

# Physical Reality - On Dirac, Quaternions and the Wave Structure of Matter in Space

## Introduction

It is well known amongst philosophers of science that pre-existing beliefs, dogmas, tend to blind us to obvious truths. In an evolutionary sense, we can say that we are genetically programmed to be culturally programmed, and once our ideas are formed they are very hard to change.

Galileo famously referred to this with his comment that the philosophers refused to look through his telescope [1] and Einstein, writing about the importance of philosophers to science, also refers to this danger. He writes;

*"Concepts that have proven useful in ordering things easily achieve such an authority over us that we forget their earthly origins and accept them as unalterable givens. Thus they come to be stamped as 'necessities of thought,' 'a priori givens,' etc. The path of scientific advance is often made impassable for a long time through such errors." [2]*

This is why, as Kuhn observed;

*"Almost always the men who achieve these fundamental inventions of a new paradigm have been either very young or very new to the field whose paradigm they change." [3]*

Thus this FQXi essay contest asks a very important question; *'Which of Our Basic Physical Assumptions Are Wrong?'*, and further, *'What are the implicit assumptions we tend to forget we have postulated, or that have become so ingrained that they have become unquestioned dogma?'*

I believe that over the past 350 years many dogmas (assumptions) have emerged that are blinding us to obvious truths, and once we rid ourselves of these we find there is a simple solution to the central problems of physics (that currently cause a great deal of conflict and confusion). Below are eight assumption that I believe to be incorrect (they will be discussed at the end of this essay).

- i) Newton's formalism of matter being made of discrete and separate particles connected across space and time by instantly acting gravity forces (action-at-a-distance).
- ii) Faraday's formalism of electric and magnetic force fields.
- iii) The wave properties of matter, discovered in the 1920s, are not real and can only be considered as 'probability waves' for finding particle locations.
- iv) This particle wave duality of both light and matter is insoluble, simply a limitation of our minds and our theories, and physical reality itself is thus beyond comprehension.
- v) The velocity of light is constant.
- vi) Both relativity and non-locality prove that there can be no absolute three dimensional space.
- vii) That metaphysics is dead (one active substance exists that causes and connects the many things we experience) and mathematics is the only language for representing physical reality.
- viii) Our theories are approximations of physical reality, merely social constructs that are necessarily incomplete and not absolutely true (i.e. logical positivism, relativism, postmodernism).

So what is the correct assumption? Let us begin by stating this (existential statement), then show our evidence for why this is correct.

## **Fundamental Assumption - Space exists and propagates waves, and all matter-energy interactions in space-time are really wave interactions in space.**

Is this assumption acceptable to science. Yes it is, for three reasons;

i) We all experience existing in space.

ii) Space is a priori to our sense of external objects which are at the heart of empirical science.

iii) This unites metaphysics (one active substance, vibrating space) with Occam's razor of science (there is no more simple foundation for science than one substance, space, existing).

Before we move on though, there is one important distinction to be made. We are describing physical reality, i.e. matter-energy interactions in space-time. However, there is clearly more to reality than just the physical, our mental experiences of pleasure and pain, love and hate, good and bad, color and taste, etc., i.e. all our higher mental experiences can only be personally experienced. So there are still clearly limitations to how much we can know about reality (postmodernism), and that it is reasonable to assume that space must have further properties that are beyond the realms of science. Thus we have moved the boundaries of postmodernism to a closer understanding of reality (space exists and is vibrating), but there is still a separation between our ideas of space and the true nature of space itself.

## **The Dirac Equation**

In 1928, Paul Dirac, using a clever mathematical trick, factorised the relativistic Schrodinger equation into complex plane waves (Schrodinger had failed to solve this and ended up publishing his non-relativistic wave equation which provided the correct results for the allowed energy states of the hydrogen atom).

So what was the clever trick? Dirac (supposedly while sitting gazing at a fire at Cambridge university) realised that he could write the relativistic Schrodinger equation (2nd order, scalar) in factor form and any terms he did not know he could put in as variables, then multiply out his factors and see what the result was. His solution was complex (vector) plane waves in the form of Clifford algebra (complex quaternions) where the variables turned out to be imaginary numbers ( $i^2 = -1$ ). There were two paired solutions (representing four particles), two with positive energy (the two spin states of the electron) and two with negative energy (two spin states of the positron). The profoundness of this discovery cannot be overstated, as it deduced two things, yet unknown and only later confirmed by experiment - spin and antimatter (positron).

However, Dirac did not understand the solution, nor does it seem that anyone else has for the past 80+ years. Thus he ignored the plane wave solutions and proposed his idea, known as the 'Dirac sea', that space contained electron particles and positron 'holes' that these electrons could 'fall into'. This is a classic example where pre-existing ideas, that matter is made of particles, blind us to obvious truths, that matter is made of waves.

Dirac was aware of the limitations of his solution, as he writes;

*"The problem of the exact description of vacuum, in my opinion, is the basic problem now before physics. Really, if you can't correctly describe the vacuum, how it is possible to expect a correct description of something more complex?" [4]*

We can now understand what this mathematical solution really meant, but to do this we first need a little background about quaternions.

## **Quaternions**

When I first studied physics many years ago I found imaginary numbers very confusing, it disturbed my

sensibility to have a relation  $i^2 = -1$ . However, it is perfectly logical and sensible once you understand that we are dealing with vectors (directions) and that  $i^2$  is just a rotation of 180 degrees so the vector points in the opposite direction, thus the -1. From this we realise that the imaginary number  $i$  just represents a rotation of 90 degrees (orthogonal).

Fortunately for physics, a brilliant philosopher, metaphysicist and mathematician, Sir William Rowan Hamilton (1843) invented quaternions, a four dimensional algebra that uses this property of imaginary numbers to represent three orthogonal planes, which allows us to represent the motion of real things in real three dimensional space. Hamilton believed this mathematics was special and would be important to physics (he carved it on a stone bridge, as he was walking when he thought of it and did not want to forget!).

And it turns out he was correct about its importance - in fact he was 60 years ahead of Einstein in connecting three dimensional space (x,y,z) with time (t) into a four dimensional space-time continuum in one quaternion equation  $q = t + ix + jy + kz$  where the  $i$ ,  $j$ , and  $k$  terms are imaginary numbers such that  $i^2 = j^2 = k^2 = ijk = -1$ . As Hamilton poetically wrote of his quaternions;

*"Time is said to have only one dimension, and space to have three dimensions. ... The mathematical quaternion partakes of both these elements; in technical language it may be said to be 'time plus space', or 'space plus time': and in this sense it has, or at least involves a reference to, four dimensions. And how the One of Time, of Space the Three, Might in the Chain of Symbols girdled be." [5]*

It is beautifully elegant mathematics that perhaps scares off many people because of its use of imaginary numbers  $i$ ,  $j$ ,  $k$ . But these just represent 90 degree rotations.

So whenever you see  $ix$ ,  $jy$  and  $kz$  you are considering vectors on orthogonal planes (90 degrees, perpendicular, at right angles) to the  $x$ ,  $y$  and  $z$  axis. This then gives us the four dimensional mathematical structure to represent plane waves flowing through 3D space.

However, we must add another level of complexity as we are working with complex (transverse) plane waves in three dimensional space so we need to add a second level of complex numbers to represent these transverse oscillations of the plane waves. This is where the work of Clifford (1878) is important as he developed quaternions into complex quaternions that do exactly this.

So now we are in a position to test this complex plane wave theory of reality by writing the wave equations in the form of complex quaternions to see if it correctly deduces the properties of matter that we observe in the space around us. We find that it works perfectly. For a general summary I recommend the following paper by Andre Gsponer and Jean-Pierre Hurni. Their conclusion.

*"If quaternions are used consistently in theoretical physics, we get a complete description of the physical world, with relativistic and quantum effects easily taken into account. Thus Hamilton's idea, which motivated more than half of his professional life, that quaternions are a fundamental building block of the physical universe, appears to be essentially correct in the light of contemporary knowledge." [6]*

## **Deducing the Dirac Equation with Complex Quaternions**

We find, by using complex quaternion wave equations to describe these complex plane waves in three dimensional space, that there are four unique solutions (two electron-positron pairs) where the complex (transverse) wave components cancel, forming scalar (longitudinal) spherical standing waves - space vibrates radially in and out around a central point - where this point like wave center is the cause of the particle effect of matter that has deceived us for so long.

Though I came to the conclusion that the electron and positron were opposite phase spherical standing waves in space many years ago (1997), this realisation that they were created by intersecting complex plane waves only occurred to me recently (May, 2011) after I had multiplied a quaternion  $Q$  by its complex conjugate  $Q^*$  (opposite vectors) and discovered that the complex / vector components all cancel

leaving a real / scalar result as follows.

$Q = t + ix + jy + kz$  and the complex conjugate  $Q^* = t - ix - jy - kz$

$$QQ^* = t^2 - tix - tjy - tkz + ixt - i^2x^2 - ixjy - ixkz + jyt - jyix - j^2y^2 - jykz + kzt - kzix - kzjy - k^2z^2 = t^2 + x^2 + y^2 + z^2$$

From this mathematics it occurred to me that the opposite vectors could be used to represent opposite phase transverse waves that are destructively interfering / cancelling one another, leaving just the scalar / longitudinal waves (space vibrates radially in and out around a central point). However, to apply this to physics we must write the above equation using complex quaternion wave equations which are much more complex again. The point is that even in this simplified form you can see that the vector / complex components (on left) all cancel leaving the four term scalar on the right. Basically the terms on the left represent the phase of the complex / vector plane waves of background space that are fundamental to quantum physics, the four terms on the right represent the scalar product of this which is in the four dimensional space-time structure of matter in Einstein's relativity and Schrodinger's relativistic scalar wave equation.

From this discovery five things occurred to me.

- i) The negative energy solutions (positrons) are not negative energy, rather, the negative sign represents the opposite phase spherical standing wave (which is why positrons and electrons annihilate due to destructive wave interference).
- ii) What we call empty space is in fact full of waves, but the transverse wave components do not cancel so space vibrates in an infinite variety of patterns, what we now call the quantum field.
- iii) Electrons and positrons will be forming in 'empty space' all the time due to the natural chance that the phase of the transverse wave components cancel. (I did not know this at the time, searched it and found it to be true - they call them virtual particles since they quickly annihilate.) [7]
- iv) Most profoundly, this then means that what we call the energy of the electron and the positron is actually the energy required to separate them once they have formed, and ignores the actual energy of the spherical standing wave, their true energy. This has profound implications for 'dark energy' and the Higgs particle!
- v) It must be possible to deduce the Dirac equation from this quaternion foundation. I searched this and was surprised to see that Lanczos had actually done this a year after Dirac (1929), and most significantly, in the same equation he also deduced isospin seven years before Proca discovered this (1936). [8]

## **Complex Quaternion Wave Equations Deduce the Central Equations of Modern Physics**

We can now show, with this mathematical formalism for the wave structure of light and matter in three dimensional space, that it deduces the main equations of quantum physics and Einstein's relativity. i.e. We have a simple unified description of physical reality that satisfies the laws of Nature. Most importantly, you will notice in the following solutions that the work is not mine, I have just used the internet to search complex quaternion wave equations and various fields of physics. I invite the reader to do the same thing for themselves. (This is not my 'pet theory / crackpot science'.)

1. Maxwells Equations. All four Maxwell equations appear in the one quaternion equation.

<http://theworld.com/~sweetser/quaternions/spr/Maxwell.html> [9]

2. Lorentz Transformations.

<http://redshift.vif.com/JournalFiles/V12NO4PDF/V12N4ACE.pdf> [10]

3. de Broglie Matter Waves. These are actually faster than light (non-local) phase waves, and are caused by a Doppler shifting of the in and out waves around the wave center when it moves relative to other wave centers. Most importantly, in the same equation you also get the relativistic mass increase of special

relativity, thus uniting quantum physics with special relativity for relative motion.

<http://www.quantummatter.com/space-resonance/origin-of-the-natural-laws-in-a-binary-universe/> [11]

(See point 19 near end of page)

4. Derivation of Dirac, Klein-Gordon, Schrodinger, Diffusion and quantum heat transport equations from a universal quaternion quantum wave equation.

[http://uofk.academia.edu/arbabarbab/Papers/474054/Derivation\\_of\\_Dirac\\_Klein-Gordon\\_Schrodinger\\_diffusion\\_and\\_quantum\\_heat\\_transport\\_equations\\_from\\_a\\_universal\\_quantum\\_wave\\_equation](http://uofk.academia.edu/arbabarbab/Papers/474054/Derivation_of_Dirac_Klein-Gordon_Schrodinger_diffusion_and_quantum_heat_transport_equations_from_a_universal_quantum_wave_equation) [12]

5. Complex Quaternions and Special Relativity

<http://onlinelibrary.wiley.com/doi/10.1002/0471713465.app3/pdf> [13]

<http://arxiv.org/pdf/hep-th/9508011.pdf> [14]

<http://gravisma.zcu.cz/GraVisMa-2010/Papers/Short/E53-full.pdf> [15]

6. Complex Quaternions and General Relativity.

[http://ckw.phys.ncku.edu.tw/public/pub/WebSources/Quarternions/www.pcisys.net/\\_bestwork.1/index-2.html](http://ckw.phys.ncku.edu.tw/public/pub/WebSources/Quarternions/www.pcisys.net/_bestwork.1/index-2.html) [16]

There is also a very interesting article on the history of physics, how quaternions were developed, and how they were replaced by vector calculus (Gibbs, Heaviside) and led to the infinite dimensions of Hilbert space - which has led to many of the problems of modern physics - working with infinite dimensions rather than a real three dimensional space.

<http://www.valdostamuseum.org/hamsmith/QOphys.html> [17]

This is just the tip of the iceberg, there is an enormous amount of work on complex quaternions and various fields of modern physics that the reader can search for themselves, just search 'complex quaternion wave equations' and any field of mathematical physics. You will find in most cases that someone somewhere on the planet has deduced the solution in quaternion wave structure. However, no one seems to have yet realised why - because this mathematics represents really existing plane waves in three dimensional space.

You can read my own work on quaternions and the wave structure of matter at;

<http://www.spaceandmotion.com/> [18]

<http://www.spaceandmotion.com/physics-quaternion-complex-plane-wave-equation.htm> [19]

<http://www.spaceandmotion.com/metaphysics.htm> [20]

<http://www.spaceandmotion.com/mathematical-physics/logic-truth-reality.htm> [21]

## **A Brief History of Physics Explained with the Wave Structure of Matter in Space**

Let us now consider the major discoveries of modern physics that occurred in the first half of the 20th century, but now we can understand their discoveries based upon this wave structure of matter. In this way we can explain why the assumptions listed at the beginning of this essay are incorrect.

At the beginning of the 20th century the discrete particle theory of matter (Newton, 1687) and the continuous electromagnetic wave theory of light (Maxwell, 1861) were well accepted in mathematical physics. However, neither of these theories were correct. The particle is formed from a spherical standing wave in space, the point like wave center causing the particle effect. The electromagnetic wave is really a repeating pattern of 'hills and hollows' on the surface of the plane waves that create these spherical standing waves - due to their interaction with other wave centers trapped in repeating motion in atoms (setting up resonance).

In 1900 Max Planck introduced the idea that light energy was discrete / quantised. In 1905 Albert

Einstein further developed this idea of light quanta with the equation  $E=hf$ , where discrete frequencies corresponded to discrete quanta of light energy.

This is correct because light is a resonant phenomena between bound resonators (electrons in atoms / molecules) where the electrons are spherical standing waves.

In the same year Einstein also published his paper on special relativity (non-accelerated reference frames) that showed that matter and energy were equivalent, and that the velocity of light was constant irrespective of the motion of the source or receiver. This is correct, matter and energy are equivalent because matter is formed from waves, and a wave is a flow of energy between two different states of the wave medium, where space is a slightly elastic (nearly rigid) wave medium and the two energy states are kinetic and potential energy. The velocity of light is constant simply because we are dealing with a purely theoretical model of non-accelerated reference frames (which do not exist in reality, gravity exists everywhere) and as explained below all forces / acceleration are due to wave interactions caused by changing velocity of waves.

In 1916 Einstein published his paper on general relativity that included acceleration. It was based on the equivalence of inertial and gravitational mass (gravity could be treated as an accelerated reference frame). The metaphysical foundation was that matter-energy could be represented as continuous fields in a four dimensional space-time continuum, where the space-time coordinates changed with energy density, known as the curvature of the 4D space-time continuum. Importantly, Einstein did not believe that either light or matter was made from discrete particles, as he writes;

*"Physical objects are not in space, but these objects are spatially extended (as fields). In this way the concept 'empty space' loses its meaning. ... The field thus becomes an irreducible element of physical description, irreducible in the same sense as the concept of matter (particles) in the theory of Newton. ... The particle can only appear as a limited region in space in which the field strength or the energy density are particularly high. " [22].*

Einstein is close to the truth, he was correct that there are no discrete and separate particles, his error was to represent matter as continuous fields in space-time, rather than real waves in continuous space that cause both matter and time. The continuous fields are incorrect, and are caused by many discrete wave interactions in space, which explains why his field theory of matter was never able to explain the discrete phenomena of quantum physics. He came to realise this at the end of his life, he writes;

*"I consider it quite possible that physics cannot be based on the field concept, i. e., on continuous structures. In that case nothing remains of my entire castle in the air gravitation theory included, [and of] the rest of modern physics." [23]*

Further, and few people seem to know this, in general relativity the velocity of light depends on the energy density of space, which is correct. Einstein writes;

*"(Special relativity is founded) on the basis of the law of the constancy of the velocity of light. But the general theory of relativity cannot retain this law. On the contrary, we arrived at the result that according to this latter theory the velocity of light must always depend on the co-ordinates when a gravitational field is present." [24]*

This is correct, the velocity of light is slower in higher energy density space, explaining why light curves as it propagates through the space of our sun. This slower wave velocity changes the shape of the surface of the plane waves into a curved surface, what is known as the curvature of the four dimensional space time continuum. However, because there is a corresponding change in wavelength, and thus dimension, we never observe this changing velocity of light, it is always measured to be the same even though it is not constant (Nature is subtle and deceptive!).

In 1924 Louis de Broglie reasoned that if light had both particle and wave properties then this could also apply to matter, and led to his deduction of the de Broglie matter wave. This explained why electrons could only exist at discrete energy states in an atom, as their 'orbits' had to be a discrete number of

wavelengths long. Thus light energy must also be discrete as this related to these discrete changes in the electron's wave orbit.

It is correct that matter is a wave structure and can only exist in discrete wave patterns in the atom, it is incorrect (and unnecessary) to assume it is also a particle. Further, the de Broglie wave is a phase wave with velocity  $c^2 / v$ . (e.g. If the relative velocity  $v$  is  $1/100 c$  then the phase wave velocity is  $100 c$ , i.e. non-local / faster than light). It was this non-locality that Einstein could never accept (he called it 'spooky action at a distance') however it is simply a natural consequence of phase waves travelling faster than the group waves which create them (the group waves are the plane waves which travel at the velocity of light  $c$ ). This also explains Bohm's pilot wave theory (based on the de Broglie wave) that showed that there was a deterministic explanation of quantum physics if there were non-local hidden variables, which is correct.

In 1925, Erwin Schrodinger, applying this idea that waves behave discretely, wrote the energy equation for the electron as a wave equation. His relativistic wave equation (also known as the Klein Gordon equation) was a second order scalar equation that did not produce correct results, and he eventually discarded it and published (1926) his non-relativistic equation that predicted the the correct light spectra for the hydrogen atom. As we have explained above, Dirac solved this in 1928, and Lanczos deduced this from the correct mathematical foundation of complex quaternion wave equations in 1929.

In 1926, Max Born discovered that the square of the wave equation could be used as a probability wave to determine the location of the particle, thus leading to the current particle / probability wave interpretation of quantum physics. While this probability interpretation works, it is simply due to the fact that matter is a spatially extended wave structure of the universe, and it is this lack of knowledge of the total system that gives rise to the probability (as Einstein believed, and reflected in his comment 'God does not play dice'). Likewise, Schrodinger never accepted the probability wave interpretation, believing the waves were real. He writes;

*"Let me say at the outset, that in this discourse, I am opposing not a few special statements of quantum mechanics held today (1950s), I am opposing as it were the whole of it, I am opposing its basic views that have been shaped 25 years ago, when Max Born put forward his probability interpretation, which was accepted by almost everybody". [25]*

## Conclusion

It is no longer correct to say that we do not have a complete realistic description of physical reality that unites relativity and quantum physics. The wave theory of matter is based on the most simple science foundation for describing physical reality and correctly deduces the laws of Nature.

I believe the difficulty is no longer science, but the scientists who have been brought up with different foundations, incorrect assumptions that are blinding us to obvious truths. As Planck wrote;

*"A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it." [26]*

In ending, a beautifully prophetic quote from J. A. Wheeler

*"Someday we'll understand the whole thing as one single marvelous vision that will seem so overwhelmingly simple and beautiful that we may say to each other; 'Oh, how could we have been so stupid for so long? How could it have been otherwise!" [27]*

Geoffrey Haselhurst  
Natural Philosopher

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