FQXi Essay Contest 2016/2017
»Wandering Towards a Goal: How can mindless mathematical laws give rise to aims and intention?«

# The Dot, the Point, the Dot — the Intention

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**Summary**

*Even if the last FQXi essay contest has triggered a lot of very interesting thoughts [1], an additional contribution will be presented here to understand the origin of intention. Mathematics is not a formalism made by humans, but the logical origin of our reality. In this essay a hypothesis will be investigated, how physical entities derive from the mathematics by an emergent leap. Inspired by HoTT mathematical induction is assumed as main impetus of the development. Finally the intention as an emergent capability for analyzing meta-information will be considered.*

## The dot and the point

At high school I once learned in my geometry lesson that the dot is representing a mathematical point without any expansion and form. This assertion – or definition – is a nice starting point to discuss the link between mindless mathematics and intention. The printed dot in a medium like a book is a combination of smaller elements called pixels. These pixels are the base of the whole visual representation of information. Their size should be below the resolution threshold of the human eye, quasi without any spatial expansion. Because the pixels are not visible at normal viewing conditions, the actual shape is not important:

- A printed pixel is typically nearly round because of the fluid dynamic conditions of the ink.
- Because of the technical reasons a screen pixel is rectangular.

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But you do not see this difference; the final picture or text is the same on both media technologies. So only the combination of a lot of pixels which themselves are not visible creates a visible information. The abstract information of a mathematical point is therefore represented by a spatial ink distribution, which itself is composed by not recognizable pixels.

Even if the printed pixel is visually undetectable, but nevertheless physical measurement tools enable us to recognize further sub-structures over several scales. Depending on the scientific community different classifications are common to describe these scales:

V. Macromolecular compounds of colorants, binders and additives
IV. Monomers and atoms
III. Atomic structures
II. Elementary particles
I. …

Each level of scale shows complex patterns composed on a countable set of basic elements. These elements refer on the one hand to the subjacent level, on the other hand they will cause the specific features of the higher level, e.g. the circular or the rectangular shape of the pixels.

In principle it might be possible to follow the simple approach and describe the chemistry of polymers or the nuclear physics as complex patterns based on basic elements like we did it for a printed dot. So what happens if we generalize this simple principle, that smallest elements (pixel) without impact (physical action) are able to create patterns with impact?

Since a long time there are a lot of considerations in quantum physics what might happen in merest dimensions, for example within the Planck space-time. For example some scholars assume a black whole within the Planck interval [2].

Yet here we do not stop at such discussions about the smallest physical structures, but move ahead with a bold assumption, based on the high school knowledge mentioned above:

Whether Plank space-time, black hole, string universe, or another approach for a smallest physical pixel, in all cases a mathematical point will exist inside of such a structure by definition. As thought experiment the following hypothesis may be formulated: The smallest physical pixel, however we may define it, again consists of a complex internal structure based itself on smallest elements. These smallest elements may be mathematical points. So the structures based on these mathematical points should be the mathematics itself.

Therefore the obvious and very interesting question arises how complex structures, which are based on mathematical points may create a physical entity. Instead stepwise from the larger to the smallest as usual, we proceed directly to the infinite smallest, the mathematical point. From there we are searching for the next level above. How and by which action mathematics becomes physics?

**Processing Dasein (Existence)**

First we have to ask what exactly mathematics is dealing with. For example the Homotopy Type Theory (HoTT) defines *„…the term **a** is of type **A**.“* and explains *„**a** is a point of the space **A**“* [3]. According to the widespread opinion mathematics allows any representation for its mathematical points and addresses only the logical relationships in between the points. In other words: The physical dot becomes a mathematical point by losing all physical features, and vice versa.

Special consideration must be given to the von-Neumann hierarchy. In contrast to the ordinary perspective points will not form a quantity (set), but logically the empty set is pre-existing, before points/elements can be found there. If we print several dots on paper and ask for the empty set of these dots, this should be the plain unprinted sheet. And this sheet of paper is always required before the dots are printed. The mathematical empty set is like a substrate, a carrier or a homogeneous background. The empty set is the precondition for the existence, the dasein of mathematical points; it is the potential existence, while the mathematical points really exist. However, if any sheet of paper is not available, the situation is different. Then existence is impossible. So we have to differentiate between two stages of non-existence: The impossible existence and the potential existence.

This leads to the conclusion about what mathematics describes: Dasein/existence itself and all its logical implications and conditions, either as mathematical point and space or as element and set or in which formulation howsoever. And philosophically there should be no doubt that dasein itself is a necessity for any physical entity.

The impossible existence (_) leads logically to its opposite, the possible or potential existence (○). From this, real existence is derived (●), and then two existences are logically true (●●). From this, an infinite amount of further mathematical points or logical existences will be created. If we trust the HoTT, not only the set of natural numbers, but the mathematics in total will be unfolded by logical induction.

The HoTT describes a concept of the mathematics based on induction rules, called constructors as an universal principle [4]. This interpretation might be different to the traditional understanding of mathematics as a constant eternal characteristic of the world. From the HoTT we can derive an idea of an emergent mathematics, which develops dynamically from the starting point of the impossible dasein towards a complex logical mesh of mathematical points. Propulsion for such a process-oriented understanding of the mathematics is the induction: Impossible dasein is only existing if dasein is also possible. And the whole mathematical universe is a direct consequence from this fact of dasein – without any auxiliary assumptions. Within this process-oriented approach we can identify the *change* as a fundamental entity of the unfolding. The change in the paper-white by a black dot defines the existence of this dot. In the static case we know "the other one", e.g. is "1" different to "2", but here we emphasize the dynamic process, whereby "2" is created by induction from "1". But a change itself is not sufficient. Is the

printed dot to small or is it only grey but not black, then the change will remain invisible. A change is only relevant if an effect happens. Is the printed dot to small or is it only grey but not black, then the change will pass unnoticed. If a change is affecting something, we might identify this as information[2]. Today's terminology supports this difference. Informatics can be understood as the process oriented variation of mathematics. A meaning of the *effect* of a change has to be analyzed later. Here we can notice the main feature: A new existence, a new mathematical point is created by information.

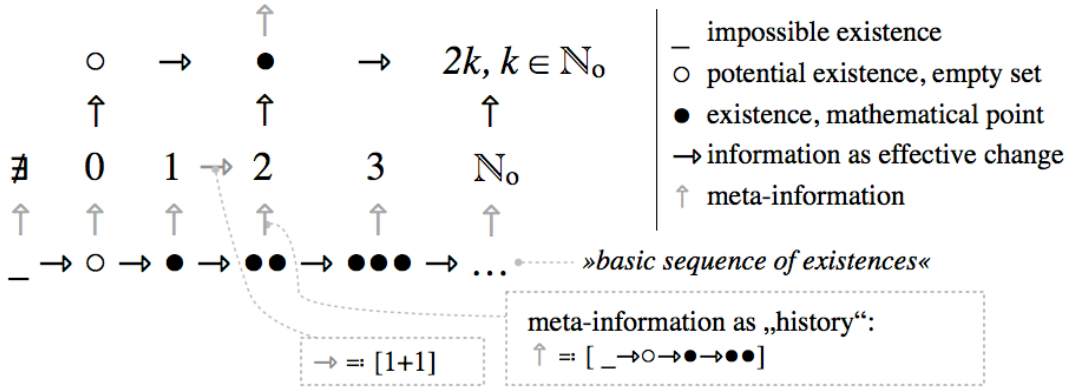


**Fig.1** Existence, information, meta-information

The idea of mathematics as a process provides a deeper understanding of the traditional static formula, that (●●) should be equal to »1+1« and equal to »2«. These differences can be formalized using HoTT. From a process perspective, the existence of 2 is only possible, if the prior history is available. This history contains the logical implications and conditions of this mathematical point. I will name this meta-information; it is a kind of presetting and "knowledge". It is the path of previous mathematical points leading logically to the current point and establish its new existence (Fig.1). It is the path of previous mathematical points leading logically to the current point and establish as result its new existence. This newly created mathematical point is again a starting point for further induction. As an example for a new type of existences Fig.1 shows the set of even numbers, which again creates new meta-information. With the ongoing unfolding not only simple connections would be expected, but a very complex mesh of existences. All this leads to a set of complex meta-information that is required to create a further new mathematical point.

## The limits of mathematics

A computer professional will remember the term "computing time" as a significant limit for the processing of information. But according to the initial hypothesis the previously mentioned mathematical processes will be located as a sub-structure within the smallest

[2] Until today information is not defined unitary. Most of the authors suggest complicated macroscopic definitions, which do not always meet its basic nature. In contrast Gregory Bateson made the casual statement, information would be the message from a change [5]. But he did not follow that approach.

physical pixel. There, e.g. inside the Planck interval, no space and no time are defined. The first idea might be the assumption, that without physical restrictions the unfolding of mathematics will happen instantly, the complete mathematics would be created by induction without any delay. Nevertheless the induction as a constructive principle ensures a sequence of existences. The mathematical point "3" is generated logically after the point "2". And the existence of "671" requires much more predefined existences. The primary existences unfold with an infinite frequency and we can term this *mathematical frequency*. But we have to consider the generation of complex mathematical points, too. As long the meta-information is simple, no important effect is expected. But as soon as the newly generated mathematical points are connected in very large dimensions and categories with a huge number of different meta-information paths, we can expect a delay of the mathematical frequency. So we anticipate an inhomogeneous unfolding of the mathematical universe in terms of the generation frequency. If we look at a certain moment on a mathematical point in the basic line of existences and identify its number, which might be quasi infinite, then we will should not find a high-complex mathematical point with the same large distance relating to the origin. This effect we may term *complexity delay*. It is based on the different cardinality. While the basic existences represent a countable set of infinite entities, the meta-information of complex existences will show a much higher cardinality. Even if this is not the place to analyze the details of that inhomogeneous unfolding of the mathematical space, we can clearly identify the reason. The inhomogeneity is triggered by the induction. Very complex mathematical points have to "wait" until all meta-information is completely created.

If we accept this presumption, we might ask for the form of the unfolding space. For low complex mathematical points we expect an undistorted mathematical frequency, shown by the green strait lines in Fig.2. With higher complexity we assume a deformation of the linearity, caused by the complexity delay. For the shape of this deformation we have to consider, that at small numbers we find close to the origin, the complexity is limited because of the amount of available points. This is illustrated by the blue curve. For higher numbers of mathematical points a very fast increase must be expected, towards infinite complexities.

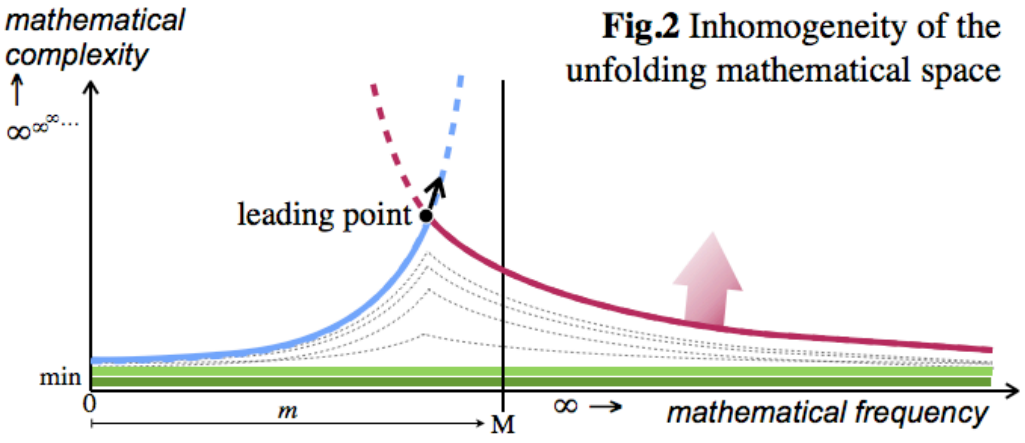


**Fig.2** Inhomogeneity of the unfolding mathematical space

The expansion of the mathematical space will happen below this curve as shown by the red curve. If the complexity delay is present, we have to find this delay at very high numbers in the infinite. The intersection between both curves might be the leading point of the expansion.

This expansion and thus the shift of the red curve will slow down because of the complexity delay. Finally it might b e possible to detect a singularity M. The leading point can not cross this singularity, because the complexity is increasing very fast (steep blue curve), while the creation rate slows down (flat red curve). Consequentially the unfolding of the mathematical space would create a singularity caused by its increasing complexity.

**The emergence of the physical universe**

In the last decades a deeper understanding of the processes of emergence was attained especially in Biology. At this macroscopic systems complexity and singularities play an important role. The idea is obvious to identify the singularity M as a candidate for the emergence of the physical universe.

Is an emergent transition from the singularity M into a new physical existence possible? Is it conceivable to define a M-cell, a mathematical cell with the absence of physical entities, which acts as total system as an emergent, initial physical entity, like the Minkowski space-time? Is there the transition from the mathematical point to the physical dot?

In view on this singularity we note two features. First, the singularity has a certain distance *m* to the origin, measured in units of the mathematical frequency. Second, the expansion, given by the movement of the leading point towards higher complexity has not an infinite rate. The relation of the expansion in the direction of the mathematical frequency to the expansion in the direction of complexity should be not infinite. Here we do not focus on a further analysis of the emergence of the physical entities, but look to the consequences for the concept of information.

**Meta-information and intention**

At the beginning we should note again the previous definition of the term information. The given definition as a change with the effect of generating a new existence is invariant against emergence. Reaching a higher emergence level by the inductive unfolding, we can suppose the same procedure again. Initially there was nothing existing, and the new level was impossible. Because of the induction the higher level became possible and later reality. If we accept our observations in the reality and nature, in higher emergence levels the singularities and inhomogeneity must be much more complex compared with the circumstances at the initial level of mathematics. But by and large we see comparable structures in every emergence level with an increasing complexity, driven by the ongoing mathematical induction of the existences themselves. This mathematical

engine of induction moves a huge flow of meta-information onwards, creating new existences within levels and new emergent levels. The whole process can be described by four categories (Fig.3, blue box). Based on the initial existence, complexity arises and finally leading towards a singularity. There an emergent physical entity was supposed.

| Mathematics | Sign & Symbol | Universal | Evolution |
|---|---|---|---|
| Physics | Pragmatics | Application | Emergence |
| Singularity | Semantics | Function | Intention |
| Complexity | Syntax | Form | Selection |
| Existence* | Semiotics | Element | Induction |

* mathematical points

**Fig.3** Suggested scheme for types of meta-information

We can translate this process in a traditional form by using the terms *semiotics*, *syntax*, *semantics* and *pragmatics*. Here another interpretation is suggested. Starting point at each level of development are *elements*. From these elements different *forms* derive. Finally forms can carry a *function*. The *application* of a function leads to a new element at an emergent level. With this scheme is is possible to characterize the development process itself, which can be termed evolution (Fig.3, green box). At the base we find the induction as logical process to generate elements. The next level of selection asks for the stable varieties. The intention directs the process towards a function. With this function the emergence of a new level can be addressed.

A mathematical formulation of the intention seems currently far away. But this consideration suggests understanding intention as a higher level of the induction. It might be possible to describe intention as a form of singularity, too.

## Conclusion

For certain this essay is highly speculative; and perhaps some statements might be not acceptable. But the purpose of this essay targets a feedback by scholars to the question, if further research in this direction is promising. Interesting fields for additional work might be: (1) A generalized theory of evolution, (2) a basic understanding of information and (3) the foundation of physics by mathematics.

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## References

**[1]** FQXi, Essay Contest »Trick or Truth: The Mysterious Connection Between Physics and Mathematics« (2015), http://fqxi.org/community/forum/category/31424

**[2]** 't Hooft, Gerard »*Nature's Book Keeping System*« in S. Wuppuluri, G. Ghirardi (Ed.) Space, Time and the Limits of Human Understanding, Springer Berlin (2017) p.133

**[3]** Univalent Foundations Program: »*Homotopy Type Theory*«, Institute for Advanced Study, Princeton (2013) p.3

**[4]** ditto p.27

**[5]** Bateson, Gregory »mind and nature«; German edition: Geist und Natur, Suhrkamp Frankfurt/M. (1984), p.40