“Reality”: a Discrete Hierarchic Duality of Finite Type

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Abstract. Modeling reality is a zoom-in/out process best implemented as a resolution of finite type, in the mathematical sense. This means that at each level of details a finite number of independent degrees of freedom are introduced: dimensions, “digits”, graphs etc. The local corresponding properties (“particle” aspects) are complemented by global properties/correlations (“wave” aspects), providing the framework for the classical-quantum duality.
Explicit examples are discussed: real numbers, Haar wavelets and Feynman graphs. The conclusion is: reality is discrete, hierarchic of finite type, well modeled by a duality of the type homology-cohomology. The development of physics, experimental and theoretical, ample supports this claims.

In contrast with the accepted theories, I will argue that “reality” is not only (quantum) digital, but also of hierarchic of finite type: potentially infinite, yet actually finite at each level of complexity [1]; of course, when we talk and think of “reality”, it is a modeling activity (“never-ending story” :) … So:

“Is Reality Discrete or Continuum?”

The answer is “Yes” :) … it “looks” *continuum,* and without computers it was inevitable to model reality as such (the “tip of the iceberg”), but the best models, from a conceptual point of view, are the *discrete* models (the deep-hidden bulk)!

I have rephrased the original question as “Discrete or Analog?”, since “analog” seems to me that it rather refers to “how reality *is* and *works*”. During the not-so-distant past (500 years or so), most of us believed that reality is divisible indefinitely, totally forgetting the impressive Greek knowledge (Democritus, Zeno etc.), and therefore getting used to think implicitly that “analog” means “continuum”.

Now continuum can be obtained basically in two ways: via Dedekind's construction of the reals, or Cauchy's construction via sequences, leads to what I will call an “amorphous line”, even though it has the integers as a “scale”; the other “incarnation” is the power series representation using some fixed set of digits, like 0 through 9 corresponding to the decimal representation. This later version is an example of a *resolution* of the structure, providing a hierarchy of “details”, in both directions, “up” towards the large scale structure, as well as “down”, in the “infinitesimal realm”. Now “this continuum” is really discrete, a *potential infinity* of *finite type* (finite in each degree), but has the power of the continuum as an *actual infinity* (see Wiki/Aristotle). The latter, the continuum *c*, is the cause of all “troubles” in mathematics!

Of course we could get “fancier”, and use an adapted grid to describe the reals, with a changing of the structure for each level of depth within the hierarchy. It turns out that this is indeed the practical way to generalize: it is the way adaptative numeric methods are implemented. But decimal representation models “empty space”, so there is no reason to favor and zoom-in more near “this” number or “that” number, even if it is close to a prime which are “special” (or is there? see Annex). But if we solve an initial value differential problem, it is a different story; the “space” (real line) is no longer “empty”, and the data, a vector field say, represents the “matter's action”, and when approximating the solution we should proceed “slower” near big curvature points/reals, and move fast where there is no notable rate of change (no derivative / “nothing happens”).

Now what if, in three real dimensions, we decide to use graphs as a grid to mark a certain resolution depth, and then, at the next level of detail insert a a graph at a point, in order to have more details (Fig.2), in a sort of a “graphical representation” of R^3, analogous to the decimal representation (a lattice structure with scaling group 10^d) … ?

Then it starts to look like what Feynman was doing, the so called Feynman Path Integral quantization method. But it was not “rigorous”, the mathematicians were complaining, so it did not “exist” :) … until recently, when D. Kreimer and A. Cones formulated renormalization of QFT as an algebraic problem [2]. I always argued that, at a deeper conceptual level, the Feynman approach says that the “paths are essential”, i.e. the interactions, not the “points” of a Newtonian-Einstein space-time [17,18]. The “Feynman legacy” [19] is the reunification of particles and fields, as not making sense independently; it is a “physics of interactions” rather then a “physics of motion”.

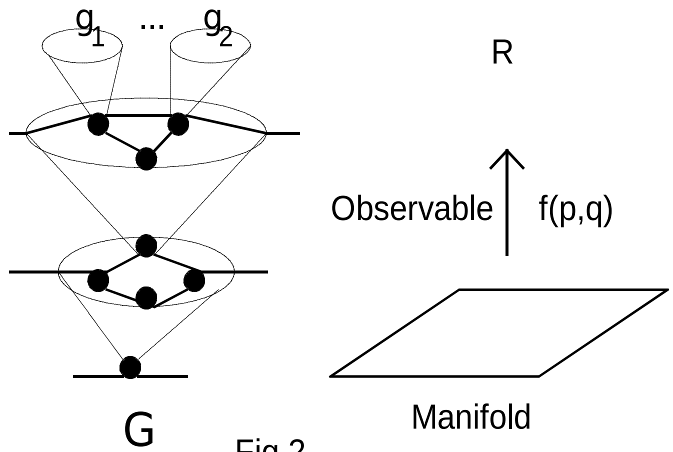
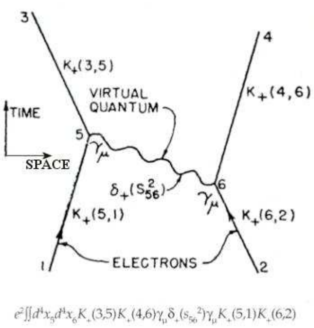
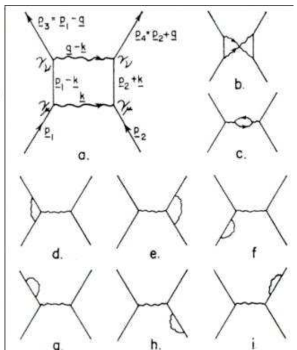


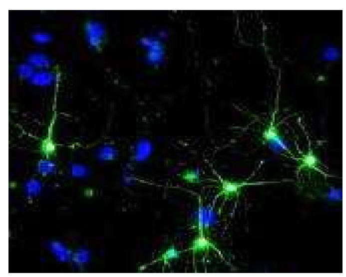
Fig.2



Second order Feynman diagram for electron-electron scattering and the corresponding mathematical expression


Fourth order Feynman diagrams for electron-electron scattering

Whether it is the "Universe's **Quantum Computer**" as a dynamically changing quantum network or a model of the brain's **Neural Network:**



the mathematical-physics formalism is the same (Simply flows on graphs, or fancier cohomology of dg-colalgebras; e.g. “salesman problem” and electric circuits, or Feynman graphs and Riemann surfaces etc. [17]). But before looking at such “technicalities” (or even better: instead!), let us review some basic principles ...

Reasoning top-down, the axiomatic way, modeling a system requires “cutting a piece of reality”, and replacing the severed interactions with the environment as boundary conditions. Then, one implements the first level of structure as subsystems with their mutual interactions. So far we have “counted” the parts, defining what the external structure is. Of course we need to decide what “digits” / graphs/ symbolism, to use.

Now there are local properties, of “particle” type, and global properties, of “wave” type, which includes the concepts of interference (or better: resonance), in a duality which accounts for correlations and entanglement. By-the-way, distance is an obsolete concept in a physics of quantum interactions, or from the Information Theory point of view (physics of quantum communications); what matters is the capacity of the corresponding channel, a much more interesting “story” ...

Duality is a pervading feature of reality itself, whether it is Yin and Yang of oriental philosophy or particle-wave duality of "modern" physics; in mathematics the the list of dualities is huge. Now we should distinguish between *classical logic*, with its binary exclusive alternative 0/1, True or False, dominated by an exclusive "*or*" XOR ("tertium non datur"), which is dual to *quantum logic* based on 0 AND 1, True AND False, of course with various nuances of "AND". Mathematically these "nuances" correspond to the three dimensions of the *qubit*, the unit of quantum information; denote them by R,G,B and think of them as basic colors; of course, reality is not "black" XOR "white", but colorful!

The above description refers to the "Master-Slave" model of quantum computing [3]; again, we recognize familiar trends of our thought process, if we think of the interplay between conscience and subconscious …

Mathematically, the local-global duality is well served by homology, with its description of subsystems as "points" and relations between subsystems as "relations", in duality with cohomology, encoding "spooky-action-at-a-distance" correlations (not actions) and global properties like electric and magnetic charges [4]-[7]. Whatever that maybe, trust Jim Stasheff [8]: modern physics is cohomological!

At a more specialized mathematical level, very much under control and unlike the Feynman path Integral "business", the continuum versus discrete dilemma has been implicitly settled quite recently by Maxim Kontsevich, in my opinion, through his Formality Theorem [9]. My interpretation of its role is the result of extensive work done towards understanding it and applying it in physics [10]; bluntly put: "Whatever you can do with manifolds, you can do better with graphs and local symmetries".

Let me explain. A famous problem in quantum physics was how to *quantize* Poisson manifolds using *Deformation Theory*. Kontsevich solved this problem by essentially "approximating" a manifold using a hierarchy of graphs, totally similar with the Feynman approach, and reminiscent of an *adaptive numerical method*, with the flavor of a finite element method from engineering (computer assisted design); then he solved the problem at the level of the discrete model, and transported the solution to the continuum framework.

I said "approximation", but in the context of an infinite formal sum, it is rather a resolution of the solution, the same way a real number, as an element of the amorphous set of real numbers coming from *Dedekind's cuts* definition or *Cauchy norm completion*, can be "resolved" into a hierarchy of structure called its decimal representation (see also [16] (1)).

If this is, I agree, not obviously similar, think of the "true" hierarchy of Haar *MultiResolution Analysis* [11], which allows to represent continuum data (say a square integrable function in the plane), as a sequence of "details", extending in both directions, in the "small" and in the "large". For those not familiar with this modern "version" of Fourier analysis, just see how a picture is uploaded from the Web through a slow modem connection …

The "difference" between the decimal numbers example, and Haar wavelets is "small": the latter involves functions, which are dual to numbers, besides the obvious difference of dimension of the "configuration space". Otherwise, the same idea of a group of scaling and translations is involved, which briefly will be referred to as "resolution".

The idea of "resolution" refers to a procedure of disassembling the object into a hierarchy of pieces and their structure. For "real" systems, these would be subsystems and the interactions between them. At a fundamental level ("macro" in computer science parlance), the universal language is that of *Quantum Information Dynamics*, the theory of the dynamics of quantum information, which is based on the *Quantum Computing* model, but allows for a dynamical change of the underlying quantum hardware; this corresponds to the difference between *Quantum Mechanics*, where the number of particles of a given type does not change, and *Quantum Field Theory* where the particles are allowed to transform from one type into another.

Why is it "universal"? Because "all we need" is a local-global duality to implement particle-wave duality; it yields the above *Master-Slave Duality* which accounts for the interface between classical and quantum. It is a misconception that “quantum” happens at small scale, and eventually at bigger scales things start to look classical. There is no classical-quantum “divide”. It all depends on what kind of questions we “ask reality”, and how we interpret the results. It will be explained elsewhere that the weirdness of quantum mechanics originates in the old assumption of an existence of an ambient space-time background. The *Mach Principle* should lead to a total removal of the “marble” geometry based on a classical point-wise space, and replace it with a geometry a la Klein, totally algebraic; the translation of the old theory is via differential Galois theory. In this way *String Theory* is “saved” (there are exactly zero landscapes!), and we will finally understand quantum mechanics: Heisenberg uncertainty relations are an indirect way of removing the physical attribute given to space-time coordinates, in order to discretize reality. Amazingly it is also the conclusion of modern symplectic geometry (see Maslov quantization and M. Gromov’s Non-Squeezing Theorem [12]), as well as the conclusion of Loop Quantum Gravity (see C. Roveli, L. Smolin etc.).

All these efforts just to start with a continuum model, only to get rid of the continuum one way or another … Isn’t it time to redesign physics on sound foundations, instead of patching it over and over again, a habit we call “unification”? This is why the Standard Model reminds me of Microsoft Windows, and why the “Big Question” of how to unify General relativity and Quantum mechanics is the wrong goal. In fact, and somehow surprisingly, General Relativity is conceptually correct, but it is built with the wrong tools (Eilenberg and MacLane were not Einstein’s classmates), and Quantum Mechanics is conceptually wrong (that's why “... nobody understands quantum mechanics” - Feynman 60s), while technically it is correct (“Do not think, just compute!”).

Moreover, irrespective of the answer, we should develop mathematical models which are discrete and countable, of finite type yet not finite, in order to improve the flow of the research cycle (Fig.1).

Discrete mathematical models, replacing the usual continuum models based on manifolds (“ambient space-times”), should not be confused with approximations of the continuum models (e.g. lattice models), used for computational purposes just because the continuum models are not solvable / tractable as is.

The “good news” is that physics is in a process of slow recovery after the “dark ages” of last half a century [13], especially due to the globalization of communication of knowledge via The Web.

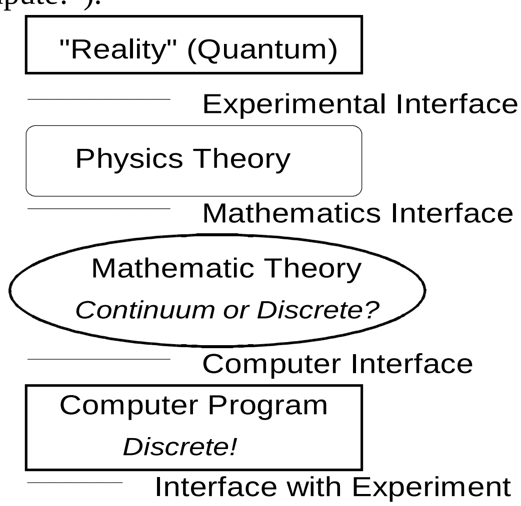


Fig.1

Documenting the above claims using the author's unpublished work [14] would make a nice project for an *Open Research Project* in the *Cloud*. Glimpses into the main ideas are provided by the author's talks [15] (… anybody interested?).

Annex – Two Types of “Discrete vs. Continuum”

The “continuum” is the *actual infinity* obtained by completion of the rationals Q (the *potential infinity* which is discrete if viewed as a shadow of the Hopf ring of integers Z), with respect to a norm. Now there are essentially two ways to proceed: using the usual absolute value $| |_\infty$, yielding the reals R; or the ultra-metric $| |_p$ associated to a prime p, obtaining the p-adic numbers as a continuum completion of “discrete & finite”, and then (why favor a prime?), take all primes and form the adeles! There is strong evidence that this is the way to proceed, allowing to formulate particle physics as number theory.

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