Evolving Unpredictability

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We introduce notions of classical and quantum predictability. We argue that it is possible that the merging of gravity and quantum theory imposes limitations to quantum mechanics and a weakening of predictability. Possible consequences for physics, mathematics and computer science are discussed.

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The universe is an unpredictable place. The Greek Epicurus proposed the idea that atoms tend to go one way but once in a while, at a random moment, one of them swerves to the side [1]. Today physics is grounded on quantum mechanics and that greatly departs from the classical notions of measurability, predictability, computability and decidability.

In principle, a classical world allows for predictability. As famously said by Laplace *"an intellect which at any given moment knew all the forces that animate Nature and the mutual positions of the beings that comprise it (...) for such an intellect nothing could be uncertain; and the future just like the past, would be present befores its eyes"* [2]. There is no room for surprise in classical mechanics. This motivates our definition of predictability in the classical sense:

Classical Predictability: given a system in some state, one can in principle predict with certainty the results of measurements for any observable quantity referring to the system.

The situation is drastically different in quantum mechanics. In a quantum universe one can in general only predict probabilities. That is not to say that the theory is completely unpredictable: we can still define a notion of predictability in the quantum sense:

Quantum Predictability: given a system in a pure state $|\psi\rangle$, one can only predict with certainty the results of measurements of observables for which $|\psi\rangle$ is an eigenstate. For all other observables, only probabilities of measurement outcomes may be predicted.

Both notions of predictability are very different, the quantum version being weaker.

Another interesting feature of quantum predictability is the presence of *entanglement*. Consider two spins in the maximally entangled singlet state,

$$|S\rangle = \frac{|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}} \tag{1}$$

For such state, quantum predictability makes no sense for a single subsystem. The "marginal" quantum state of any of the two subsystems consists of a state of maximal entropy. For state $|S\rangle$, one may predict only the results of measurements of joint observables for which $|S\rangle$ is an eigenstate. Thus, if we have access to only one subsystem, we can't predict anything but probabilities regarding measurements on the whole system. It is only after gathering results of measurements on the whole system that one can make some certain predictions.

Is it possible that the jump from quantum mechanics to a more fundamental theory might require weakening predictability even further? I will argue that this is plausible, at least in a practical sense, by presenting three scenarios in which this may happen. In particular, the theory that incorporates quantum mechanics and gravity in a single structure could be even less predictable than pure quantum mechanics.

The first way quantum predictability could be loosened up considers black holes. As described by the general theory of relativity, these objects consist of a curvature singularity protected by a causal horizon such that things may cross from the outside to the inside, but never the other way around. Following Hawking [3], suppose that part of an entangled state crosses the horizon: the predictability of measurements of any observable of the entangled system will be lost, at least temporarily. The fallen share of the entangled system can never cross the horizon back. As an entangled state only allows for predictions about joint observables, the information will be trapped within the black hole.

Hawking also showed that when quantum mechanics and curved spacetime are combined, black holes radiate thermal quanta. It is plausible that the information originally lost in the black hole might be restored once the black hole completely evaporates: it would be imprinted on correlations among the thermal quanta emitted during the singularity's history. Thus, in principle, information might be restored, but this is unclear, as we don't have a full theory of quantum gravity validated by observations.

Perhaps the notion of quantum predictability must change in a complete microscopic description of black holes. Still, information in these systems diffuses so quickly into inaccessible degrees of freedom that predictability is lost for all practical purposes. As Hawking explained in one of his last papers: *"The chaotic collapsed object will radiate deterministically but chaotically. It will be like weather forecasting on Earth. That is unitary, but chaotic, so there is effective information loss. One can't predict the weather more than a few days in advance"* [4].

The second way in which quantum predictability may have to be abandoned also involves a causal horizon, but this time on the scale of the entire universe. It is known beyond reasonable doubt that the universe is undergoing a period of accelerated expansion due to what is effectively described as a cosmological constant [5]. The further things are from us, the faster they recede. There is a horizon beyond which we will never be able to receive a single bit of information coming from an object behind it. In our universe things are constantly crossing this horizon.

Once again, part of an entangled state that crosses it will cause a loss of quantum predictability. Furthermore, a quantum state $|\psi\rangle$ could decohere so fast that correlations would build up with neighboring systems that would cross the horizon *before* we could possibly retrieve the information. A microscopic theory of the cosmological constant may also require a depart from the quantum notion of prediction. As for black holes, it is believed that a temperature can be attributed to the horizon and it is likely that the information could be retrieved in the long run, literally the entire history of the universe. Again, for all practical purposes, quantum predictability is gone, unless an appropriate microscopic description of the cosmological constant rescues it.

The third scenario involves unavoidable leaking of information to inaccessible degrees of freedom. The universe is filled with a thermal background of gravitational waves at a temperature T of about $1K$ [6]. As there is no negative gravitational charge or mass, one can never shield from the interaction with these random ripples in spacetime [7]. It has been shown that a quantum system in a superposition of different energy states,

$$\frac{|E\rangle + |E+\Delta E\rangle}{\sqrt{2}} \tag{2}$$

decays into a statistical mixture within a time scale of $\tau = \frac{\hbar}{k_B T}\left(\frac{M_{\rm pl}c^2}{\Delta E}\right)^2$ [8]. To observe the quantum superposition in state (2), one needs to perform an interference experiment, and the time necessary to observe one oscillation of the interference is on the order of ΔE^{-1} [9]. If the decoherence time τ is smaller than this oscillation time, quantum predictability is once again lost. In this case, the system decoheres faster than one could possibly measure its coherence. This is the case if the energy difference satisfies $\tau < \Delta E^{-1}$, that is,

$$\Delta E > \frac{M_{\rm pl}^2 c^4}{k_B T} \tag{3}$$

This inequality imposes limits on what we can measure and compute in the universe.

As an example, consider the optimal quantum metrology protocol for measuring a relative phase. A maximally entangled state of the GHZ form is prepared with N quantum bits,

$$\frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}, \tag{4}$$

where $|0\rangle, |1\rangle$ form the computational basis, and there is an energy difference ϵ between the two qubit states. Application of a phase θ on the system leads to,

$$\frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}} \rightarrow \frac{|0\rangle^{\otimes N} + e^{iN\theta}|1\rangle^{\otimes N}}{\sqrt{2}} \tag{5}$$

and subsequent measurement of the qubits allows determining the phase with a resolution given by the so-called Heisenberg limit, $\delta\theta_{\rm min} = \frac{1}{N}$ [10]. Combining $\Delta E = N\epsilon$ with equation (3) we derive a bound on the maximal number of qubits one can employ in a useful way. For a number of qubits above the threshold,

$$N > \frac{M_{\rm pl}^2 c^4}{k_B T \epsilon}, \tag{6}$$

the GHZ state decoheres faster than one could possibly measure its coherence. It is safe to assume that the 'quantum energy' ϵ must be larger than the minimum thermal energy set by the gravitational wave background $\epsilon > K_B T$. The maximum number of qubits then becomes,

$$N > \left(\frac{M_{\rm pl} c^2}{k_B T}\right)^2 \approx 10^{64} \tag{7}$$

Sure this seems an absurd limit, but one has to compare it with estimates on the number of baryons in the comoving portion of the universe that intersects our past light cone; this is estimated to be $N_{\rm total} \approx 10^{80}$ [11]. From this perspective, not much of the observable universe can be entangled. With the Heisenberg limit we also get a bound on the minimum measurable phase in today's universe, $\delta\theta_{\rm min} \approx 10^{-64}$ rad.

Adding up, the general theory of relativity imposes constraints to quantum theory. This is the converse of the standard point of view. As noted by Wigner [12] and Osborne [13], quantum uncertainty is the usual culprit, by limiting one's ability to measure spacetime.

Inequality (6) depends on the gravitational waves' temperature. As the universe expands, it gets cooler and the threshold above which superpositions cannot be observed becomes lower in the past. In particular, measurability and predictability effectively improve as time goes by. At the big-bang, $T \approx M_{\rm pl} c^2$ and the maximum number of qubits an entangled state could have becomes of order one: quantum predictability becomes a delicate issue. If we combine this argument with the conviction that the early universe underwent a de Sitter phase with a causal horizon, the issue becomes even more pressing as the random gravitational waves that carry information could rapidly cross the horizon.

Loss of quantum predictability in the physical world, even if only in practice, would certainly have an impact on all of science and in particular computer science. Indeed, quantum metrology can be seen as a particular case of quantum computation [14]. The inability to verify quantum superposition presents limits to quantum computing and consequently limits to what problems one can solve in practice, assuming that BQP $\notin$ BPP and, consequently, that there are problems quantum computers can efficiently solve that would never be tractable in a classical device [15]. Gravity might preclude the answer to otherwise tractable questions in a purely quantum mechanical universe.

From classical to quantum mechanics, the notion of predictability had to be relaxed. The unification of gravity with quantum mechanics may lead to a depart from the current notion of quantum predictability, at least in practice. In particular, quantum superposition of extremely large systems turns out to be unobservable due to the presence of a thermal gravitational wave background. This could have profound implications for physics, mathematics and computer science. Quantum computers face an ultimate noisy environment that limit their computing power. These limits are very far from today's experimental capabilities, and probably will remain so for the history of our civilization, but their existence is interesting in itself. The weakening of predictability may be an important clue to the next fundamental theory. A unification of gravity, quantum mechanics and complexity theory may turn out unavoidable.

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