

“What is (the) time?” “Get a clock, it will tell you!”

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Abstract: MANY COMPLICATIONS ENCOUNTERED IN DISCUSSIONS OF TIME CAN BE AVOIDED USING A PURELY OPERATIONAL DEFINITION OF TIME IN TERMS OF CLOCKS. THIS CIRCUMVENTS ISSUES STEMMING FROM THE GAUGE INVARIANCE OF COORDINATE CHOICES IN DIFFEOMORPHISM INVARIANT THEORIES LIKE THE VANISHING OF THE HAMILTONIAN. TIME ITSELF AS A LOCAL COORDINATE IS UNOBSERVABLE BUT MEASUREMENTS OF CLOCKS EVIDENTLY ARE. THIS, HOWEVER, REQUIRES A DEFINITION OF WHAT KIND OF SYSTEM CONSTITUTES A CLOCK WITHOUT REFERENCE TO THE CONCEPT OF TIME. IN THIS ESSAY, WE ATTEMPT SUCH A DEFINITION AND ARGUE THAT THE POSSIBILITY OF DEFINING TIME CONSISTENTLY IN TERMS OF CLOCKS IN AN UNAMBIGUOUS WAY INDEPENDENT OF THE PHYSICAL MATERIALISATION OF THE CLOCKS REVEALS A DEEP PROPERTY OF NATURE UNDERLYING THE CONCEPT OF TIME.

Everybody has an intuitive understanding of what “time” is. At least as long as she does not try to explain what it “really” is. Right now, the time is 4:49p.m. And now it is a bit later. But what is it that flows? This question supposedly becomes even more subtle once one includes relativity (special and then general) and quantum physics. But the main difficulty seems to originate from the fact that there seems to lack an intrinsic time: you cannot just go and measure the time of a certain event as you cannot measure the x -coordinate of that event unless you have (“subjectively”) introduced some reference coordinate system.

The generally accepted approach appears to be to use some relative definition of time: The national institute of standards gives me a reference time and this is what I measure. This, however, does not really tell us a lot about the “nature of time”, especially how we know that there is really an objective principle of nature that we refer to when we mention “time” rather than time being a completely man-made subjective concept that does not correspond to an objective property of nature. It is this objective principle that I would like to discuss here.

First of all, there is a simple, operational and relational but somewhat tautological definition that any child might come up with:

Time is what clocks measure. (T1)

This definition at least has the virtue that it does not directly refer to me or us and therefore is at least somewhat objective rather than subjective. But this definition already

presupposes an important property of “time”: We could have said

$$\text{Time is what my clock measures.} \quad (T2)$$

This obviously is a much more subjective definition as it refers to only one specific clock. Definition (D2) is really tautological: It defines one thing in terms of one other of which we know little and therefore is not more than a renaming. It is however an important property of time which objectifies it that it does not matter which clock one refers to when one defines it: I can measure time either with my watch which is based on some vibrating piece of silicone oxide or I can measure it with a pendulum clock or I can measure it with microwaves resonating in a cavity (Einstein’s light clock) or I can measure it in terms of the oscillation of light that was emitted in a specific atomic transition. Time is something which seems to exist independent of the material and working principle of the clock I use to measure it. It is deep fact that (T1) is actually well defined.

Of course there are some important qualifications: The clock actually has to be functional (I cannot use the pendulum clock after I hit it with a sledge hammer) but a broken clock should not be called a clock and we will investigate this a bit more later when we discuss what actually constitutes a clock. In addition, relativity tells us that the clocks will only measure the same (“proper”) time if they are sufficiently close together throughout the whole experiment and feel not too different gravitational forces (including forces resulting from acceleration). We should really envision the limiting situation of the two clocks coinciding which includes that the clocks should be small enough such that tidal forces are not important. This we will call that the two clocks are “close”.

Another important clarification is that this time is only defined up to affine transformations: Which time I call 0 is arbitrary and what my clocks actually measure are differences in time between two events. Furthermore, my pendulum clock ticks once per second whereas the microwaves in the cavity oscillate much faster. Therefore we see that different clocks in principle measure time at different rates. However, if the conversion factor c_{AB} for a pair of clocks A and B has been fixed (by measuring the time between two events with both clocks) once it will remain the same for all other measurements with these clocks. In addition, for any third clock C we have $c_{AC} = c_{AB}c_{BC}$. Therefore, for each clock we only have to know the conversion rate compared to one reference clock. And of course identical clocks have a relative conversion rate $c = 1$. This fact is reflected in the SI definition of a second: It singles out caesium clocks as the reference clocks in this sense.

Thus we find that a more precise formulation of the existence of an objective entity “time” can be given as the following observation which I will refer to as “equivalence of clocks”: *If two close clocks A and B measure the time between two pairs of events 1 and 2 then $T_{A1}/T_{B1} = T_{A2}/T_{B2}$.* As in mathematics, where a definition is good if it leads to strong theorems, this strong observational statement is the manifestation of a property of nature that is referred to as “time”.

The fact that we can define time independent of the material realisation of the clock used to measure it is the non-trivial part of our definition. This makes time not a property of clocks but in fact an objective property of nature. The observational fact that time

exists can be formalised as the possibility abstracting the measurement of a clock from its material realisation.

This should be contrasted with the elevation of a floor in a building defined as the number of steps one has to climb to get to that floor from street level. As long as there is only one staircase this is a valid definition. However, this is no longer well-defined when there are several staircases with varying heights of individual steps. Then the count of steps will depend on which staircase you took and in general there is not even an affine relation between the counts from different staircases if the height of a step is not uniform on staircases. In the world of an M. C. Escher painting, there wouldn't even be a monotonic relation.

Another example is from thermodynamics: Imagine doing experiments with a Carnot machine. There, the heat Q I extract from a process depends on the process and not just on the initial and final state. Therefore, the heat is not an objective property of the Carnot machine in the sense above. To obtain a "state variable" I have to divide the heat by the temperature to get the entropy which only depends on the initial and final state and not what I did in between. In a more formal way: the δQ is a one form in state space which is not closed while the one form $dS = \delta Q/T$ is.

Dropping the closeness requirement we can make a weaker statement that is usually called the "arrow of time": *If two clocks A and B (not necessarily close except at the events when the time is measured) measure the time between two pairs of events 1 and 2 then T_{A1}/T_{B1} has the same sign as T_{A2}/T_{B2} .*

I believe that this equivalence independent of the physical nature of the clock is what constitutes time as an objective aspect of nature.

Still of course all this discussion has of course been somewhat empty as we have not discussed what constitutes a clock and what distinguishes for example clocks from rulers or thermometers. In order to make the statement of the comparability of the measurements of all clocks as general as possible we should have a very broad idea of the concept of clock:

A localised subsystem of the universe that is sufficiently decoupled from
the remaining degrees of freedom and that is not constant but periodic
constitutes a clock (C)

Note well that we did not refer to "periodic motion" since "motion" probably presupposes a concept of time that we are here after. However we assume that some notion of "the universe" as a (differentiable) manifold but without singling out some of the directions as "time-like" a priori.

With this very broad definition of a clock, a measurement of time amounts to just counting the number of cycles of the periodicity of the clock. The fact that periodic systems always "run" at the same ("constant") rate compared to each other is really a highly non-trivial statement about nature. It can be traced back to the well-posedness of the initial value problem (which therefore we see as equivalent to the existence of time) or the Markov-ness of dynamics when described in phase space.

This means that, again, we have to convince us that this definition is well-defined: We

have to convince ourselves that such clocks tick at a uniform rate compared to any other clock. But this follows from the form of evolution equations that govern such subsystems: If the state of the clocks is such that they ticks right now and the evolution of this state is such that they come back to this state (and thus both click again, we can assume that the rates are in a rational relation) then since that state is then the same, the following evolution will also be the same and they will keep on indefinitely ticking coincidentally after always the same number of intermediate ticks of the individual clocks.

So far, this discussion has been classical but I made sure that it includes at least special relativity and general relativity situations of weak gravity in the sense that it is still possible to have clocks in extend much smaller than the radius of curvature. Our discussion was also general enough to include situations that are inherently quantum mechanical when sufficient care is taken to understand the proper notion of periodicity (“the subsystem coming back to the same state”).

Thus, we do not expect any further complications even in quantum theories of gravity and matter: As soon as the theory is capable of describing periodic sub-systems (like for example harmonic oscillators of some sort) we can use observations of those sub-systems as measurements of time thereby having an unambiguous concept which is not susceptible to any kind of subtleties coming from coordinate invariance. There is no need to model an unobservable “true” time, the only thing that has to be modelled are clocks and those can be sufficiently simple.

However, there might be a limit to this discussion of the nature of time in situations where on one side gravity is so strong that the clocks should be smaller but there are quantum mechanical limits on the minimal size of the clock (at least without inducing even stronger gravity). It is not clear that our definition of time can be extended to those quantum gravity situations and at least our description breaks down. But it is not clear that the concept of time should at all have a meaning in that context: It could well be that time (and space) are only emergent concepts that make sense only for weak enough gravity like for example in string theory. However, as soon as such a theory is in a regime where quantum gravity effects are not strong our above definition should be applicable and the comparability of clocks should hold. Otherwise the theory would be in conflict with our observations that suggest there is an objectifiable concept of time.