Mathematics is the Link Between Physics and Metaphysics

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Abstract

Mathematics allows us to develop physical theories that make accurate predictions. These theories relate a primitive ontology to nomological variables. All of our physical theories make metaphysical commitments that are directly or indirectly related to mathematical models. After a brief introduction to some of the fundamental issues associated with the use of mathematics in developing physical theories, I discuss how the link between physics and metaphysics was achieved in classical mechanics, special relativity and quantum theory. I argue that mathematics may not be sufficient for developing a theory of everything due to incompleteness of formal systems or the existence of unknowable truths.

1. Introduction

In the history of physics there is a long list of failed mathematical models. Examples include Gottfried Leibniz's formulation of the principle of conservation of energy, pushing gravity theory with a faster-than-light material graviton, the emitter theory of light propagation and even certain aspects of Newtonian mechanics. The time symmetry of the Newtonian equation of motion is a mathematical result that is not part of physical reality but only of mathematical reality. In order to see that, consider Newton's second law, which reduces to **F** = m**a** for constant mass m, where **F** is the applied force and **a** is the acceleration. Solving for the acceleration **a** we get:

$$\mathbf{a} = \frac{\mathbf{F}}{\mathrm{m}} \qquad (1)$$

Assuming that **F** and m are constant and initial conditions are equal to zero, we can integrate both sides of equation (1) twice to obtain the following equation for position as a function of time t in the case of linear motion:

$$\mathrm{s} = \frac{\mathrm{F}t^2}{2\mathrm{m}} \qquad (2)$$

Equation (2) is time-reversible since position is a function of time squared. There is nothing in Newton's second law to demand that when a force is applied to a particle, a distance will be covered in positive time, meaning in the future, or in negative time, meaning in the past. Since motion in negative time in the macrocosm is not empirically verified, the solution of equation (2) for negative time is not acceptable. This example illustrates how mathematical models used in physics can predict phenomena that fail verification and are part of mathematical reality only.

One possibility is that mathematics can model the totality of physical reality, as shown in Figure 1(a). It is also possible that mathematics cannot model the totality of physical reality, as shown in Figure 1(b). If the latter is true, then the program of using mathematics in the search of a *theory of everything* is an exercise in futility.

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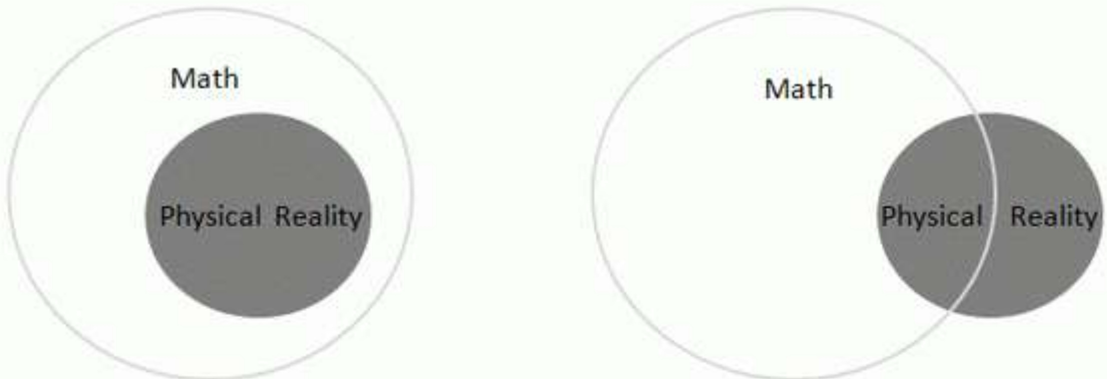

(a) Math can model all of physical reality (b) Math cannot model all of physical reality

Figure 1. Math as a tool for modeling physical reality

Logical positivists claim that everything can be described mathematically and they also adhered to the *knowability principle*, i.e., to the notion that every truth is knowable. However, the position that all of physical reality is knowable through mathematics can lead to the puzzling result that we are omniscient (Fitch, 1963). To see that, let us consider a variation of *Fitch's paradox of knowability* that uses the sentence p, which is an unknown mathematical truth about physical reality. The sentence p is true but it is not known that p is true. Using this sentence and the knowability principle, one can prove that all mathematical truths about physical reality are known by physicists collectively at some point in time[2]. The proof goes as follows: if p is an unknown truth and all truths are knowable, it should be possible to know that p is an unknown truth. But as soon as we know that p is an unknown truth, we know p. Apparently, Gödel's Theorem challenges this result. This theorem states that any consistent formal system within which a certain amount of arithmetic can be carried out is incomplete. An acceptance of Gödel's Theorem leads to a questioning of the knowability principle and to the conclusion that there exist truths about physical reality that are unknowable via mathematics. A further consequence of this is that the unification of consistent theories in different domains of application, such as general relativity and quantum mechanics for example, maybe incomplete. This does not necessarily mean that physical reality has unknowable aspects to it but it could only mean that modeling it using mathematics cannot lead to the desired omniscience implied by the knowability principle and Fitch's proof. Note that questions have been raised about the meaning of both Gödel's

[2] For details see: "Fitch's Paradox of Knowability" in Stanford Encyclopedia of Philosophy, http://plato.stanford.edu/entries/fitch-paradox/

Theorem and Fitch's paradox. The objective in this paper is to provide a brief account of the philosophical issues arising from the use of mathematics in developing physical theories.

2. Mathematics and metaphysics in classical mechanics

Isaac Newton was the first to use mathematics successfully in a physical theory. He claimed that the metaphysical commitments his theories made were directly related to empirical facts. An example of this effort was the rotating bucket experiment that he used to argue about the existence of absolute space[3]. Newton's mathematical approach to physics gained popularity and the metaphysical commitments related to his mechanics continued to be of interest only to philosophers of science. I will demonstrate below that Newton's second law of motion is a link between empirical physics and metaphysics and that such link was achieved through the use of mathematics and assumptions about the nature of physical reality.

Newton's second law as it originally appeared in the *Principia*: "The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed." (Newton, 1952)

Newton defined *motion* as follows:

> "The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjunctly. The motion of the whole is the sum of the motions of all the parts; and therefore in a body double in quantity, with equal velocity, the motion is double; with twice the velocity, it is quadruple." (ibid)

From this definition we infer that Newton defined *motion* as equal to the product of mass and velocity, a vector that was later called *momentum*. In mathematical notation

$$\mathbf{p} = m\mathbf{v} \qquad (3)$$

where $\mathbf{p}$ is the momentum vector, m is the mass and $\mathbf{v}$ is the velocity vector. According to the second law then, the alteration (change) of motion is proportional to the motive force impressed:

$$\mathbf{p_f} - \mathbf{p_i} = k\mathbf{F_m} \qquad (4)$$

where $\mathbf{p_f}$ is the final momentum, $\mathbf{p_i}$ is the initial momentum, $\mathbf{F_m}$ is the motive force impressed and k is a constant. Furthermore, Newton defined an *impressed force* as follows:

> "An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line. This force consists in the action only; and remains no longer in the body when the action is over. For a body maintains every new state it acquires, by its vis inertia only. Impressed forces are of different origins as from percussion, from pressure, from centripetal force." (ibid)

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[3] "Newton's Views on Space, Time, and Motion", " in Stanford Encyclopedia of Philosophy, http://plato.stanford.edu/entries/newton-stm/

In the second law Newton made use of the term *motive force impressed*. Obviously, there must be some difference between an *impressed force* and a *motive force impressed.* Without an essential difference between these two types of forces, the modern expression of the second law that equates force to the time rate of change of linear momentum cannot be deduced.

Let us rearrange equation (4) as follows:

$$\mathbf{p_f} = k\mathbf{F_m} + \mathbf{p_i} \qquad (5)$$

Since k is the proportionality constant, $\mathbf{F_m}$ must be also a momentum vector in equation (5). Therefore, momentum causes a change in momentum, which is a tautology. Based on Newton's original statements and definitions, it is not immediately clear how to derive the fundamental law of mechanics. However, that can be achieved after assuming that *motive force impressed* is force **F** times the interval of time in which it is impressed and equation (5) becomes (Barbour, 2001):

$$\mathbf{p_f} - \mathbf{p_i} = k\mathbf{F}\Delta t \qquad (6)$$

After making this assumption we can write equation (6) as follows:

$$\mathbf{F} = \frac{\mathbf{p_f} - \mathbf{p_i}}{k\Delta t} \qquad (7)$$

Taking the limit of the right hand side of equation (7) as Δt approaches zero and choosing a system of units in which k =1, the familiar form of Newton's second law emerges:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad (8)$$

To arrive at this form of the second law from the original statements and definitions of Newton, we had to equate the *motive force impressed* to a quantity known in mechanics as i*mpulse*, which is equal to the product of force and time. As a result, in equation (6) the left hand side is an empirical quantity, since both the rate of change of position and the quantity of mass can be measured in some unit system, but the right hand side is a commitment to the metaphysics of time and force. In essence equation (6) relates empirical physics to metaphysics as follows:

$$\text{Empirical physics } \{\mathbf{p_f} - \mathbf{p_i}\} = \text{Metaphysics } \{k\mathbf{F_i}\Delta t\} \qquad (9)$$

The notion of force is metaphysical, i.e., an assumed feature of physical reality, since forces are observed only by their effects. This notion has been criticized by various scientists and philosophers of science as an intellectual construction (Greenwood, 1965). Similarly, time is a metaphysical notion, since it can be measured indirectly using clocks that either involve harmonic motion, such as a pendulum, or count events, such as atom transitions. Time is defined in terms of motion and, in turn, motion evolves in time. Newton understood this cyclicality and defined *true time*, i.e., metaphysical time, as "*something absolute that flows independently of the other three spatial dimensions*." (Newton, 1952).

Thus, we have seen that there are certain difficulties in deriving Newton's second law that are resolved as soon as we commit to the metaphysics of time and force. Without the metaphysical commitments the law reduces to a tautology, equation (5), and force is just another measure of the empirically determined quantity of momentum. The metaphysical commitments made in deriving the fundamental law of mechanics do not reduce its significance but instead show how mathematics can provide a link between physics, in the sense of empirical observations, and the metaphysics of what exists in physical reality. Without the metaphysical commitments a transition from kinematics to dynamics is impossible. Note that the term *dynamics* already makes an appeal to metaphysics. Newton made a bold step and established this connection via his second law that became the foundation of mechanics and facilitated the birth of modern physics. On the contrary, Leibniz tried to define an innate property of matter that would be a measurable cause of all interactions and an empirical quantity. He defined the *living force*, or *vis viva*, as the quantity mv^2, which he argued is conserved (Roberts, 2003). This effort failed because velocity is a frame-dependent quantity. For example, velocity has a zero value for a stationary passenger in a moving train but a non-zero value for an observer on the ground. As a result, Leibniz's *vis viva* could not serve the purpose of an innate active force because its value varies depending on the reference frame. Therefore, dynamics without any metaphysical commitments but based on pure empirical quantities was an impossible task.

Harokopos (2005) has shown that if the metaphysics of force are replaced by the metaphysics of power, then a different set of laws of motion can be derived that support causeless curvilinear motion at constant speed, such as for example uniform circular motion. In Newtonian mechanics a *centripetal force* is required for uniform circular motion but when power is considered as the cause of motion, no active cause is necessary for that type of motion because power is equal to zero for that particular path given that kinetic energy is constant. The fundamental law of motion when power is used in the place of force is expressed as follows:

$$P = \frac{d(E_K)}{dt} \qquad (10)$$

where P is the (instantaneous) power and E_k is the kinetic energy. The laws of motion that emerge after considering power as the metaphysical cause of motion support a non-autonomous universe that is manifested in discrete spacetime by a higher reality (Harokopos, 2011). These metaphysics are different from those of Newton that support an autonomous universe in which interactions generate forces and in turn forces cause interactions. Specifically, the metaphysics of power support a universe in which interactions occur due to the transfer of energy from a background spacetime that acts as a giant mechanism to bring about the phenomena. Note that although the two different sets of laws of motion are computationally equivalent, the mathematics is different and the metaphysical commitments made assume different physical realities, an autonomous world in the case of laws of motion based on force and a non-autonomous world in the case of the laws of motion based on power. This difference emerges due to a richer *law of inertia* when power is assumed to be the cause of motion that allows causeless motion in curvilinear paths at constant speed. As a result, the cause of this type of inertial motion could be attributed to another reality that is different from physical reality.

In this section, we saw that Newton's second law links physics and metaphysics is and mathematics is the tool for achieving that. In the next section, we will see how the mathematics of relativity theory is the link between physics and some different metaphysical commitments than those made by Newton's laws.

3. Mathematics and metaphysics in special relativity

Special relativity is the accepted theory for studying phenomena where gravitational forces are negligible and speeds get close to the speed of light. All the predictions this theory makes have been confirmed in many experiments around the world. Special relativity replaces the common sense view of a three-dimensional space and absolute time of Newtonian mechanics with a four-dimensional spacetime where different observers measure different times and locations for the same event depending on their state of motion. Special relativity converges to Galilean relativity, which is the basis of Newtonian mechanics, when speeds are low compared to the speed of light.

A central notion in special relativity is the *spacetime interval* between the origin event (0,0,0,0) and an event (x,y,z,t), defined as follows:

$$s^2 = x^2 + y^2 + z^2 - (ct)^2 \qquad (11)$$

The spacetime interval is invariant for all observers in inertial reference frames irrespectively of their speed. Note that in three-dimensional space the term that involves time in equation (11) is not present. In special relativity, time is an imaginary coordinate and part of a four-dimensional spacetime, also known as *Minkowski spacetime*.

Equation (11) looks like an ordinary mathematical object but when used in a theory of physics, it establishes a link between the empirical world of rulers and clocks that allow making local measurements in inertial reference frames and the metaphysics of a four-dimensional spacetime. This equation reflects a metaphysical commitment made by a more accurate model of physical reality in its domain of application. This metaphysical commitment comes at a high price that some scientists are still not convinced it must be paid. Specifically, special relativity commits to a tenseless theory of time and existence and to events instead of substances (Esfeld, 2009). That represents a significant departure from the metaphysical commitments of classical mechanics. According to special relativity, simultaneity is relative to a reference frame and is not objective. As a result, an event that it is the past of an observer can be in the future of another observer depending on their relative state of motion. Everything that exists in the past, present or future, exists in a four-dimensional *block universe*. The objects in the block universe are different from the objects of classical mechanics and of a three-dimensional universe. The objects of relativity theory have temporal parts, not just geometric, and their extension is not fixed but is dependent on the reference frame of the observer. In relativity theory physical objects are modeled as continuous sequences of spacetime points, also known as *processes*. According to this radical departure from classical mechanics, motion in the block universe of relativity can be defined as a continuous sequence of spacetime points that have similar physical content. Change is a continuous sequence of spacetime points with different physical content (ibid).

One could argue that the notion of three-dimensional objects with extension is compatible with special relativity. This view leads to inconsistencies, like the bug-rivet paradox[4]. One is forced to accept only four-dimensional objects in special relativity so that they maintain their four-dimensional configuration relative to all inertial reference frames. Thus, objects must also have temporal parts to avoid the inconsistencies.

[4] See for example: http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/bugrivet.html#c1

An extension to the familiar three-dimensional space interval in special relativity, equation (11), along with the principles of that theory, led to one of the most successful theories of all times. Mathematics established a link between the empirical world of physics and the hypothetical metaphysics of what exists. One could ignore the metaphysics and just use the theory as an instrument but that does not purge the metaphysical commitments made if what we are after is a true theory of physical reality.

In the next section, I briefly discuss how the mathematics used in quantum theory establishes a link between physics and metaphysics and the problems that emerge in that theory due to the large number of interpretations.

4. Mathematics and metaphysics in quantum theory

The fundamental law of classical mechanics, discussed in Section 3, is useful in determining the time evolution of a classical system given its initial conditions. The law was deduced from Newton's original laws and definition after making some assumptions and metaphysical commitments. In quantum theory, the fundamental law is the time–dependent form of the Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(r,t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2 + V(r,t)\right]\psi(r,t) \quad (12)$$

where ψ is the wave function, i is the imaginary unit, $\hbar$ is the Planck constant divided by 2π, μ is the reduced mass of the particle, and V is the potential energy. This law is expressed in the form of a partial differential equation that describes the time evolution of a quantum system's wave function. How this equation originated is not fully known. Richard Feynman thought the equation was an intellectual construction and that it was impossible to derive it from anything that was known (Hey, 2009). Despite that, quantum theory is one of the most successful theories of all times. The predictions that this theory makes are precise and have been confirmed to high accuracy many times in laboratories around the world. It is believed that about one third of the gross national product of the USA is directly or indirectly related to advancements in quantum theory (Lyre, 2010).

There are many interpretations of quantum theory and some of them are related to the meaning of the wave function. As with Newton's second law and its commitment to the metaphysics of force, in a similar but more intricate way, Schrödinger's equation is a mathematical object that relates empirical measurements to the metaphysics of a wave function. Although there is the *standard interpretation* of quantum theory in which the wave function is an intellectual construction that is useful for analyzing quantum systems, there are several other interpretations that assign a real existence to it, for example the *causal* and *many-worlds*. It is beyond the scope of this paper to discuss in more detail these interpretations as the aim here is to support the thesis that mathematics allows a link between physics and metaphysics, or said in another way, between *nomological variables* and the *primitive ontology* of physical reality. All of our physical theories use mathematics this way, i.e., by linking a primitive ontology to nomological variables. In the case of quantum theory a nomological variable is the wave function and in the case of Newtonian mechanics it is the momentum. The link to nomological variables is necessary because physics goes beyond assumptions about a primitive ontology and aims at describing how reality evolves in time. In simpler words, since we do not know the true nature of our physical reality, we conceive certain mathematical laws that link some (hypothetical) features of it that we think are primitive, such as for example particles, to some other features that are not primitive, such as for example momentum or wave function. The non-primitive features are necessary for describing how physical reality evolves in time. Without the notion of momentum it would be impossible to derive Newton's second law despite of an assumed primitive ontology. However, the choice of primitive ontology also plays an important role and determines the nature of the link between physics and metaphysics. Allori (2012) has put that concisely as follows: "Any fundamental physical theory must always contain a metaphysical hypothesis about what are the fundamental constituents of physical objects. We will call this the primitive ontology of the theory." The price paid for this approach to developing physical theories is the large number of alternatives that are based on different primitive ontology and nomological variables but with equivalent predictions. Another price paid is the large number of different interpretations of the same theory. The laws of motion based on power discussed in Section 3 are an example in classical mechanics and Matrix Mechanics is an example in quantum theory. Despite these realities, mathematics is a powerful tool for establishing the link between the empirical world of phenomena and the (hypothetical) metaphysics of what exists in physical reality.

5. Discussion

Since the seventeenth century mathematics has played an increasingly important role in physics and has advanced to levels that allow solving complex problems. The connection between mathematics and physics is nowadays so powerful and any theory that is not expressed mathematically is considered pseudoscience. This connection will always be important on the computational level. However, it is likely that a *meta-mathematical* framework will be developed in the future that will allow a transition to the next level of understanding of our physical reality. Assuming that Gödel's Theorem is true, a complete understanding of our physical reality cannot be obtained using mathematics alone and we must rely on more powerful instruments and more complex experiments. But performing complex experiments without knowing what we are looking for may not be an efficient way of achieving our goals. It is also possible that all truths about our physical reality are knowable and we are potentially omniscient, as Fitch's proof shows, but we have not achieved this omniscience due to *a singularity* that occurred in the past and delayed our progress.

Another possibility may be that our reality is not autonomous but it is continuously created in discrete spacetime by a higher reality, a sort of a virtual reality that is not necessarily similar to a computer generated virtual reality. In this case there may be unknowable truths and we may never become omniscient through our theories and the use of mathematics. Both Descartes and Leibniz stumbled on this possibility. Descartes concluded that this world is possible only if it is continuously recreated at each moment by an *immutable God* (Franco, 2001). Leibniz failed to find an empirical quantity that would be the measure of a *living force* that was necessary for the *autonomy* of the world. Newton took a bold step and used mathematics to commit to the metaphysics of force, offering physics a chance to move forward. Using mathematics as a tool for establishing the link between a *primitive ontology* and *nomological variable,* just as Newton first did, gave physics the necessary boost to progress. We have now reached a point that our mathematics, as powerful as they may be, cannot provide any guidance about choosing the most relevant interpretation of our physical theories. Although this is not required on the computational level, it is desirable to know whether we live in a three-dimensional space with absolute time or in a four-dimensional spacetime and whether there is only one universe or infinite multi-universes, amongst other things. Only when we know all the answers we will have a *theory of everything*. At this point a unification of general relativity with quantum mechanics appears unattainable. Possibly we lack the *meta-mathematical* framework that defines the appropriate connection between physics and metaphysics, i.e., the appropriate choice of primitive ontology and nomological variables. I will make in this paper no hypothesis of what that framework may be like or even if it exists at all. But if such framework is possible, a curious lack of its knowledge is what may be holding us from achieving omniscience about our physical reality and delays our transition to the next level of existence and civilization.

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