

The Other Half of Physics

Michael Goodband
email: michael@mjgoodband.co.uk
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Abstract. The physical dynamics of “bit from it” defines Object Physics, whereas identifying the causation of “it from bit” defines the other half of physics, Agent Physics. The two halves of physics are distinguished by a proof that scientific theories in Agent Physics can be undecidable, whereas those of Object Physics are always complete. Identification of the “it from bit” character underlying quantum theory enables a realisation of Einstein’s geometric unification of physics.

1. Its and Bits

In “It From Bit” [1] Wheeler proposed a physics paradigm of a computational universe, where quantum information “bits” determine the dynamics of material “its”. The next future state of the particle “its” is envisaged as being “computed” from their current state by a network of computational processes that obey, or implement, the fundamental laws of physics [1, 2]. This computational view of reality makes some implicit assumptions, not all of which stand up to the cold hard glare of observations in reality. It is assumed that the information and computational processes are fundamentally digital in character, which appears true for fermions as a quantum state for a fermion can only be occupied by 1 particle. But this overlooks the fact that for every baryon (composed of fermions) in the universe there-exist more than 1 billion photons (bosons) [3], and unlike fermions, any number of bosons can occupy a quantum state. So for the dominant form of particles, or “its” in the universe, the occupancy of quantum states is not digital.

Even for the 1 billionth of matter that are baryons, all is not as digital as it first appears. An electron occupying a particular orbital in an atom, labelled by a set of quantum numbers, is in a digital state of occupancy. But in a neighbouring atom there will also be an electron occupying the corresponding atomic orbital of that atom, with exactly the same set of quantum numbers. In any collection of atoms, the number of electrons in different atoms with the same set of quantum numbers can be counted: 1, 2, 3 ... etc. This beyond-digital character also applies to particle interactions. Consider 2 electrons in neighbouring atoms both being in atomic orbitals of an excited state. Now imagine that 1 electron drops to a lower energy orbital and emits a photon that collides with the neighbouring atom. This photon can cause the electron of that atom to also drop to the lower energy orbital, and emit a photon in the same quantum state. This is the stimulated emission of radiation process of a laser [4], in which the occupancy of the photon quantum state is increased from 1 to 2, and for N atoms it is increased from 1 to N. In computational terms, each stimulated emission adds 1 to the number of photons in the quantum state, and so any underlying computational process will obviously be over the natural-numbers.

The issue being considered here is whether the basic processes of the computational universe paradigm would be fundamentally digital or not. The billion to one dominance of bosons over fermions in the universe, and physical interactions between fermions being mediated by bosons, decides the issue: physical computational processes would be fundamentally over the natural-numbers. Particle interactions change the way in which the energy packaged up in particles is distributed amongst the different particle states. The number of different ways that these “its” of countable energy packets can be arranged is naturally given by the multinomial coefficient,

which for large numbers of particles is just the configuration entropy of statistical mechanics [5]. Attempts to define what information a collection of “its” contains on average also arrive at the conclusion that configuration entropy provides the key statistical measure [6], where its discrete character means each unit of information will be a “bit”.

So the input of some physics observations identifies particular meanings for “it” and “bit”. With a distribution of packets of energy (“its”) encoding the information content (“bits”), the essence of the question “It From Bit or Bit From It?” is the relationship between energy and information. Does energy determine information, “bit from it”? Or does information determine the distribution of energy, “it from bit”? But who said it had to be “or”? Stop the progression away from thinking about physics in terms of material objects and their interactions, and come back to reality. When we look closely at the nexus of physics and information in the physical terms of objects and their interactions, then not only do we find that the two options of “bit from it” and “it from bit” are physically realised, but they actually divide physics clearly in half.

2. Agent Physics

The energy-information relationship for objects is familiar from the thermodynamics of particles, where energy input drives the particle dynamics and increases the configuration entropy: the “its” get hotter and the “bits” of information decrease. However, the reverse case is also familiar. Take out some coins and scatter them on the table. Now pick up the coins 1 by 1 and stack them until there is a stack of 10. You have reduced the configuration entropy of the coins, and increased their information content. In the process of creating the stack, your actions were controlled by the number of coins already in the stack. The “it” of the coin stack was determined by the information “bits” of the stack itself: “it from bit”.

A drop in the configuration entropy of the coins cannot occur spontaneously in the physics of objects because it violates the 2nd law of thermodynamics. The energy you expended stacking the coins is the only reason why their information content could increase without violating the laws of physics. It could only happen because of your agency. This observation alone is enough to draw the simple distinction between an object which reacts, and an agent which energetically responds [7]. If we denote an object or agent by a , some stimulus by s , the response by r , and energy input by e , then we can denote agent behaviour by:

$$a(s, e) \rightarrow a', r \quad \begin{array}{ll} e = 0 & \text{Object Physics} \\ e \neq 0 & \text{Agent Physics} \end{array}$$

The energy you expended stacking the coins and decreasing their configuration entropy gives the rather odd feature of being described by a negative temperature. This is because of the way temperature T is defined as the change of entropy S with energy E :

$$\frac{\partial S}{\partial E} = \frac{1}{kT}$$

The external input of energy into a subsystem of objects causes the configuration entropy to increase, giving a positive thermodynamic temperature. In contrast, when energy input decreases the configuration entropy of a subsystem, such as for coin-stacking, the temperature is negative. The number of objects occupying a state with energy E_i at temperature T is given by the Boltzmann distribution $\exp(-E_i/kT)$, for which the number of objects decreases with increasing

energy. The reverse case of the number of objects increasing with increasing energy is called a population inversion, and occurs in lasers and our coin-stacking. In such cases where the distribution is reversed $\exp(E_i/kT)$, this gives a negative temperature T that is strangely hotter than a positive temperature. Although unusual, such negative temperature features are in undergraduate thermodynamics textbooks that deal with population inversions [5]. These odd features can be removed by defining an organisational temperature T_O as [7]:

$$\frac{\partial S}{\partial E} = -T_O$$

With this change there isn't the discontinuity in temperature between $T=-\infty$ and $T=+\infty$, and positive organisational temperature is hotter than negative organisational temperature. It is important to register that the laws of physics mean that only an open subsystem of some larger system can actually achieve positive organisational temperatures. In a laser for example, it is only the electron states experiencing the population inversion that are at positive T_O ($T<0$), the rest of material will be at normal temperatures ($T_O<0$). A laser also illustrates the general principle that energy must be constantly pumped into the subsystem to keep it at the extremely high temperatures of $T_O>0$. When the external energy supply is removed, the subsystem will “cool down” to normal temperatures. The same is true of any collection of objects that have been highly organised by agent action, such as your body.

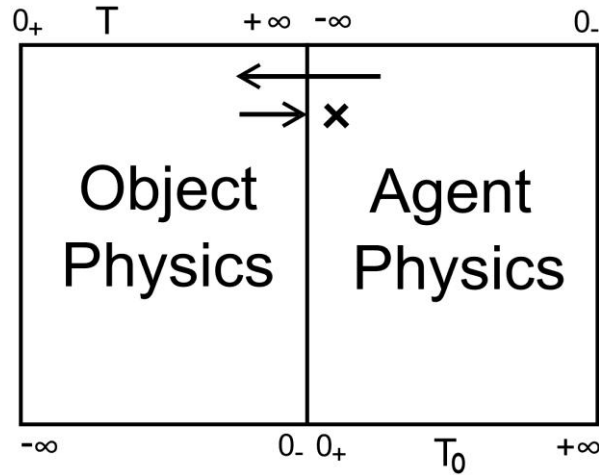


Figure 1: Temperature scale: Object Physics for $T_O < 0$, Agent Physics for $T_O \geq 0$ [7]

As is clear from the full temperature range shown in Figure 1, Agent Physics is literally the other half of physics. Object Physics is the domain of “bit from it” where the physical dynamics of distributed energy packets (objects) determines the information present. Information causally determining the pattern of energy expenditure in the physical world is the defining characteristic of *agency*, and makes Agent Physics the domain of “it from bit”.

The coin-stacking example illustrates the key feature of agent behavioural control, where the information contained in a collection of objects controls how many times an agent repeats the same behaviour. The individual behaviour of placing 1 coin on the stack adds 1 to the total number and was repeated 10 times to produce the 10 coin stack. This simple number control of repeating behaviour can then be cascaded. For example, every time you create a stack of 10 coins place 1 coin on a separate control stack and repeat the whole process again until there are 10

coins on the control stack. You will produce 10 stacks of 10 coins, plus the control stack. Repeating the stacking behaviour results in +10 addition, and cascading the behavioural control results in multiplication $10 \times 10 = 100$. Obviously, simple variations of this pattern of behavioural control for an agent can produce full arithmetic behaviour over object numbers.

Such object arithmetic is a feature of living cells in biology (in terms of molecule numbers), also occurs at higher levels in biology, and can be found in many other agent systems [7, 8]. The presence of fully arithmetic behaviour over the natural-numbers of objects within the domain of Agent Physics means that the science theories of Agent Physics can possess a mathematical feature that the science theories of Object Physics **never** possess. The critical step in identifying this feature is to grasp what the digital character of existence really means.

3. Undecidability Everywhere

Despite spurious claims to the contrary, no experiment in 4 centuries of the Scientific Age has ever contradicted that an object “it” either exists or “it” doesn’t. This specifically includes **all** experimental tests of quantum theory. The weirdness of quantum theory has tempted many to abandon reality, and fantasize about particles both existing and not existing at the same time. But the experimentally-measured fact of reality is that even for fundamental particles, a particle either exists or it doesn’t. And in fact, quantum theory predicts you will **never** measure a particle as both existing and not existing at the same time [4, 9, 10].

The mathematical complexity of quantum theory, and its subtlety of being one step removed from physical observation, has no doubt contributed to the subject becoming dominated by mathematicians. Associated with this has been a shift from the Galilean physics view of using maths to describe the physical world, to the Pythagorean maths ideology that “all is numbers”. This is a reversal of the epoch-defining shift that began the Scientific Age. In a sense, physics began with the recognition that “all can be **described** by numbers”, and that this is not the same as the pre-scientific belief that “all **is** numbers”. Progress in science has so often been dependent upon asking exactly the right question, and by stepping back from mathematical fantasy into the light of reality we can identify the right question: described by *which* numbers?

The first step in using maths as the *language* to describe different types of object in reality is to assign a mathematical symbol to denote each type of object. The symbols can be grouped into sets by the type of object they denote, and the cardinality of the sets defines the natural-numbers [11] of objects within a scientific theory. This already marks a split between maths and physics, as this requires a set theory with urelements (symbols denoting real things) and types, which maths avoids by using the empty set as the basic mathematical entity contained in other sets [11]. This urelement notation of objects gives a more precise expression of Einstein’s arguably confused notion of physically-real terms [12]. The type of science theory that Einstein advocated as being physically-real can be more concisely defined as being one where each mathematical symbol (urelement) denotes an object, and where every mathematical operation in the theory corresponds to a physical interaction between objects. The digital character of existence means that as long as this process of denoting objects and object interactions is strictly followed, then the resulting physically-real scientific theory will be mathematically consistent. The theory will never contradict itself, such as by saying an object exists and doesn’t exist at the same time.

The computational universe paradigm implicitly raises issues of mathematical completeness and computability. Correctly identifying what the digital character of existence means for science doesn’t make these issues go away. Instead, the natural-number basis of physically-real scientific theories of object state changes identifies such theories as being **exactly** the type of

arithmetic systems considered by Gödel [13]. If you construct a mathematical theory “in the air”, disconnected from observations of reality, then you can’t know if Gödel’s theorem applies because it only applies to certain types of theories that are **known** to be consistent, but if a theory is incomplete then you cannot prove the theory is consistent [13]. A mathematician who constructs an arbitrary theory that meets all the other conditions of Gödel’s incompleteness theorem is stuck in a catch-22 on this consistency issue. A physicist on the other hand, who rigorously follows the procedure for constructing a physically-real scientific theory [7, 8], can **know** that their theory is consistent because reality is observed to be consistent. So when you have a physically-real scientific theory that meets all the conditions of Gödel’s theorem, the theory is **proven** to be incomplete by Gödel’s proof. This happens in Agent Physics, whereas **all** physically-real scientific theories of Object Physics are mathematically complete over the numbers of objects, because there can be no object arithmetic without agency [7, 8].

Although Gödel’s theorem is briefly described as applying to arithmetic systems over the natural-numbers, this is insufficient in itself. Gödel’s proof technically requires the mathematical system to contain all possible number-theoretic functions over any number of natural-number valued variables [11, 13]. This is not as demanding as it appears, as such functions can be created from the 4 initial functions – successor (+1), predecessor (-1), zero and projection functions – by the repeated application of the function creation rules of substitution and recursion. Every scientific theory of object interactions must contain the 4 initial functions in order to be physically-real, but special conditions are required for the function creation rules to be physically-real [7, 8]: substitution requires a tree-like hierarchy of object reactions; and recursion requires the creation of a new object type not previously present. These conditions are only met for a growing network of object reactions that include processes which implement arithmetic changes in the numbers of some objects, such as in the metabolic network of a biological cell.

The final technicality is Gödel’s proof demands every recursive number-theoretic function be expressed *within* the theory. This can only be met by an infinite network, or indefinitely growing network [7, 8]. The latter is the case for the numbers of molecules in the metabolic network of a biological cell. This is because a scientific theory has to model the random mutation of genes and natural-selection of evolution, which together make the metabolic network indefinitely growing **in** theory. The conditions of Gödel’s proof are then met, and a physically-real scientific theory of the numbers of molecules and cells in biology is **proven** incomplete [7, 8].

The critical condition of unending variation in a potentially indefinitely growing network is repeated at higher levels in biology, such as for the network of organisms of an ecosystem. The economy of a nation has a network of agents (people and corporations) with countable numbers of traded goods flowing over the network links. Innovation provides the condition needed for the economic network to be indefinitely growing **in** theory. The same conditions can be found in the financial-markets in terms of the numbers of contracts and “innovation” of financial derivatives. They can also be found in the functional form of the neural-networks of the brains of higher animals. In all these cases, a physically-real scientific theory of the numbers of agents and objects in the respective networks can be **proven** incomplete [7, 8].

Although these agent systems have been studied outside of physics, at the fundamental level all science is ultimately physics, specifically Agent Physics in these cases. The incompleteness of physically-real scientific theories is ubiquitous in Agent Physics (e.g. biology and economics), but never occurs in Object Physics. This feature clearly distinguishes between the two halves of physics: Object Physics is decidable, whereas much of Agent Physics is undecidable.

4. The Power of Observation

Much rubbish has been said about what Gödel's incompleteness theorem means, some it by those who should have known better. It is after all part of the foundations of first-order logic and number theory taught to 19-year old undergraduate maths students. Given its significance it should be taught to all aspiring physicists and philosophers, many of whom haven't grasped that Gödel's incompleteness is a feature of discrete logic over arithmetic natural-number systems. If a natural-number system isn't fully arithmetic by not supporting multiplication, then the reverse is true and the system can be proven complete [11]. If the arithmetic system is over the real-numbers, then Gödel's proof doesn't apply. This specifically means that if you have a system over the natural-numbers that is proven incomplete, such as a physically-real scientific theory in Agent Physics, then just switch to the real-numbers and Gödel's theorem no longer applies. But the biggest difficulty of all seems to be with understanding the English word **in**.

It is not as if this is some hidden point either. In an English translation of Gödel's original paper [13], the corollary that is now called the incompleteness theorem is translated as:

Proposition VIII: In every one of the formal systems referred to in Proposition VI there are undecidable arithmetic propositions.

The guts of the proof specifying the maths system and Gödel numbers culminates in Proposition VI which Gödel then succinctly expresses in the given corollary, for which the first word is **in**. It says that in the specified type of maths system K there-exists at least one proposition P that is **in** K but cannot be derived **within** K . The interpretation that Gödel's theorem proves some things to be beyond maths and science is drivel of the highest order. The proposition P supposedly beyond reach is specifically **in** K . Now a mathematician might get upset about not being able to derive P within K , but a scientist should know better. Given that Model Theory is also taught in undergraduate maths, mathematicians should actually know better as well. It is the point that launched the entire Scientific Revolution in the first place: observation.

If the process for constructing a physically-real scientific theory is rigorously followed then every term, or proposition P , in the theory, or mathematical system K , will directly correspond to an object or process in reality. As described earlier, there-exist certain physical systems where the physically-real theory is **proven** incomplete by Gödel's theorem. This means that there will be at least 1 proposition P in the theory K that cannot be derived within K . But for a physically-real theory, every proposition P directly corresponds to something observable in reality. Now a mathematician might try to tell you that being unable to derive something in maths makes you blind, but in reality it doesn't kill your powers of observation. The maths problem becomes a science problem of observing something you cannot derive in your scientific theory.

The mathematical reason for such undecidability is that your theory is over the natural-numbers of objects in reality, which presents you with a very simple resolution to the apparent impasse of undecidability: change numbers. If you switch using natural-numbers to describe the numbers of objects to using real-numbers, Gödel's theorem no longer applies to your theory. You are then free to add an extra term for your observation P and there is no barrier to your theory being made scientifically complete, i.e. being able to predict all your observations of reality. Of course, your real-number valued terms are no longer physically-real terms, because object numbers don't take decimal values in reality, and there is 1 and only 1 possible interpretation of using a real-number value to denote discrete objects: as a probability.

This should sound familiar. Such switching from natural-numbers to real-numbers in theory and back again, is at the heart of what quantum theory is really about. Despite appearances to the

contrary, all experimental measurements of quantum theory are in the domain of classical physics. This includes all the particle reactions observed in cloud chambers etc. The scientific method is to construct a scientific theory in physically-terms that can predict all the observations. The well-known problem observation is particles travelling as waves, but the experimental measurements of the Casimir effect, where metal plates are pulled together by the vacuum between them [14, 15], cannot be accounted for in classical physics either. If we ignore the wave property for now, and attempt to construct a classical physics theory of particle reactions in physically-real terms we will produce a theory over the natural-numbers of particles. However, vacuum effects like the Casimir effect cannot be accounted for without converting Heisenberg's uncertainty principle into the following *Vacuum Reservoir Hypothesis* [7]:

A physical system with energy E and particle number N , can increase its energy by ΔE and increase particle number ΔN during some physical process of duration $\Delta t > 0$, such that both the energy and particle number return to the values E and N when the process has finished.

This is a feature of quantum field theory, but here we are considering what happens when we add it to a physically-real classical physics theory. Will it finally result in a classical physics theory that succeeds where all others have failed? Don't be silly, of course it fails. The interesting point is that the resulting theory is **proven** mathematically incomplete by Gödel's theorem [7, 8], and the given hypothesis is directly the reason for the incompleteness. The hypothesis postulates that the vacuum can supply energy to a particle, which it can use in particle behaviour as long as it returns it back to the vacuum. In classical physics this property means that a particle meets the definition of being an agent, and the return condition of the vacuum state acts to control the particle behaviour. The behavioural control imposed by the information "bits" of the vacuum state allows fully arithmetic particle behaviour, giving the causation "it from bit". And the moment that happens, the otherwise physically-real scientific theory is **proven** incomplete.

If we then switch from natural-numbers to real-numbers, add in the observation of the wave property, our theory becomes mathematically identical to quantum theory [7, 8, 16]. The origin of the commutator relations of quantum theory is the Hamiltonian mechanics formulation of classical physics [17], and the spin statistics theorems for fermions and bosons come from applying the space-time symmetries of Relativity to real-number valued fields [10]. The heart of quantum theory is **just** the switch from natural-numbers of particles to real-number valued fields in theory and back again. Every other feature of quantum theory comes from applying the Hamiltonian formulation of Relativistic physics to real-number fields denoting particle numbers.

The power of observation is to bypass the undecidability of "it from bit" in Agent Physics, but it can only be implemented if the type of number used for description of the natural-numbers of objects in reality is changed to a real-number description. To grasp that this is valid in science you have to let go of the pre-scientific belief that "all **is** numbers", and join the Scientific Age where "all can be **described** by numbers". This gives us the answer to the question: described by *which* numbers? In Object Physics the physically-real natural-numbers are just fine because the scientific theories can be complete; whereas in Agent Physics the undecidability means that a number-type switch is needed. This inevitably gives non-physically-real scientific theories with probabilities, but where the theories are nonetheless scientifically complete. This gives the origin of quantum theory, but quantum theory is **merely** an example of a much larger pattern in Agent Physics, due to the causation of "it from bit".

5. Science is Not a Belief System

The energy-entropy relationship reveals Agent Physics to be a well-defined discipline that is **obviously** the other half of physics. It is the domain of information controlling energy expended, of “it from bit”, that has been studied under non-physics disciplines until now. But at the fundamental level all science is ultimately physics, mostly Agent Physics as it happens, whereas at the most fundamental level of physics ... it’s still physics. In the half century since the Standard Model was completed, the pre-scientific maths ideology of “all **is** numbers” has come to dominate the leading edge of physics. This has brought claims of maths theories being advanced science, when they are not even science. The last time this ideology was dominant, intellectual progress stalled for 18 centuries and this failure would no doubt have stretched into eternity were it not for the birth of physics and the Scientific Revolution.

The proof that Gödel’s incompleteness theorem applies to physically-real scientific theories in Agent Physics challenges this belief, making it an ideologically unacceptable proof. This is possibly why it has been greeted with nothing more than “I don’t believe it”. Everyone in an advanced society with public education is supposed to know by age 16 that statements of “belief” have no place in science. The proof is based in the foundations of scientific modelling and the maths of arithmetic over the counting numbers ... physicists are supposed to know this stuff. Scientists are supposed to know that a statement of “belief” about a **proof** is irrelevant, unscientific and un-mathematical as well. The proof says that in Agent Physics “the map is not the territory” [18], that a mathematical theory in science is a *description* of reality in the *language* of maths, not reality itself. As such, you’re free to change your *description* of reality, because it won’t change reality. To believe otherwise is equivalent to a medieval belief in magic. The proof makes it possible to succeed [8] where Wittgenstein [19] the philosopher failed, where he explicitly failed because he asserted an ideological belief in maths over reality.

There has been a view that Einstein was wrong about quantum theory not being fundamental, but the ability to derive quantum theory by changing number type in the *description* of physics proves Einstein to essentially be correct [7, 8, 16, 20]. What Einstein was wrong about was his assertion that quantum theory could be replaced by a non-probabilistic theory. Instead, the incompleteness proof gives yet another proof that there is no complete physically-real scientific theory that replaces quantum theory. Once you’re prepared to step-up to the science standard of **proof**, drop the ideological belief that maths **is** reality, you find Einstein was right in his vision that a purely geometrical theory could achieve physics unification [21, 22]. There-exists 1 and only 1 possible extension to General Relativity with compactified dimensions, and no arbitrary fields, that succeeds in unifying gravity with the 3 particle forces through quantum field theory being derived by a change in number type description [20]. It derives **all** known particles, and **only** all known particles, all the correct coupling constants and boson masses, including predicting the Higgs boson mass [23].

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