamentality of our best theories of spacetime is perhaps not as well-established as usually believed. Contrarily to the usual classification, the scenario that I have just discussed would imply that general relativistic curved spacetimes give way, as the energy increases above the Planck scale, to flat Minkowski spacetime, which would therefore be ‘more fundamental’. At even higher energies, above the Lorentz violation scale, the truly fundamental theory of the microconstituent atoms of spacetime might either be timeless, for example in the mathematical sense described earlier, but it could also simply be Newtonian, with an absolute time defined by the clock on the wall of ‘God’s laboratory’, as in a real condensed-matter system in a real world laboratory.

Rather than finishing on this provocative note, there is one more mysterious aspect of time that deserves looking at from the condensed-matter perspective: the cosmological arrow of time.

IV. THE COSMOLOGICAL ARROW OF TIME

Until now, I have focused on the essence of time as residing in the Lorentzian character of the low-energy laws of physics. This Lorentzian character in itself is time-symmetric and so does not explain the arrow of time. However, from the condensed-matter point of view, the cosmological arrow of time might not be too hard to understand. Just like dark energy, the arrow of time should be understood in macroscopic thermodynamic terms. For a condensed-matter system, any thermal or other excitation, such as a spatial inhomogeneity, is a perturbation from the homogeneous zero-temperature equilibrium state. At a macroscopic level, nature always tends towards equilibrium, unless it is externally kept from doing so. So an out-of-equilibrium condensed-matter system will always try, by all means available to it (for example, by radiating quasi-particles), to dissipate its excess energy, leading to an expansion and

flattening out of the effective low-energy spacetime. This excess energy could for example be a remnant from an original big-bang-like phase transition. Generally speaking, complete energy-dissipation up to ‘thermal death’ and hence a completely flat effective spacetime at zero temperature⁴ is only possible when the environment is also at zero temperature. In the case of the whole universe, it is not really clear then whether similar boundary conditions are justified and so whether the final state will be thermal death or a tiny but non-zero temperature. But in any case, it is clear that the quasi-particles that have been created by the perturbation of the quantum vacuum will not ‘re-collapse’ into a far-out-of-equilibrium and highly curved state.

V. SOME FINAL COMMENTS

Is the spacetime of our universe really composed of a condensed-matter system? It might be completely wrong to think of spacetime as composed of material atoms localised in a Newtonian inertial frame with an absolute time. Even so, it could still be very instructive to consider the possibility that, whatever the constitution of the fundamental degrees of freedom of the quantum vacuum, they are nearly completely frozen out at the extremely low temperatures that reign in most of the present universe. The physics at such ultracold temperatures might then well be governed by collective excitations and their emergent symmetries in a way very similar to the physics of real laboratory condensed-matter systems.

In such a condensed-matter scenario, the low-energy Lorentz symmetry of the spacetime that we live in, and in particular our relativistic conception of time, is expected to break and give way to the microscopic theory of the ‘atoms’ of spacetime at an energy scale many orders of magnitude above the Planck scale. This means that any modification to the Lorentzian character of spacetime at high energies would be extremely hard to detect experimentally, much harder than in the usual quantum gravity scenarios based on a single characteristic Planck scale for all quantum gravitational effects. This should of course not lead to a defeatist attitude, but on the contrary provide an additional stimulus to further develop ingenious experiments, at both extremely low and extremely high energies. The bottom-line might then seem a bit disappointing: Although we have a reasonably good understanding of time in our low-energy relativistic world, we are still far away from understanding its true origin, and many experimental advances are still needed. Even if it is far from certain that these experimental advances will ever wholly uncover the origin of time, they will definitely offer us plenty of interesting surprises about the physics of our universe.

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⁴ Note that zero temperature would not necessarily imply *complete* thermal death, in the sense of the total absence of phononic and quasi-particle excitations. Quantum pressure could still induce excitations, even at zero temperature.

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