Trick or truth, or just a treat? Is the connection between physics and mathematics mysterious?

Phillip Helbig

February 2015

Mathematically (but not necessarily otherwise), I am a Platonist, which means that I believe that mathematical objects exist in a mathematical mindscape [1, p. 37 ff.] which is independent of human thought. To me, and, I think, to most physicists and mathematicians, this is obvious, because anything else would imply that non-human beings have some different sort of mathematics, if indeed they have any mathematics at all. Penrose's triangle of three worlds [2, sect. 1.4] in which the physical world is a part if Plato's mathematical world (Penrose's 'first mystery'), the mental world part of the physical world (his 'second mystery'), and the Platonic mathematical world part of the mental world ('third mystery') breaks down at the third step: mathematics is not a product of the mind, but rather something which, perhaps incompletely, is perceived by the mind. (Nevertheless, it seems that Penrose also believes in an objective mindscape outside of human existence, similar to Rucker's mindscape [2, sect. 1.3].) For similar reasons, physical reality also exists independently of human thought. This is even easier for most people to believe since, unlike mathematics, the question as to *where* it exists is not as pressing. Tegmark [3] has claimed that physical reality *is* mathematics. Whether or not that is true, I think there is another explanation for what Wigner called the unreasonable effectiveness of mathematics in the natural sciences [4].

What is mathematics? In essence, mathematics is a set of rules, by which one thing follows from another. Things are true if they can be shown to follow from, ultimately, a handful of axioms and common notions accepted by all. What is physics? Like chemistry or biology, it is also a set of rules, but in particular one which, like mathematics, can be used in practice. Even if chemistry ultimately follows from physics, and biology from chemistry, a description of biology or even chemistry in terms of rules in the way that physics is described by rules is at best unwieldly, and in practice impossible. As a result, emergent properties of such systems are described more heuristically. This is even more so in the case of social sciences and so on. Unless one believes that true randomness exists on some level, every effect has a cause and everything is governed by rules. If some physical process is governed by rules which are clear enough to us, we classify it as part of physics.

Physics is well described by mathematics because both are simple enough

for us to understand at the level of rules and their applications.

There are other things which are as well described by mathematics as is physics. One example are games. Whether games of chance or games of skill, the common property of all games is that they have clear rules. Of course, games are invented by humans so one could argue that the effectiveness is not mysterious. In a somewhat similar position is music, also described very well by mathematics, though one could argue that, in some sense, no mathematical description of music can be complete since it does not contain the emotional impact of music. However, the same argument could be made for physics, since the physical world also has an emotional impact.

Should we postulate some mysterious connections between mathematics and physics, mathematics and games, mathematics and music? Or would it make more sense to argue that such connections are due merely to the fact that all of these systems are characterized by rules which are easy enough for humans to understand?

Things which are not well described by mathematics might be simply too complicated for us to undertand their rules. Certainly there is no evidence for any sort of vital principle; emergence seems capable of explaining properties of more complex systems. Pre-scientific societies often saw the external world in terms of living beings [5]. Scientific societies are those capable of understanding the rules.

In summary, if there is no true randomness, then every process is determined by rules. Mathematics is the most abstract set of such rules. If something is simple enough that we can understand the rules, then we can describe it *via* mathematics. The connection between mathematics and physics has attracted more attention than that between mathematics and, say, games or music, because physics is perceived to be more fundamental, but these can all be described by mathematics for the simple reason that we can understand the rules by which they operate.

References

[1] Rudy Rucker. *Infinity and the Mind*. Bantam, New York, 1982.

[2] Roger Penrose. *The Road to Reality*. Jonathan Cape, London, 2004.

[3] Max Tegmark. *Our Mathematical Universe*. Allen Lane, London, 2014.

[4] Eugene P. Wigner. The unreasonable effectivenes of mathematics in the natural sciences. *Communications on Pure and Applied Mathematics*, 13(1):1–14, February 1960.

[5] Edward Robert Harrison. *Masks of the Universe*. Cambridge University Press, Cambridge, UK, second edition, 2003.