The irrational side of reality

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Not all aspects of reality are ruled by "mindless mathematical laws". I argue that "aims and intention" are happening in just those aspects of reality, which are not subject to mathematical laws.

Can mindless mathematical laws give rise to aims and intention? May be we should upfront ask: Are there aims and intentions active in this world, which somehow impact the course of objective events? My answer to the latter question is a clear *yes*. But I will argue in this essay, that aims and intention are active in aspects of reality, which are not accessible to scientific analysis and the "mindless mathematical laws" of physics. Hence my answer to the first question is a clear *no*.

Actually for thousands of years the point of view has been prevalent in science and philosophy of nature, that aims and intentions are not ruled by any laws. Of course this conception had almost sunk into oblivion for two and a half centuries in classical physics, from Newton to Heisenberg, but then it was revived due to the analysis of quantum phenomena.

Why does a phenomenon show up the way it does show up? According to Aristotle, a reasonable and complete answer must consider four aspects of the phenomenon, often called four causes. To get a quite simple sketch of this idea, consider a potter manufacturing a vessel. First of all the potter needs clay, to start his work. The clay is the "material cause" of the vessel. The potter could use the clay to form a statue, or a teacup, or whatever, or nothing. His decision to create a vessel is the second cause of the vessel. Next the potter applies his art, and actually forms the vessel. This operation is the third cause of the vessel. Eventually there is the completed work, which has the form of a vessel. This is the fourth cause of the vessel. Clearly "cause" is not really an appropriate translation of the greek word αιτία used by Aristotle. We might better translate it as the four "aspects" of a phenomenon.

Not all phenomena are tangible things like vessels. Also events, like an eclipse of the sun, or the meeting of two people, or a dog jumping over a fence, are phenomena. A rainbow is a phenomenon, a thought in the mind of a human being is a phenomenon, the smell of a blossom is a phenomenon, the blossom is a phenomenon,

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the sound of a hammer cleaving a stone is a phenomenon. In short: Phenomena are the building blocks of reality. In each of them Aristotle identifies the four aspects.

For the first aspect (the clay in the vessel example) he used the greek word δύναμις = capability or potentiality. For those two aspects, which were in the vessel example the decision of the potter to create a vessel, and his artful realization of it, "choice" and "realization" may be appropriately generalized notions. And finally, the fourth aspect – the form in case of the completed vessel – may appropriately be generalized to the notion of the "obvious surface" or "form" of a phenomenon.

In a 1958 lecture Heisenberg explained [1] (my translation): "Thereby a decisive step away from classical physics had taken place, which essentially resorted back to a conception which played an important role in Aristotle's philosophy. The probability waves [...] can be understood as a quantitative version of the δύναμις, the possibility, or in the later latin translation the "potentia" in the philosophy of Aristotle. The conception, that the course of events is not inevitably determined, but that the possibility or "tendency" for some course of events has by itself some kind of reality, — a certain intermediate layer of reality, midway between the massive reality of matter, and the spiritual reality of ideas or pictures — , this conception plays in Aristotle's philosophy an essential role. It gains a new shape in modern quantum theory, as now exactly this notion of possibility is quantitatively comprised as probability, and subject to laws of nature, which are accessible to mathematical formulation. The laws of nature, formulated in the language of mathematics, here don't determine the course of events themselves, but the possibility for some course of events, and the probability that some course of events will happen."

When Heisenberg identifies Aristotle's "capability" or "potentia" with that aspect of reality, which is described by the state vector of quantum theory, then it seems quite natural to identify Aristotle's "obvious surface" or "form" with that aspect of reality, which is described by classical physics.

Both quantum theory and classical theory are strictly deterministic: State vectors evolve unitary according to the Schrödinger or Dirac equation, and classical systems evolve deterministic according to the classical (relativistic or not relativistic) equations of motion. Thus "capability" and "form" both are ruled by "mindless mathematical laws". But capability and form do not exhaust the aspects of reality.

Are there counterparts for Aristotle's "choice" and "realization" in physics? Choice and realization are neither parts of quantum physics, nor parts of classical physics. Choice and realization stand in-between, they are located at the transition from the possible (ruled by the laws of quantum theory) to the actual (ruled by the laws of classical physics). In physics, this transition is called measurement.

At superficial consideration one might guess, that "choice" is subject to Born's rule, which is not a deterministic rule, but at least it is a rule. This consideration is wrong, however. Born's rule is an integral part of quantum theory, applicable to the "capability" aspect of reality, and to no other aspect. Born's rule deals with the selection "capability" is offering to "choice", to choose from. Sometimes "capability" is offering just one item. Then Born's rule tells us that "choice" will pick this item with probability $P=1$. In most cases "capability" offers a variety of items, but with some bias. Born's rule is nothing other than the mathematical description of the bias of the selection offered by "capability". Imagine you are offered 5 face-down playing cards, and are asked to choose one of them. If four cards are kings and one card is a jack, then Born's rule tells us that with $P=0.8$ you will pick a king, and with $P=0.2$ a jack. But Born's rule does not guide your hand to pick this or that card. Born's rule merely describes the selection offered by the "capability" aspect of reality, but it doesn't give us any clue about the "choice" aspect.

Is there any chance that we might find in future some laws of nature, which are applicable to the "choice" aspect[2] of reality? It has been proved experimentally, that such laws do not exist. To understand the prove, consider this simple (simple to understand, not simple to perform) experiment, performed and reported in 1995 by Kwiat, Mattle, Weinfurter, Zeilinger, Sergienko, and Shih [2]:

Pairs of correlated photons, named photon_1 and photon_2 in the sequel, were produced due to type II spontaneous parametric down conversion in a β-bariumborat (BBO) crystal, as sketched in figure 1 on the next page.

Due to appropriate cut and alignment of the BBO crystal relative to the direction and polarization of the pump laser, the photon pair is prepared in the entangled state

$$|\text{photon pair}\rangle = \sqrt{\frac{1}{2}}\Big(|\,0^\circ\rangle_1\,|\,90^\circ\rangle_2 + |\,90^\circ\rangle_1\,|\,0^\circ\rangle_2\Big)\ , \qquad (1)$$

in which 0° means linear polarization in the paper plane of fig. 1, and 90° means linear polarization vertical to the paper plane.

By means of $\lambda/2$ plates, the polarization plane of photon_1 was rotated by γ_1, and the polarization plane of photon_2 was rotated by γ_2. Thus after the $\lambda/2$ plates, the state vector of the photons

[2] As this is an essay on aims and intention, but not an essay on the interpretation of quantum phenomena, I will not comment on the aspect of "realization". I merely mention that with regard to "realization", modern physics differ basically from Aristotle's conception. An aristotelian phenomenon somehow realizes itself, while in quantum measurement the realization of the result is fundamentally impacted by the arrangement of measurement devices chosen at will by the experimenter.

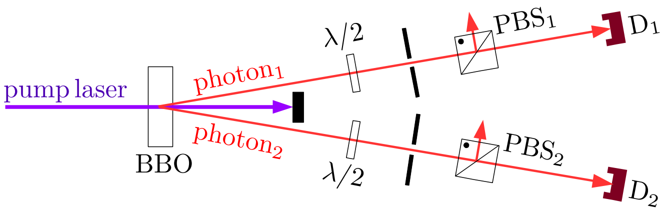

Fig. 1 : The experiment of Kwiat et. al. [2]

was

$$|\text{photon pair}\rangle = \sqrt{\frac{1}{2}}\Big(|\,\gamma_1\rangle_1\,|\,90^\circ\!+\!\gamma_2\rangle_2 + |\,90^\circ\!+\!\gamma_1\rangle_1\,|\,\gamma_2\rangle_2\Big)\ . \qquad (2)$$

In state (2) the photon pair reached the polarizing beam splitters PBS_1 and PBS_2 . Both beam splitters were aligned for transmission of 0° polarized light, and reflection of 90° polarized light. Reflected photons were ignored, transmitted photons were observed by the detectors D_1 and D_2 . Only coincidences[3] of both detectors were counted. Thus

$$|\,0^\circ\rangle_1\,|\,0^\circ\rangle_2 \qquad (3)$$

is the only eigenvector of the pair of detectors.

According to Born's rule, the modulus square of the projection amplitude of the photon vector (2) onto the detector vector (3) is just the probability $P_{\mathrm{TT}}(\gamma_1,\gamma_2)$ for the transmission of both photons:

$$\begin{aligned} P_{\mathrm{TT}} &= \frac{1}{2}\Big|\,{}_1\langle 0^\circ|\gamma_1\rangle_1\ {}_2\langle 0^\circ|\,90^\circ\!+\!\gamma_2\rangle_2\ +\ {}_1\langle 0^\circ|\,90^\circ\!+\!\gamma_1\rangle_1\ {}_2\langle 0^\circ|\,\gamma_2\rangle_2\Big|^2 \\ &= \frac{1}{2}\Big|-\cos(\gamma_1)\sin(\gamma_2)-\sin(\gamma_1)\cos(\gamma_2)\Big|^2 = \sin^2(\gamma_1+\gamma_2) \end{aligned} \qquad (4)$$

Kwiat et. al. checked (4) with 12 different settings of $\gamma_1\!+\!\gamma_2$, spread approximately evenly over the full circle. The experimental counting rates differed by less than 3 % from the expected sin-square distribution, thereby confirming the prediction (4) of quantum theory.

The remarkable thing about (4) is, that this result does depend only on the sum $\gamma_1\!+\!\gamma_2$, but not on the absolute values of γ_1 and γ_2 . The polarizing beam splitter PBS_2 must transmit photon_2 with certainty (probability $P\!=\!1$) at some setting γ_2 , if the correlated photon_1 is reflected at $\gamma_1\!=\!\gamma_2$ or transmitted at $\gamma_1\!=\!\gamma_2\!\pm\!90^\circ$. But photon_2 must be reflected with certainty at the same setting γ_2 , if photon_1 is transmitted at $\gamma_1=\gamma_2$ or reflected at $\gamma_1=\gamma_2\pm 90^\circ$. This perfect correlation must exist for any arbitrary angle γ_2 . It

[3] Kwiat et. al. missed to document in their publication [2] the time window within which detected photons were acknowledged as "coincident". Usually the time window is chosen 2 to 10 nanoseconds in this type of experiments.

seems that photon_2 must be informed about the setting of γ_1 and the outcome (reflection or transmission of photon_1) at PBS_1, to behave correctly at PBS_2 and realize the correlation (4).

Weihs et. al. [3] repeated the experiment with γ_1 and γ_2 set by random number generators such that photon_2 could not know — provided there was no superluminal communication — the setting of γ_1 nor the result (transmission or reflection) of photon_1 by PBS_1 before it had made it's own decision at PBS_2 and had been registered by the detector, and vice versa (the distance between the two groups of polarization rotators, beam splitters, and detectors was about 400 m in this experiment). Still the correlation (4) was confirmed.

Einstein, Podolski, and Rosen [4] concluded that each photon must arrive at the beam splitter with a complete program for correct behavior (transmission or reflection) at arbitrary values of γ_1 and γ_2, because — if superluminal communication or some non-local coordination are excluded, which seemed self-evident to EPR — no other explanation is possible for the perfect correlation (4). As quantum theory does not display these complete programs implemented into photon_1 and photon_2 for arbitrary γ_1 and γ_2, but only the correlation (4) for $\gamma_1 + \gamma_2$, EPR deemed quantum theory incomplete.

Lets introduce this code for the photon pair polarization measurement results:

$$r_{\gamma_1} = +1 \quad \longleftrightarrow \quad \text{photon}_1 \text{ was transmitted @}\,\gamma_1 \tag{5a}$$
$$r_{\gamma_1} = -1 \quad \longleftrightarrow \quad \text{photon}_1 \text{ was reflected @}\,\gamma_1 \tag{5b}$$
$$r_{\gamma_2} = +1 \quad \longleftrightarrow \quad \text{photon}_2 \text{ was transmitted @}\,\gamma_2 \tag{5c}$$
$$r_{\gamma_2} = -1 \quad \longleftrightarrow \quad \text{photon}_2 \text{ was reflected @}\,\gamma_2 \tag{5d}$$

Thus the doublet $(r_{\gamma_1}\, ,\, r_{\gamma_2})$ completely documents the settings γ_1 and γ_2 and the observed result of one experimental run.

As $r_{\gamma_2'}$ can be predicted with probability $P = 1$ by simply measuring r_{γ_1} with $\gamma_1 = \gamma_2'$, and $r_{\gamma_1'}$ can be predicted with probability $P = 1$ by simply measuring r_{γ_2} with $\gamma_2 = \gamma_1'$, EPR asserted that $r_{\gamma_1'}$ and $r_{\gamma_2'}$ are "parts of reality" even if they were not actually measured.

If the EPR argument should be correct, then not only the results (5) of the actually performed measurements, but also the results

$$r_{\gamma_1'} \text{ with } \gamma_1' \neq \gamma_1 \quad \text{and} \quad r_{\gamma_2'} \text{ with } \gamma_2' \neq \gamma_2$$

of the not performed measurements are real. For the following argument it is sufficient to assume, that in addition to the measured results (5) one $r_{\gamma_1'}$ is real for some certain $\gamma_1' \neq \gamma_1$, and one $r_{\gamma_2'}$ is real for some certain $\gamma_2' \neq \gamma_2$. Thus we assume that in each experimental run not only the doublet

$$(r_{\gamma_1}\, ,\, r_{\gamma_2})$$

of the actually observed results, but the full quartet

$$(r_{\gamma_1}\,,\, r_{\gamma_2}\,,\, r_{\gamma_1'}\,,\, r_{\gamma_2'})$$

is realized. In table 1 the 16 possible quartets are listed.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
r_{γ_1}	+1	+1	+1	+1	+1	+1	+1	+1	−1	−1	−1	−1	−1	−1	−1	−1
r_{γ_2}	+1	+1	+1	+1	−1	−1	−1	−1	+1	+1	+1	+1	−1	−1	−1	−1
$r_{\gamma_1'}$	+1	+1	−1	−1	+1	+1	−1	−1	+1	+1	−1	−1	+1	+1	−1	−1
$r_{\gamma_2'}$	+1	−1	+1	−1	+1	−1	+1	−1	+1	−1	+1	−1	+1	−1	+1	−1
q	+2	+2	+2	−2	−2	−2	+2	−2	−2	+2	−2	−2	−2	+2	+2	+2

Table 1: The 16 quartets $(r_{\gamma_1}\,,\, r_{\gamma_2}\,,\, r_{\gamma_1'}\,,\, r_{\gamma_2'})$

In the bottom line of the table, for each quartet the value of

$$q = r_{\gamma_1}\cdot r_{\gamma_2} + r_{\gamma_1}\cdot r_{\gamma_2'} + r_{\gamma_1'}\cdot r_{\gamma_2} - r_{\gamma_1'}\cdot r_{\gamma_2'} \tag{7}$$

is displayed. As the table is exhaustive (there doesn't exist any further quartet, which is not listed in this table), Nature must choose in each experimental run one of these 16 quartets, and hence one of these 16 q values. With regard to the sum of N values of q chosen by Nature in N experimental runs, we know for sure:

$$\begin{aligned} & -2 \leq S \leq +2 \qquad (8) \\ & \text{with } S \equiv \frac{1}{N}\sum^{N} q \overset{(7)}{=} \frac{1}{N}\sum^{N}(r_{\gamma_1}\cdot r_{\gamma_2}) + \frac{1}{N}\sum^{N}(r_{\gamma_1}\cdot r_{\gamma_2'}) + \\ & \qquad + \frac{1}{N}\sum^{N}(r_{\gamma_1'}\cdot r_{\gamma_2}) - \frac{1}{N}\sum^{N}(r_{\gamma_1'}\cdot r_{\gamma_2'}) \end{aligned}$$

These mean values of products of results are the correlation functions C. They are related to the probabilities P of the various results due to

$$\begin{aligned} C(\gamma_1\,,\gamma_2) &\equiv \frac{1}{N}\sum^{N}(r_{\gamma_1}\cdot r_{\gamma_2}) = \\ &= P(r_{\gamma_1}=+1\,,\, r_{\gamma_2}=+1) + P(r_{\gamma_1}=-1\,,\, r_{\gamma_2}=-1) - \\ &\qquad - P(r_{\gamma_1}=+1\,,\, r_{\gamma_2}=-1) - P(r_{\gamma_1}=-1\,,\, r_{\gamma_2}=+1) = \\ &= P_{\mathrm{TT}}(\gamma_1\,,\gamma_2) + P_{\mathrm{RR}}(\gamma_1\,,\gamma_2) - P_{\mathrm{TR}}(\gamma_1\,,\gamma_2) - P_{\mathrm{RT}}(\gamma_1\,,\gamma_2)\ , \end{aligned}$$

with the indices $_{\mathrm{TT}}$ encoding transmission of both photons, $_{\mathrm{RR}}$ encoding reflection of both photons, $_{\mathrm{TR}}$ encoding transmission of photon$_1$ and reflection of photon$_2$, and $_{\mathrm{RT}}$ encoding reflection of photon$_1$ and transmission of photon$_2$. Thereby (8) can be written in the form

$$\begin{aligned} & -2 \leq S \leq +2 \qquad (9) \\ & \text{with } S \equiv C(\gamma_1\,,\,\gamma_2) + C(\gamma_1\,,\gamma_2') + C(\gamma_1'\,,\gamma_2) - C(\gamma_1'\,,\gamma_2')\ . \end{aligned}$$

This is a variant of Bell's inequality [5]. The proof presented above has been published by Peres [6]. It is stronger than Bell's original proof, because it is simpler. Peres' proof is based on the one and only assumption, that the results $r_{\gamma_1'}$ and $r_{\gamma_2'}$ of the not performed measurements share the same status of reality as the results r_{γ_1} and r_{γ_2} of the actually performed measurements. Once this assumption is made, the conclusion (9) is inevitable.

Kwiat et. al. ignored reflected photons, and only observed transmitted photons. But they could deduce

$$
\begin{aligned}
P_{\mathrm{RT}}(\gamma_1, \gamma_2) &= P_{\mathrm{TT}}(\gamma_1 + 90^\circ, \gamma_2) \\
P_{\mathrm{TR}}(\gamma_1, \gamma_2) &= P_{\mathrm{TT}}(\gamma_1, \gamma_2 + 90^\circ) \\
P_{\mathrm{RR}}(\gamma_1, \gamma_2) &= P_{\mathrm{TT}}(\gamma_1 + 90^\circ, \gamma_2 + 90^\circ)
\end{aligned}
$$

from the observed $_{\mathrm{TT}}$ events, and thereby were able to check (9) experimentally. They measured the polarization correlations at

$$
\gamma_1 = -22.5^\circ \ , \ \gamma_1' = +22.5^\circ \ , \ \gamma_2 = -45^\circ \ , \ \gamma_2' = 0^\circ \ ,
$$

and found

$$
S = -2.649 \pm 0.006 \ .
$$

This result violates Bell's inequality (9) by about 108 standard deviations.

The violation of Bell's inequality disproves the basic assumption of Peres' derivation. It demonstrates that the results of not performed measurements are *not* real. While still in flight, the polarizations of the photons are not merely unknown, but they do not exist. Only in the very moment of measurement, Nature makes her *choice* and non-locally *realizes* polarizations of photon_1 and photon_2 . Note that I am alluding to the aspects of "choice" and "realization" in Aristotle's phenomenology.

Nature is not free to create arbitrary polarizations at will. Instead she must choose from the selection (4) offered by the "capability" aspect of reality, and thereby is subject to the bias (the correlation) encoded in this selection. This does not mean, however, that Nature's choice is subject to any law of physics. The laws of quantum physics determine the selection offered by the "capability" aspect. But there are no laws that rule which item Nature will choose from that selection.

What is not pre-determined, that can of course impossibly be pre-computed. If the polarizations of the photons are not pre-determined but created only in the moment of measurement, then there does not exist any law by which they could be computed. If Nature decides for $_{\mathrm{RT}}$ (i. e. reflection of photon_1 and transmission of photon_2) in one particular experimental run with settings γ_1 and γ_2 , then she could have decided as well for $_{\mathrm{TR}}$ in the same run with the same settings. The violation of Bell's inequality experimentally proves that the reason of Nature's decision is not merely unknown to us, but that there exists no reason for this or that decision. Nature's choice, happening in the process of measurement, is truly irrational, and hence out of reach of the laws of physics.

I guess that even in the centuries of classical physics most physicists assumed that the application range of physical laws is limited. They clearly felt that questions like "does God exist?" or "is there some kind of life after death?" can not be appropriately handled by the methods of physics. And of course they always pondered about the question "do we have a free will, or is this world strictly deterministic? Are we free to decide, or are all our allegedly free decisions after careful consideration of alternatives merely illusions?"

But it came as a surprise, that the limits of the application range of physical laws are not somewhere far-off, but are encountered in the midst of physics. Due to analysis of quantum phenomena, and in particular due to the experimentally confirmed violation of Bell's inequality, we learned that irrational aspects of reality are ubiquitous: The *choice* aspect of reality, present in each and every phenomenon, and showing up in each and every measurement process, is not subject to laws of physics.

Each time an instable atomic nucleus chooses to decay or not to decay within the next second, each time two nearby atoms choose to form or not to form a molecule, each time the molecules of a gas choose to absorb or not to absorb a traversing photon, each time a synapse somewhere in my brain chooses to fire or not to fire, an irrational aspect of reality is active, and shaping the course of forthcoming events.

Aims and intentions, and free will, can evolve in these irrational aspects of reality. They can evolve, but we can not prove that they do. We vividly feel that they do, but as physicists we are not used to accept feelings as proofs. On the other hand, as the irrational aspects of reality are not subject to the laws of physics, we will definitively never be able to compile a sound physical proof for the existence of free will, aims, and intentions, and their effects onto the course of objective events. The violation of Bell's inequality merely proves that there really is an irrational aspect of reality, and that consequently free will, aims, and intentions, which may impact the course of objective events, can not be excluded.

Acknowledgment:
My point of view regarding the irrational aspects of reality owes much to the beautiful book of Jos Verhulst [7]. Though I didn't have a look into it since many years, it should appropriately show up in the references.

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