The Discreet Charm of the Discrete

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In 1900, Lord Kelvin delivered an instrumental speech to the Royal Institution of Great Britain delineating that era's accomplishments and gaps in physical understanding. While heralding the tremendous progress made in the field of physics in offering a comprehensive picture of nature, Kelvin pointed to two "dark clouds" obscuring the view forward [1]. Why couldn't Earth's motion through the aether, the substance supposed to be the conduit of light waves, be measured? Why couldn't classical statistical mechanics, with its equipartition of energy theorem, model the thermodynamic behavior of solids? Kelvin was optimistic that these remaining mysteries would somehow eventually be resolved.

Indeed, within the next half-decade, brilliant proposals by Planck and Einstein swept away the nebulous issues, while placing limits on the applicability of classical physics. Planck's notion of the quantum, posited to resolve the classical prediction of an "ultraviolet catastrophe" in which blackbody radiation would skew toward the lowest wavelengths, offered a clever way of modeling a physically realistic distribution. Its revolutionary concept that light's energy is allocated in frequency-dependent packets dispelled the long-held supposition that luminous energy depended only on intensity and could thereby hold a continuous range of values. Einstein's 1905 paper on the photoelectric effect rendered the quantum notion even more tangible, showing how light's frequency affected the energy of its bundles, which in turn set bounds on the release of electrons from metals. Remarkably his special theory of relativity resolved the aether conundrum during the same "miracle year" that he solidified the idea of the quantum.

The notion of discreteness had ample philosophical precedent. Greek atomism, advanced by Democritus and later by the Epicureans, upheld a lower limit on the size of nature's constituents. In 56 B.C., the Roman philosopher Lucretius wrote the epic poem De Rerum Natura (On the Nature of Things) that advanced the idea of minute building blocks making up all matter. Because the materialism of that work contradicted spiritual ideas, it was little regarded by European clerical authorities during the Middle Ages, and restored to prominence only in the 17th century through the writings of the French philosopher Pierre Gassendi—an ardent atomist familiar with Lucretius. Later that century Robert Boyle, one of the principle founders of the field of chemistry, demonstrated the existence of elements—in the modern sense, replacing the Greek quartet of fire, earth, air and water. John Dalton, in the century that followed, advanced atomism even further through his cogent argument that all chemicals are composed of various combinations of elements, and that each element is made up of a unique type of atom with a distinct atomic weight. In the nascent decades of the 20th century, the quantum idea led naturally to a much more mathematically precise and predictive form of atomism—model atomic theory—in which what we call atoms are themselves composed of more basic elementary particles.

Today, physics is faced with perhaps an even murkier dilemma than the riddles that led to quantum theory and relativity. The mystery of the dark energy that drives the universe's acceleration, coupled with the conundrum that the calculated energy of the vacuum is far too large to be physically realistic, has spurred heated efforts toward a solution to both issues. I will argue that the answer may lie with a more fundamental form of discreteness—a minimal length scale for elementary particles. In other words, at its most basic level, the universe may be digital, rather than analogue.

Dark energy is the name coined by Michael Turner to characterize the engine driving the accelerated expansion of space. In 1998, two teams of astronomers employed observations of Type-Ia supernovae, along with Doppler shift data, to establish the recessional rates of extremely distant galaxies, showing that the expansion of space has been speeding up over time. Both the Supernova Cosmology Project, headed by Saul Perlmutter [2], and the High-Z Supernova Search, headed by team leader Brian Schmidt and researcher Adam Riess [3], had been looking to measure the rate by which the expansion of space has been slowing down. Instead, they were astonished to find substantial evidence that spatial growth has been increasing its pace. Further astronomical observations, including the results of the Wilkinson Microwave Anisotropy Probe, have confirmed this startling conclusion and mapped out the accelerated dynamics of the universe with greater precision.

In the absence of a known agent for such acceleration there have been numerous models of dark energy. One natural way of representing dark energy is by including a repulsive cosmological constant term in Einstein's field equations of general relativity. Ironically, Einstein had originally proposed the term to stabilize his equations, and had removed it after Hubble's 1929 discovery of the recession of galaxies (subsequently interpreted as the expansion of space). Yet, as first shown by Willem de Sitter [4], one set of solutions of Einstein's field equations with a cosmological constant includes a spatial scale that not only expands, but also speeds up in its expansion.

Field theorists have long been familiar with a different use for the cosmological constant term: a representation of the repulsive effect of vacuum energy within space. As Hendrik Casimir predicted in 1948 [5], fluctuations of the quantum vacuum can lead to detectable forces—for example, an otherwise unexpected attraction between two electrically neutral, parallel conducting plates. The bubbling froth of quantum fluctuations produces a negative pressure overall, equivalent to a de Sitter cosmology with a significant cosmological constant. Such a result would be auspicious, except that the theoretical cosmological constant according to standard field theories is about120 orders of magnitude larger than the observed value connected with cosmic acceleration. This fine tuning problem offers a baffling challenge—to produce, through theory, a cosmological constant that is small but non-zero.

Potential solutions to the fine-tuning problem of the cosmological constant include the hypothesis of supersymmetry, namely that each elementary particle of the fermion category (quarks and leptons) has a boson counterpart, and *vice versa*. Calculating the vacuum energy density, the contributions of fermion and boson terms would at least partly cancel, potentially producing a realistic value for the cosmological constant.

Another possible solution is the Randall-Sundrum "braneworld" hypothesis [6, 7]. In that scenario the observable universe is housed within a three-dimensional membrane, or brane, that is itself floating in a warped, higher-dimensional anti-de Sitter space, called the bulk. (Warping refers to the shape of the extra dimension, and anti-de Sitter space is one with a negative cosmological constant.) The large, positive cosmological constant associated with the vacuum energy density of the quantum fluctuations of fields confined to the brane are partly cancelled by the bulk's negative cosmological constant, leading to the observed modest value.

Both supersymmetric and braneworld models require a precise cancellation of terms to replicate the measured acceleration of the expansion of space, and are based on physics yet to be verified experimentally. Analysis of data from the Large Hadron Collider may yield proof of these hypotheses. In the meantime, it is worthwhile to consider an alternative view: the notion that the universe on its fundamental level is digital, rather than analogue.

Just as the ultraviolet catastrophe of blackbody radiation inspired the notion that energy is bundled as quanta proportional to frequency, I would argue that the overwhelming discrepancy between the calculated and observed values of the cosmological calls for an upper limit to the frequency of photons and other elementary fields. As this would, in the case for example of the photon, mandate a lower limit of wavelength, it would consequently set a minimal length scale. While the frequency cap would place an upper bound on the vacuum energy density, potentially matching it with its observed value, the minimal length scale would impose graininess on the microscopic arena. Space would have a voxel (three-dimensional pixel) structure, akin to a multistory office tower filled with tiny cubicles on each floor. As a bonus, the existence of a lower limit on length would render field theories such as quantum electrodynamics finite and without the need for cancellations of infinite terms.

Borrowing a term coined by Arthur Koestler in his 1967 work, The Ghost in the Machine [8], I call the fundamental limiting quantities of energy and information, "holons." A holon is a photon (or other fundamental field) of highest energy and lowest wavelength. Given that each field carries a cache of information about its quantum state, we might also think of holons as the bits of data that fill the spatial vacuum. For example, the polarization states of the holons (right-circular or left-circular) could represent the equivalent of "0" and "1" in a string of information. The vast amalgam of holons would represent the digital configuration of the quantum froth that underlies all of reality, akin to a quantum computer.

The concept that the essence of reality is pure information has a venerable history. In 1870, William Clifford suggested that matter could be bumps in the fabric of space. Einstein sought, in his speculations on unified field theory, to replace the "low grade wood" of matter with the "fine marble" of geometry [9, 10]. Greater interest in these ideas was sparked by the speculations of John Wheeler about the nature of spacetime foam. In the 1950s, Wheeler proposed the idea of geons [11], elementary particles derived from geometric twists in the solutions of Einstein's field equations. He later suggested that a quantum geometrodynamics governed the ebb and flow of geometric structure, with the standard general relativistic dynamics following a least action path through a jumble of possibilities.

In the early 1970s, Wheeler 's PhD student Jacob Bekenstein revolutionized the concept of how thermodynamics applies to gravitation by identifying the entropy of a black hole with the area of its event horizon [12]. He suggested that black holes, by accruing entropy, increase their surface area in a version of the second law of thermodynamics. Later, he speculated that the maximum information content of a black hole grows in proportion to its area as well, rather than its volume [13].

In line with the remarkable advances in computation during that period, along with Bekenstein's work, interest grew in models of fundamental interactions based on information, rather than matter and energy, as the principal agent. Edward Fredkin proposed that the interactions of elementary particles could be replicated through the discrete dynamics of cellular automata [14]. In a cellular automaton, each site is assigned a binary value. Upon iteration, each site is updated through simple algorithms that consider its value, along with the values of its neighbors. Fredkin's conjecture became the basis of "digital physics." Wheeler himself came to advocate that information is the most fundamental element of the universe, proposing the idea of "it from bit."

Indeed there is an efficient way of modeling geometric dynamics through digital algorithms: the concept of structurally dynamic cellular automata (SDCA), proposed by Andrew Ilachinski [15]. Unlike standard cellular automata, SDCA update connections between sites, as well as site values, with each iteration. Based on simple rules, new links might be added to unite next-nearest neighbors into nearest-neighbors, or removed to turn neighbors into disconnected sites. Investigations of SDCA by numerous researchers (including this author) have revealed a variety of behavior, including monotonic growth, steady decay, stable structures and periodicity [16].

As Wolfram and others have shown, cellular automata with certain types of rules can replicate computations in a universal fashion, akin to a Turing machine [17]. It is entirely possible that the field equations governing fundamental interactions could be reduced to discrete form by means of binary systems and basic, iterative operations. Unlike standard cellular automata, SDCA would offer the possibility of evolving network complexity along with the emergence of field equations from value rules. Thus the geometric structure of space could evolve in tandem with the emergence of particle-like interactions, potential reproducing the gravitational dynamics of general relativity along with the mechanisms of the other types of forces.

Given the promise of simple, discrete algorithms, it is interesting to consider a dynamic digital model of fundamental interactions based on holon states. Because holons would have the minimal wavelength of all fields, they might be thought of as "cells" in a dynamic grid. It is possible that simple digital rules, akin to SDCA algorithms, could transform both the values (quantum states, such as polarization) and linkage of sites. This would render this stratum geometrically dynamic, offering a digital basis for distortions in spacetime. One might imagine a very early universe, right after the nascent moment called Planck time, marked by increasing connectivity. With the growth of structure, interactions of longer and longer wavelength would be possible, enabling fields with lower and lower energy to emerge. Certain configurations might appear over time that could be identified with cooler, less energetic fermion and boson states. Such would correspond with the overall expansion and cooling of the universe. Meanwhile, the holon states themselves, because of their size constraints and reluctance to be squeezed into smaller bundles, would offer a subtle negative pressure corresponding to the observed cosmological constant.

The origin of the cosmological arrow of time is a longstanding mystery. One measuring of entropy in network theory is the growth of complexity in link structure. Envisioning the nascent cosmos as a digital system with the simplest possible link structure, one might postulate that connectivity rules produced increasingly intricate webs of bonds between sites. These would correspond to an increasing topological entropy that would set a uniform direction of time's arrow. In short, time's arrow would be a function of linkage rules that mandate growth of form.

One macroscopic implication of the existence of holons is a limitation of the amount of information enclosed within a boundary. That is because the voxel structure would place a cap on the quantity of data housed within any given volume. Just as a computer has a finite memory capacity that depends on the minimum size of storage units, any region of space would be able to contain only a limited amount of information governed by the amount of holons that might fit. Given that longer-wavelength fields would convey less information per unit volume, the amount of smallest-wavelength units—holons—would offer the maximum capacity. Thus the holon hypothesis sets the information capacity of a region to be finite, rather than infinite.

The existence of such an upper bound matches the predictions of the holographic principle, the hypothesis put forth by Gerard 't Hooft and Leonard Susskind that the information within a volume is encoded on its surface [18]. This principle extends Bekenstein's hypothesis that a black hole's information content depends on the surface area of its event horizon, rather than on its volume, to much greater regions and the universe itself.

In the case of black holes it is straightforward to imagine a boundary containing its total information content. That is because, from an outside observers' vantage point, objects falling toward the event horizon appear to slow down. Ultimately, they appear as frozen visages on the surface. Thus, everything that entered would make its mark on the surface and "deposit" its information for all outsiders to observe. Consequently, the area of the event horizon, not the volume inside, would be the suitable measure of maximum information storage.

Applying this idea to the universe as a whole, however, is less clear, as its boundary is not unequivocally defined. Researchers investigating how the holographic principle applies to cosmology generally pick a definition. For example the 2004 article by Miao Li, "A Model of Holographic Dark Energy" [19], selects the boundary to be the cosmological event horizon. The cosmological event horizon is defined as the farthest potentially observable region in space, taking into account the accelerated expansion of space. Under this constraint, Li matches what he calls the holographic dark energy to a reasonable value of the cosmological constant. Nevertheless, Li recognizes in the paper that other definitions for the boundary would yield different results.

Indeed, one might argue that an infinite universe would lack a physical boundary altogether, rendering the holographic principle ineffective for describing space as a whole. However, the holon conjecture that there is a minimum wavelength and maximum energy for the fundamental building blocks of the universe is independent of the idea that an actual boundary exists. It matches both finite and infinite conceptions of the cosmos. Thus a digital picture of reality could well exist within an endless universe.

Starting microscopically, we might use the holon conjecture to find a natural boundary for the information surface of the universe independent of whether or not it has an actual physical boundary. By choosing a holon wavelength that leads to the vacuum energy density that needed to reproduce the observed cosmological constant, one can calculate the size of the boundary for which the complete set of information within might be housed on its exterior. This offers a reasonable bound on the surface area that could contain the information content of the universe. Such a region would not represent any kind of physical border of the universe, but would constitute a kind of horizon by which a hypothetical outside observer could observe the sum total of information within the volume.

One might imagine the connection between the size of the minimal wavelength and the size of the information boundary as analogous to the conditions imposed on a hypothetical kind of mailbox that displays an image on its exterior of every letter it contains. If the mailbox is full of large envelopes, and these were somehow scanned, one could imagine displaying all their contents on the outside of the mailbox. However, the smaller the letters, the more could fit in the box, and the greater the quantity of information that would have to fit into the exterior display. Eventually, below a certain size limit (a postcard, let's say), although ample items could fit into the volume of the box, there would be insufficient room on the surface area of the box to exhibit all their contents. Given the size of the information bundle—the letters—there would be a particular threshold size of the information-displaying surface—the mailbox. Such is the linkage between the size of the minimal wavelength and the extent of the surface area corresponding to the information boundary.

Considering that a minimal wavelength could well be less than a billionth of a billionth of the dimensions of an atomic nucleus, it would seem an extraordinary feat to prove its existence. Nevertheless, Craig Hogan, Director of the Fermilab Center for Particle Astrophysics, has suggested that the "noise" associated with the graininess of space might be detectable through high-precision interference experiments [20].

When in 2007, researchers from GEO600, a gravitational wave experiment based near Hanover, Germany, reported anomalous noise, Hogan conjectured that such static could be signs of subtle length changes in the paths taken by light beams because of a kind of pixelation of space. If spatial distances are discrete, light might in certain cases be forced to take a path that is not the very shortest, but rather is the minimum length that fits the graininess of the smallest scale. It is like taking a walk through a city with a grid pattern of streets; those keeping to the discrete lattice would be forced to choose routes that are somewhat indirect compare to, for instance, the direct path taken by a helicopter hovering above the buildings.

Hogan and his colleague Aaron Chou have designed a high-precision instrument called a holometer, with the purpose of trying to measure luminous interference due to small differences in the transverse components of their paths. They expect such discrepancies would occur because of the discrete structure of space at minute scales. Such differences would manifest themselves as fringe patterns, which could be analyzed for the purpose of measuring the graininess of the universe at its most fundamental level. To rule out other types of noise, the device will include sensors designed to pick up even the slightest rumblings due to environmental factors, such as planes flying overhead. That way, they would be more confident of positive results.

It will be exciting to see if Hogan and Chou discover, through their holometer experiment, evidence that light's motion follows discrete steps. If so, such results could be interpreted as proof of the existence of a smallest length interval, which could, in turn, point to the existence of holons. Such a discovery would augment our understanding of space at its most fundamental level, and could possibly help resolve the dark energy enigma.

In conclusion, I have argued for a foundational interpretation of the universe in which the fundamental building blocks, called holons, are units of finite size and maximum energy. Although this interpretation lends itself well to the holographic principle, I have shown that it is independent of that conjecture and does not require delineating a boundary of the universe. Rather, it could well be a microscopic constraint that is part of the fabric of reality, similar to the quantization of other physical quantities. The impetus for introducing such a hypothesis is to cap the vacuum energy of the universe and bring it in line with the observed dark energy required to accelerate the expansion of the universe. Limiting the smallest wavelength of fields, thus placing an upper bound on their energies, offers a natural step beyond the Planck conjecture of quantized energy.

I have also illustrated how holons, with their cell-like qualities, could match well the concept that the universe, on its most fundamental level, is a digital system with simple rules, akin to a cellular automaton. I have argued that a structurally dynamic cellular automaton might be an ideal method of introducing geometric dynamics, as well as modeling the emergence of fundamental interactions. The growth of form in such a system could match increasing topological entropy that would provide a natural arrow of time. Moreover such a model would be independent of whether or not the universe is finite or infinite.

As in the time of Kelvin, murky clouds occlude the way toward a fuller understanding of the universe. However, it could be the case that a novel type of discreteness could dissipate these clouds by providing a way of accounting for dark energy and offering a means of understanding how nature operates on its smallest scale. As new experiments test the very limits of the tiniest components of reality, revealing perhaps if they operate according to digital or analogue mechanisms, exciting times for science surely lie ahead.

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