Sources of Unpredictability in Quantum Mechanics

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Abstract

Contents

1 Metaphysical Considerations

1.1 A criterion for distinguishing science from dogmatism

What is in science epistemology that distinguishes it from religion/dogmatism? Sure, they have different methods of investigation, but not always; for example, they can both sometimes refer to aesthetic judgements.
Sure, they have different domain of discourse, but not always; for example, both has something to say about causality.
Yet if we ask scientists[1] what makes them apathetic about religion, I believe most of them –if not all– will agree that it is because in religion

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[1]I am implicitly assuming here that a scientist who takes his/her curiosity and investigations seriously cannot be sympathetic about religion and religious reasonings.

there are dead-ends, every now and then, which block further questioning. Eventually, a curious child is stopped by 'Some issues are beyond our understanding'. so
In science there shall be no dead-end for questioning.
If we accept this principle, then as a consequence, there shall be no end to scientific pursuit. We can never claim to have a final complete theory of physical reality; for if we have one, we have to answer to the obstinate child, at the end, 'because these are the laws of nature' and the obstinate child should be able to ask why, otherwise it would have not make much difference if we said 'because this is the will of the Almighty'.

1.2 Epistemological and Ontological existence

But if we cannot ever reach a final complete theory how do we know it is there at all?! We cannot be sure, unfortunately. The physical reality may not exist at all. One thing we know with absolute certainty is that we can never know whether reality is there or it is all an illusion. this is more of a free choice: we can choose solipsism and never leave our beds or we can be pragmatists about this choice, and for all practical purposes[2] *assume* the existence of an independent physical reality. Therefore physical reality is *ontologically*[3] *existent* but *epistemologically*[4]*inaccessible.*

1.3 Can one ever deduce ontological conclusions from a physical theory?

By 'ontological conclusion' I mean conclusions about the physical reality itself, irrespective of our models to describe it. This is possible only if we have accessed the reality itself. or equivalently, an absolute, final and complete theory of physical reality; but according to the previous section, that is not possible in our worldview.
How are we sure about what future theories may have for us? We cannot simply take –incomplete and inconsistent– quantum theory and project what it says to the very reality of our physical world; to do so would be as naïve as considering time absolute because Newtonian mechanics says it is! I venture to conclude that theorems like Gödel Incompleteness have **no** implication for physics!

2 Reflections on the theory of quanta

2.1 What exactly is ν in $\epsilon = h\nu$?

From the considerations of Planck [1] and Einstein [2] we can at most say *Monochromatic* radiation of low density (*within the range of validity of Wien's radiation formula*) behaves thermodynamically as though it consisted of a number of independent energy quanta of magnitude $R\beta\nu/N$.[5]

[2]it is famous by FAPP in quantum mechanical contexts.
[3]Ontology is the study of meaning of existence.
[4]Epistemology is the study of how we acquire knowledge.
[5]from the english translation of Einstein's paper

There are two critical assumptions

- The wave under consideration is a monochromatic (harmonic) one.
- We are within the range of validity of the Wien's radiation law (Wien's approximation). i.e. the frequencies with which we are working are bounded below by $\nu_{\min}$.

We should *in principle* be able to construct a generic wave from its modes whose energy is proportional to $h\nu$.
According to the investigations of Planck and Einstein, $\epsilon = h\nu$ gives the energy of a quantum of electromagnetic waves. According to Planck, a monochromatic electromagnetic wave is composed of finitely many quanta, say n quanta. Take a general electromagnetic wave; after Fourier decomposition, the energy of each mode with frequency ν, according to Planck and Einstein should be

$$\epsilon_m = nh\nu$$

the application of time-frequency fourier analysis (inverse transform) to this equation is problematic and not, in general, well-defined.
A wave in general does not know any of the concepts *frequency* or *wave number*. these are concepts which appear when we choose a basis for our function space. by the principle of relativity, **A physical theory should not depend on our choice of basis**, so $\epsilon = h\nu$ must be *a* special case of *the* –still unknown– quantum 'filter'[6] condition. To dispense with the problematic n we can focus on a monochromatic (harmonic) wave and that is what usually done in the course of development of quantum mechanics.
There is however, an alternative approach: we can look locally. Let us suppose to be a photon. we see an electromagnetic wave coming, we ask its frequency when it is passing us and we declare our energy to be $\epsilon = h\nu$. this approach is not limited to harmonic waves but has severe conceptual problems. most importantly, the electromagnetic wave should *shake something (us)*, hence the Luminiferous aether...
There is yet another possibility: ν has nothing to do with waves! it is about rotation and at the same time it is intrinsic. one can hardly think of anything but spin.
It is conceivable that $h\nu$ is the quantum filter for the spin of massless particles only. if we want to generalise it for all particles either we should find a new route or we have to build on an abuse of notation:
ν as the frequency of rotation (angular velocity) $\rightarrow$ ν as the frequency of oscillation of *some* vague wave
this *some wave* is the de Broglie wave and developing its dynamics brings us to orthodox quantum mechanics.

2.2 Local definitions for the familiar wave concepts

What should we do then in a world where we are forced to assume every wave as harmonic? There are two approaches: In a first temporary ap-

[6] I believe –and am developing theories which support this claim– that we cannot have a theory which is inherently quantised, by itself. Rather, quantisation is a *filtering process* which one applies to a *background theory*. The background theory is essentially classical.

proach we accept this as a bare *fact* and we say if everything is harmonic so be it. We *brute force* everything to be harmonic. How? by giving harmonicity to all waves. How? by defining for all waves what all waves do not possess: Frequency and Wavevector.
Let us cheat and see how we can find an expression for wavevector in terms of the wavefunction itself[7]

$$\psi(\mathbf{x}, t) = e^{i(\mathbf{k}\cdot\mathbf{x} \pm \omega t)}$$
$$\nabla\psi = i\mathbf{k}\psi$$

we have two possibilites for a definition. first

$$\mathbf{k} := -i\frac{\nabla\psi}{\psi}$$

but this is not beautiful and has singularities.
The second (better) approach would be to promote wavevector to an operator

$$\hat{\mathbf{k}} = -i\nabla \tag{1}$$

and say instead that we are dealing with an eigenvalue problem, which is exactly what people do in orthodox quantum mechanics. [for some details you can see Technical notes]
Here it is crucial to notice two important radical changes that are *forced* upon us once we assume all waves as harmonic:

1. i, bringing about complex numbers, which are hard to interpret physically and were sofar assumed to be only *tools*. Now they are more than tools. we cannot assume all waves as harmonic unless we pay the price: complex numbers.
2. **Eigenvalue problem and Linear Operators**, bringing about Hilbert spaces.

Now that we have this we can have many things for free. for example, without appealing to any opto-mechanical analogies (like Schrödinger[3]) we can get some equations.
From the special-relativistic energy relation

$$E \cdot E = c^2\mathbf{p}\cdot\mathbf{p} + m^2c^4$$

Applying the de Broglie operator equation we have

$$(\hbar\omega)^2 = \hbar^2c^2\mathbf{k}\cdot\mathbf{k} + m^2c^4$$

$$-\hbar^2\partial_t^2 = \hbar^2c^2\nabla^2 + m^2c^4 \tag{2}$$

which is the Klein-Gordon operator equation.
For a massive non-relativistic particle in a potential V, approximately we have

$$E = \frac{\mathbf{p}\cdot\mathbf{p}}{2m} + V$$

[7] Note that if we restrict ourselves to real-valued harmonic functions like $\sin\phi$, this approach is no more viable; in this situation, only a 'second-derivate approach' is available. [see Technical notes]

which gives Schrödinger equation.
In a second approach we may say perhaps this is all unnecessary, only if we can generalise our quantisation condition to all particles[8]...

3 Probabilities and Unpredictability in Quantum Mechanics

As we saw in the previous subsection, if we insist on considering only harmonic waves, it is *inevitable* that we should allow for complex numbers. What is the implication for our theory in relation to the real external world? The primary implication is that we cannot interpret what the wave function represents, because it is not possible for us to make physical sense of a $\mathbb{C}$-valued function and consequently we cannot know what the theory *means*. Is this not the main reason that we are forced to content ourselves with the modulus of the wavefunction as a probability density function? This, I propose, is the ultimate cause for probabilities and uncertainties in quantum mechanics.
It is worth noting that some physicists, like Gerard 't Hooft, lower the ontological state of mathematics to rescue quantum mechanics:
'*Complex numbers are nothing but man-made inventions, just as real numbers are. [...] Note that quantum mechanics can be formulated without complex numbers, if we accept that the Hamiltonian is an anti-symmetric matrix. but then, its eigenvalues are imaginary.***'**[4]
Are not eigenvalues –indeed a critical– part of quantum mechanics?! thus quantum mechanics **cannot** be formulated without complex numbers!

4 Technical notes

If we insist on the solution discussed in section two, we must as well have a definition for the propagation velocity of a general wave. It must be

$$\hat{c} := \partial_t \nabla^{-1}$$

which is not so beautiful and well-defined. An ugly speculation here can be that the propagation velocity of a general wave is not a constant number. Another less-ugly possibility is

$$c := \pm i \frac{\partial_t \psi}{|\nabla \psi|} \tag{3}$$

which has a sign ambiguity. Two possibile solutions: Either we should have only one kind of waves (backward-in-time *or* forward-in-time) but not both; which treats time asymmetrically and is ugly to some while beautiful to Eddington!
or to use second derivatives

$$c := \sqrt{\frac{\partial_t^2 \psi}{\nabla^2 \psi}} \tag{4}$$

[8]Pursuing this path is irrelevant to the topic of the essay contest.

which is not good either due to the all headaches of multivaluedness of complex functions.
In the first case we can forget about the sign ambiguity and insert the definition in the generic wave equation. It would give a non-linear and rather-beautiful equation. I am not currently interested in that because of the economy of explanation and prediction (it cannot explain/predict much, at least as I understand).

References

[1] Planck, Max (1900c). 'Entropie und Temperatur strahlender Wärme'. Annalen der Physik. 306 (4): 719–737.

[2] Einstein, Albert (1905). 'Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt'. Annalen der Physik. 17 (6): 132–148.

[3] Schrödinger, Erwin (1926). 'An Undulatory Theory of the Mechanics of Atoms and Molecules'. Physical Review, 28, 1049-1070.

[4] 't Hooft, Gerard (2016). 'The Cellular Automaton Interpretation of Quantum Mechanics'. Springer. Page 17.