There Are No Pea-Shooters for Photons

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ABSTRACT: The case for the particle nature of light is well established, just as the wave theory of light has been discredited. Historically, the decisive arguments against the wave theory were developed in the early years of quantum mechanics. The need to consider the wave nature of matter only became fully evident following the advent of the Schroedinger equation in 1926. In this essay we consider whether the old arguments against the wave theory remain fully justified.

1. INTRODUCTION

Feynmann famously advocated for the "shut-up-and-calculate" school of physics and it's safe to say nobody wants to second-guess Feynmann. But the cold hard formulas, no matter how well verified, remain just that – formulas. Physicists have always felt drawn to seek out more human, more pictorial representations – in a word, *anschaulichkeit*. Even Maxwell attempted to rationalize the ether by equating it to a mechanical body with certain properties of stiffness and inertia. We have never really gotten away from the urge to devise pictures to guide us in the use of our equations, and we probably never will.

Quantum mechanics, from the very outset, has created unprecedented challenges to physicists in our quest for *anschaulichkeit*. We need look no farther for evidence of these difficulties than to consider the theme of this essay competition: "Is reality digital or analog?" The very notion that one could even contemplate asking such an audacious question testifies to the depths of the problem.

The difficulty for quantum mechanics goes back to the age-old question of waves versus particles. It ought to be noted that if one truly believed in the idea of "shut-up-and-calculate", there should be no need to agonize over the distinction. If quantum mechanics truly relies only on the mathematical formalism to derive results, then where in the process is it necessary to identify specific elements of the theory as being "waves" or "particles"? And yet we persist in doing just that.

In this essay, we are going to argue that, in the ongoing debate over the true nature of reality, the wave interpretation has gotten the short end of the stick. We are going to ask the question: how good are the arguments which have traditionally been used to debunk the wave theory?

2. WAVE-PARTICLE DUALITY

The notion of duality predates the advent of quantum mechanics. There are some picturesque examples from the world of mathematics: within certain constraints, it is possible to develop a subset of geometry wherein the words “point” and “line” are interchangeable, as in “two lines define a point” (or “two points define a line”). In circuit theory, results derived for voltages, resistance, and node elements become equally valid when properly transformed into theorems about current, conductance, and loop elements.

The wave-particle duality of quantum mechanics is a horse of a different color. It would be extremely satisfying if we said that one could analyze any physical process in terms of particles, and then re-analyze the same system from the wave perspective, with the identical result being achieved in both cases. This unfortunately is not the way it works. In the beginning, the “wave-particle duality” was taken to mean that for some phenomena, we had to use waves; and for others, we needed particles. This type of equivocating was clearly not going to be sustainable; and indeed, in the present day, the world of physics has come down decisively on the side of the particle interpretation.

The literature, both popular and scientific, is replete with any number of decisive examples showing the failings of the wave interpretation. Here and there we find an acknowledgement that this or that phenomena, previously held to be proof of the particle theory, can in fact also be explained by waves. What is sorely missing from the debate, and what I am going to try and shed some light on in the remainder of this essay, is just how easily and convincingly the wave interpretation can be applied to the very experiments which are most often used to shoot it down. It is not hard to find citations indicating that such arguments exist. What is much harder to find anywhere is a clear sketch of how these arguments go.

3. THE BLACK-BODY SPECTRUM.

The ultraviolet catastrophe inherent in the Rayleigh-Jeans formula is an inevitable consequence of the equipartition theorem in classical mechanics. It is interesting, however, to think through the actual mechanism in detail. Why exactly to all frequencies of the radiation field get the same share of energy?

The equipartition theorem is especially easy to understand for the case rigid diatomic molecules, where the energy is shared equally between the five modes: three translational and two rotational. If the average translation velocity of a molecule is 500 m/sec, then the average tangential velocity of a spinning molecule, taken about its center of mass, is also 500 m/sec. That is how equipartition works for mechanical energy. The question then becomes: how is this mechanical energy converted to radiant electromagnetic energy?

The simplest way is to allow the molecules to have a dipole moment. Species like O2 and N2 will of course be electrically balanced (that's why light passes through them so easily) but pretty much any molecule composed of two different atoms will have some dipole moment. When it it given rotational motion, it becomes an antenna. And as an antenna, it radiates.

What is the frequency of the radiation? It is simply the rotational frequency of the molecule: in other words, the tangential velocity divided by the radius. The problem occurs if we let the radius become very small. The smaller the interatomic distance, the higher the frequency radiated by the spinning molecule. In theory there is no limit to how small the molecule might be, and how high the resulting frequency.

There is however a well-known example to show that molecules do not in fact spin with arbitrarily high-speed. I am referring to the anomalous specific heat of hydrogen (and other light molecules) at very low temperatures. It is sometimes said that the rotational motions are "frozen out". The interesting thing is that we can identify a mechanism which causes this: it derives from the de Broglie notion of matter waves.

In order for the rotational motion to be driven independently of the translational motion, we rely on a clean hit between two molecules. This only works if the molecules are made of hard little billiard balls. What happens if the molecules are moving so slowly that their de Broglie wavelength becomes comparable to the interatomic spacing?

When the incoming atoms are that big, you don't get a clean strike which sets the target molecule spinning. You can't help but strike both atoms at once, which imparts translational energy only. You can no longer drive the rotations, and that's why the specific heat goes down.

The law of specific heats breaks down at low temperatures because the equipartition theorem does not take into account the wave nature of matter. Without the equipartition theorem, there is no black body catastrophe.

4. THE PHOTO-ELECTRIC EFFECT

It is fairly well known that if you take a superposition of the s and p states of the hydrogen atom, you get a tiny oscillating dipole. What is generally ignored is the fact that you can understand at least some of the spectral properties of hydrogen simply by treating these oscillating dipoles as classical antennas. For example, one of the easier elements to calculate is the Einstein A coefficient for spontaneous emission. From the Schroedinger equation you have a dipole moment and a frequency; you simply put these numbers into the classical antenna formulas and the result is a radiated power. Is it the same power you get from considering an ensemble of atoms in the excited state and using the Einstein A coefficient to calculate the rate of emission? For now, I am simply going to answer this question with another question: why wouldn't it be?

The problem is that among physicists, antenna theory is not the best known subject. We are therefore going to make a brief digression in an attempt to bring peope up to speed.

The crystal radio is a device which absorbs electromagnetic energy from a passing wave and converts it into mechanical energy in a small speaker. While much is made of the importance of the crystal in detecting the signal, for our purposes I am going focus on the question of energy transfer. The question the becomes: how much energy can you collect with a small antenna?

The word "small" is used here in its technical sense; that is, since AM radio broadcasts in a band whose wavelength is on the order of hundreds of meters, and home-made antennas are general not much longer than ten meters, the ratio of lengths falls within what is considered to be the domain of "electrically small" antennas. The parameter of interest is called the "radiation resistance" and for small antennas it is known to be proportional to the square of the length. Since voltage goes as length, and power as voltage squared over resistance, we get the seemingly paradoxical result that for a small, tuned, lossless antenna, the maximum theoretical absorbed power is *independent* of the size of the antenna.

This result seems absurd; and it must seem presumptuous or worse to declare it without any corroboration. However, it is against the entire spirit of my purpose here to rely on authority to validate my arguments; therefore, I will try and explain the result in such a way that one can see that it must be so.

The key is to understand the mechanism of power absorption. From a naïve point of view, we see the power of the broadcasting antenna spreading out through space, so that it arrives at the distant receiving antenna with a much reduced intensity. Since in order to be absorbed, the power from the travelling wave must first strike the antenna, it appears obvious that the maximum available power must be limited by the physical size of the receiving antenna.

What appears to be obvious turns out in this case to be false. It must be false because the wires of the antenna are so thin, and their cross-sectional area so small, that for typical radio signals it would be impossible to build a working antenna capable of driving even the smallest earplug with an audible signal.

We are then tempted to look for an explanation which relates the absorbed power to some natural function such as the square of the physical length. Wrong again. The best way to understand what is going on is to look at the composite wave patterns generated by the two antennas.

Remember that for an antenna to receive power, current must flow in it; and once current flows, it becomes a transmitting antenna. Looking down from above (fig. 1), we see two sources of waves interfering with each other: the nearly-plane waves from the transmitter, travelling from right to left, and the spherical waves of the receving antenna, radiating outwards in all directions.

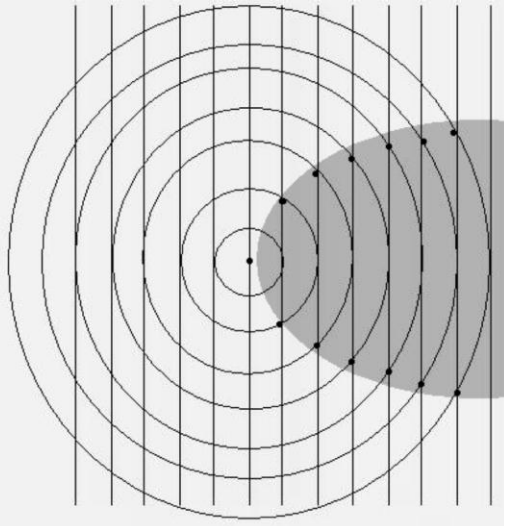

Fig. 1

The calculation for available power is based on the combined interference pattern. In almost all directions, the power radiated by the receiver simply represents wasted energy scattered from the incoming wave. But within the shadow region in Fig. 1, there is a possibility of something more interesting. If the phase of the spherical waves is correctly lined up with the phase of the incoming waves, power can be drawn out of the system. Under certain circumstances, this power can be more than that which is wasted in all the other directions, and it is then available to do useful work in an electric circuit. How do we achieve those circumstances? We control the phase by adding the correct impedance to the antenna circuit, and we control the amount of current by adding a suitable resistor. This is called “tuning” and “impedance matching”.

It should be clear that according to this analysis, the maximum available power depends only on the geometry of the two wave patterns. Since all antennas much smaller than a quarter wavelength have identical radiation patterns, it is clear that the length of the antenna can have no bearing on this calculation.

Due to the imperfections of real materials – specifically the electrical resistance of copper – the ideal theoretical results cannot be easily achieved by the typical do-it-yourselfer. However, on the atomic level, we are in fact dealing with truly lossless systems. If we consider the hydrogen atom as a miniature antenna, and attempt to drive the s-p transition with a wave of the correct frequency, then the effective absorption cross-section has absolutely no relation to the physical size of the atom. In fact, according to classical antenna theory, it is on the order of the square of the radiation wavelength, about a million times greater.

That is why it is so disturbing to read, again and again, explanations of the photo-electric effect wherein the wave theory of light is “debunked” largely on the grounds that the diffuse power in a wave cannot possibly be concentrated in the small volume of a single atom. By the same logic, a crystal radio should not work. But it does work, and it works without the need to assume mysterious “corpuscles” of radio energy. It works because waves are able to do things that you wouldn’t easily guess at.

To be sure, the photo-electric effect is not normally applied to single hydrogen atoms; but the photo-ionization of hydrogen, if not exactly *the* photo-electric effect, is surely at least (note the indefinite article) *a* photo-electric effect.

What about the classic metal cathode effect? Here the argument becomes even more misplaced. We now know that the wave function of the conduction electron is spread through the entire body of the metal. The size of a single atom could not be more irrelevant to a consideration of the rate at which energy is delivered. And yet that is exactly how the case is argued in any number of textbooks, and not just at the elementary level.

I have not yet mentioned the frequency-versus-amplitude arguments which are also used to make the case for particles. These are of course a holdover from the early days of quantum mechanics. Based on what we now know about the role of frequency in the Schroedinger equation, I find these arguments as applied to the photo-electric effect to be so silly as to be unworthy of further comment.

5. THE COMPTON EFFECT

The Compton effect is widely held to be the final "nail in the coffin" of the wave theory of light. One can search the usual sources in vain for a convincing explanation of how one might reconcile the Compton effect with the wave theory of light. And yet there is an explanation, and it is due to Schroedinger himself no less.

In its simplest form, the mathematics of the explanation is surprisingly simple. It draws on nothing more complicated that the familiar potential well analysis for a single electron. It is well known that solutions for the one-dimensional well take on the form of sine waves. The form remains the same if you extend the geometry to three dimensions while constraining the electron to travel along one axis. When you then square the wave to get the charge density (yes, Schroedinger interpreted it as *charge* density, not probability) you get parallel sheets of charge.

It is perfectly obvious what happens when you shine light waves onto parallel sheets of charge. When the spacing of the sheets is just right – that is, when it is half the wavelength of the light – the reflections from successive layers constructively re-inforce each other. Even when the reflection from each layer is very small, it does not take long before the cumulative effect results in total reflection.

In quantum mechanics, of course, wavelength is the measure of momentum. So you get total reflection when the momentum of the light is equal to the momentum of the electron. You can always get this to happen by transforming your experiment into a center-of-mass coordinate system. It's just like billiard balls.

The physics of what is happening is most clear when you take a snapshot at the exact midway point of the interaction. You have an electron wave function composed of two equal components, one moving to the left and one moving to the right. It is the superposition of these components that gives you the standing wave sheets of charge. This system of charge sheets provides an impenetrable barrier to the passage of light arriving from the opposite direction and having the same wavelength.

There are some who might object to this picture on the grounds that it does not show how the reflection process is initiated. When the electron starts of as a pure wave in one direction, there are no multiple sheets; so the interaction should never get started. I consider these objections to be ill-founded. Mathematically speaking we can work both directions from the midway point. Once we are halfway there, the process certainly goes to completion. And rolling the film backwards, we get exactly the same mathematical form for the first half of the process. I admit that it may be tricky to wrap our human minds around the question of how the process kicked off, but the question at some point becomes philosophical. I am not at all sure that we should demand of physics that it provide an answer to these things.

6. THERE ARE NO PEA-SHOOTERS FOR PHOTONS

In the three cases I have dealt with in this essay, I have nowhere shown anything close to a sound mathematical analysis of these phenomena using the wave picture. This is not the right forum for arguments of that kind. What I have attempted to do is to respond to the type of arguments which are normally put forward against the wave theory. The arguments go, in each case, that the wave theory is not only incapable of providing any kind of explanation for the experimental phenomena in question, but in fact gives answer that are wildy incorrect.

What I believe I have shown is that, on the contrary, all of these effects are well within the scope of things that you can do with waves. I haven't shown that the calculations would give precise agreement with experiment, but I believe I've shown that there are reasonable calculations that at least lead to correct results on a phenomenological level.

If I am correct, then how do I explain that the greatest minds in the history of science found those arguments to be convincing? It is a case of historical accident. At the time the particle theory was being hammered out, nobody could have imagined that electrons themselves would turn out to be governed by a wave equation. The straw-man arguments for the photo-electric and Compton effects were grounded on a picture of the electron as a kind of tiny, charged ping-pong ball. The Schroedinger equation made these arguments obsolete, but by then the ideas were so deeply entrenched that they found new life in the Copenhagen interpretation.

It is for me something of a miracle that those early particle theory arguments turned out to be so robust that they (or their successors) now constitute the central paradigm of what is widely acclaimed as the most successful theory ever devised by man. The particle theory is here to stay. The question is: why is there not also a wave theory which stands on equal footing with the particle theory – a true "wave-particle duality"?

If it were true that the wave theory was inherently incapable of accounting for any number of observed phenomena – as has been argued in the past – then that would be the end of the wave theory. If on the other hand the argument has become that the wave theory and the particle theory diverge after so and so many decimal places, to the advantage of the particle theory, that is a different matter.

There is a new class of argument that has arisen in the last ten or twenty years, based on the notion of photon bunching and antibunching, which claims to provide direct observation evidence for the photon as a particle. The argument goes, more or less, as follows: it is true that you can calculate all kinds of phenomena based on the wave theory of light, and get answers in agreement with experiment. But if you actually look closely enough, you can see individual photons. So the wave theory is wrong.

The experiment which demonstrates that light is a particle is simple. You shoot one photon at a time at a beam splitter. There are two detectors. The efficiency of the detectors is, for sake of argument, 20%. According to any wave theory, if one or the other detector registers 20% of the time, you should get simultaneous detections 4% of the time. The absence of simultaneous clicks proves that the photon went one way or the other.

It's a very simple experiment and it's very decisive. Note, by the way, that it cannot be done simply by taking an ordinary thermal light source or even a laser source and making it very weak so that there is, on average, only one photon at a time. For reasons I won't get into at this time, the statistics don't work. It has to be one photon at a time.

There is one small but vexing problem with this experiment. It has never been done; at least, it has never been done just as described. The reason it has never been done is that we still do not have such a thing as a pea-shooter for photons.

There are, however, closely related experiments (Grainger, Thorn et al) which, we are assured, come to the exact same thing, making use of entangled photons, coincidence counters, and autocorrelation analyzers. Make no mistake about it - these are very good experiments indeed, and they raise vexing problems for the wave theory. But they are not demonstrative in the simplest possible way, as described above. And the reason is that ultimately, we still do not have a pea-shooter for photons.

My real problem with these experiments that claim to observe individual photons is: to what purpose? In the past, theorists have pointed to one phenomenon after another and claimed that these things could not be explained by any wave theory. Most of these arguments were predicated on the interaction of Maxwellian waves with classical particles. When the analysis is redone using Maxwell's waves and quantum matter as governed by Schroedinger's equation, we find that the wave theory works surprisingly well. Why would nature provide us with two different but complementary means of describing how things work, only to frustrate us at the very end by providing an experiment which does nothing for the sake of our greater understanding at large, save to come down arbitrarily on one or the other side of the argument?

If the measurement of the precession of the perihelion of Mercury had come down in favour of Newton instead of Einstein, it would have had devastating implications for our understanding of the universe, taking us all the way back to Michelson and Morley without any explanation as to how we should understand their null result. What on the other hand would be the negative consequence of living in a universe where we had no direct means to distinguish between the wave and particle theories of light? If we really don't need photons to explain the black body spectrum, the photo-electric effect, and the Compton effect – then just what *do* we need them for?