

What Is Possible in Physics Depends on the Chosen Representational Formalism*

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Abstract All of science is built on the foundation of the millennia-old numeric forms of representation and the associated measurement processes. Hence, the most promising way to approach physical reality (and physics) afresh is to shift to a *non-numeric representational formalism*. I discuss here one such formalism for structural/relational representation—evolving transformations system (ETS)—developed by our group. In particular, the adoption of ETS obviates the introduction of consciousness into physics, since under the formalism, the two *forms* of object representation—by an agent (subjective) and in Nature (objective)—agree. Moreover, ETS suggests *the primacy of the new temporal representation over conventional spatial representation*, and it is not difficult to envisage that the latter is actually instantiated on the basis of the former, as has also been suggested by some quantum gravity researchers.

1. The urgent need for a non-incremental development of science

Today, many natural sciences, including physics, are in a peculiar state. On the one hand, it is generally understood that we are faced with many major questions requiring *radical* rethinking of the existing paradigms. On the other hand, we continue speeding up *along the same ‘road’*, business as usual. Certainly, very few of us are willing to take seriously the possibility of existence of *another scientific road*, fundamentally different from the numeric-based ‘road’. The latter has been *the only road* science has been following so far and is associated with *the numeric forms of representation*—numbers (including vector and Riemannian spaces) and the numeric measurements.

So here is the big question: *Is it safe to expect to arrive at the wished-for new destination on our fast train following the same ‘numeric’ road?* Even if we continue to improve this road with new mathematical developments, it simply may not help; and as unrealistic as the thought may seem at first, we should at least *consider* the possibility of existence of an *entirely* different road, *going in a completely different direction*. By not considering the tangible possibility that *we are reaching the limits* of what can be profitably squeezed out of the underlying numeric form of representation, we might be making a serious mistake. It is imperative to keep in mind that *all scientific data depends* on the chosen representational formalism, i.e. on the chosen *form of data representation*.

Thus, what is not so well understood is that the needed paradigm change *might be of much more radical nature than physicists and even mathematicians have been prepared to think about*. More specifically, we should consider the possibility of replacing the ubiquitous numeric form of representation by its *structural, or relational*, generalization. Of course, *such transition in science would be of an absolutely unprecedented nature*, since, for example, complex numbers, quaternions, octonions, etc. are not such generalizations, as they are still numeric-based. Nevertheless, despite the unparalleled difficulties, *we simply may not have any other choice*.

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To remind that I am in good company in this respect, I devote some space to the relevant views of the three well known 20th-century scientists—Schrödinger, von Neumann, and Einstein—which were already expressed many decades ago. Schrödinger's and von Neumann's observations are particularly relevant. Here is Erwin Schrödinger, writing in 1951 [1]:

If you envisage the development of physics in *the last half-century*, you get the impression that the discontinuous aspect of nature has been forced upon us *very much against our will*. We seemed to feel quite happy with the continuum. Max Planck was seriously frightened by the idea of a discontinuous exchange of energy Twenty-five years later the inventors of wave mechanics indulged for some time in the fond hope that they have paved the way of return to a classical continuous description, but again the hope was deceptive. Nature herself seemed to reject continuous description [p. 158]

Now I shall try to give you an idea of the way in which physicists at present endeavour to overcome this failure ["of our attempts to use continuum for a precise description of nature"]. One might term it an 'emergency exit', though it was not intended as such, but as a new theory. I mean, of course, wave mechanics. (Eddington called it 'not a physical theory but a dodge—and a very good dodge too'.)

. . . The observed facts (about particles and light and all sorts of radiation and their mutual interaction) appear to be *repugnant* to the classical ideal of continuous description in space and time. . . . [He then illustrates this by reminding that Bohr's theory of spectral lines had to assume that atom makes a "*sudden* transition" to a new state while emitting "a train of light waves several feet long, containing hundreds of thousands of waves and requiring for its formation a considerable time" and "[n]o information about the atom during this transition can be offered."]

So the facts of observation are irreconcilable with a continuous description in space and time On the other hand, from an incomplete description—from a picture with gaps in space and time—one cannot draw clear and unambiguous conclusions; it leads to hazy, arbitrary, unclear thinking—and that is the thing we must avoid at all costs! What is to be done? The method adopted at present may seem amazing to you. It amounts to this: we do give a complete description, continuous in space and time without leaving any gaps, conforming to the classical ideal—a description of *something*. But we do not claim that this 'something' is the observed or observable facts; and still less do we claim that we thus describe what nature (matter, radiation, etc.) really *is*. In fact, we use this picture (the so called wave picture) in full knowledge that it is *neither*. [pp. 143–44]

Next, one of the outstanding mathematicians of the last century John von Neumann, trying for a number of years to come to grips with the *formal* encapsulation of the nervous system, came to the following conclusions about the *mathematical road* we have taken so far, which are particularly instructive given his superb abilities in discerning various mathematical structures:

It is only proper to realize that language is largely a historical accident. . . . Just as Greek or Sanskrit are historical facts [i.e. accidents] and not absolute logical necessities, it is only reasonable to assume that logics and mathematics are similarly historical, accidental forms of expression. They may have essential variants, i.e. they may exist in other forms than the ones to which we accustomed. Indeed, the nature of the central nervous system and of the message systems that it transmits indicates positively that this is so. . . . Thus logics and mathematics in the central nervous system, when viewed as languages, must structurally be essentially different from those languages to which our common experience refers.

. . . [W]hen we talk mathematics, we may be discussing a *secondary* language, built on the *primary* language truly used by the central nervous system. . . . The above remarks about reliability and logical and arithmetical depth prove that whatever the [i.e. this] system is, it cannot fail to differ considerably from what we consciously and explicitly consider as mathematics. [2, pp. 81–2]

And finally, here is Einstein himself [3, p. 467]:

in a 1941 letter to Infeld: "I tend more and more to the opinion that one cannot come further with a continuum theory."

in a 1954 letter to his friend Besso: "I consider it quite possible that physics cannot be based on the field concept, i.e., on continuous structures. In this case, *nothing* remains of my entire castle in the air, gravitation theory included, [and of] the rest of modern physics."

2. From the numeric to structural representations

Once again, I believe *we have reached the end of the first scientific era* and are about to enter the next one, and the level at which the transition must occur is *the most fundamental*, yet still unexplored, level: it is the *representational* level, the no-man's scientific land. Representational formalism deals with *the form of data representation*, i.e., it specifies the abstract structure for representing data, which, once introduced, allows theoreticians to proceed with its refinement and elaboration. So I am suggesting that *we are poised to shift from the ubiquitous numeric representation and the associated measurement process to the structural representation and the associated structural 'measurement' process*.

On a cautionary note, the term 'representation' has become popular in cognitive science and computer science but *without due considerations*—as it would have received in physics—to the suitability of the corresponding *concept* to serve as a true generalization of the classical counterpart.

2.1 The present language of mathematics and the path to the future

The consolidation of foundations of mathematics in the form of set theory at the end of the 19th – beginning of the 20th centuries allows us to focus on what needs to be overcome. The concept of set—a collection of (*unstructured, point-like*) elements—abstracted and entrenched the ubiquitous *spatial form of representation, 'the point'*; so that at present, a mathematical structure (e.g. a vector space) is defined axiomatically *starting with some underlying set of elements of unspecified structure*, and this structure is 'revealed' only later, during the analysis of the formal consequences of the chosen axiomatic system (e.g. the algebraic structure of vectors). Thus, in applications, the chosen *postulated (numeric-based) axiomatic structure*, e.g. the inner product vector space, *is simply being imposed on reality*, generally *without our full awareness*.¹ Is there an alternative?

I submit that it is wise (and healthy) for us to believe there is. And to be more or less on sure ground, when looking for it, I believe we should carefully follow the 'safe' path. Ironically, but quite logically, it appears that *the only safe path* is the path of the 'right' generalization of the most basic and ancient numeric structure, the natural numbers, about which Kronecker aptly remarked: "God made the integers; all else is the work of man."

Which further methodological assumptions can guide us in developing a fundamentally new kind of representational formalism? First, I propose to replace the concept of a *set of objects* as a foundational concept in mathematics by that of a *class of objects*, where a class is a set of objects that *have 'common' formative, or generative, history* (see below); so that these objects are 'similar' to each other precisely because of the common informational mechanism responsible for their production.

Second, I am convinced that the accumulated philosophical wisdom—the tradition from Heraclites to Hegel, Bergson, and more recently Whitehead, Collingwood, Whyte, and Čapek—should also serve as an important guide. I am referring to the view, supported now by discoveries in physics, of an object as a process, a structural process, to be sure. In other words, we need a formalism in which *objects are viewed and represented as (structured) processes* (see also [4, Chapter 4]).

Thus, on the formal side, our starting point is Peano constructive definition of natural numbers [5], in which the 'successor' operation plays the critical role.² Generalizing this successor operation to the fundamentally new concept of (structured) *successor event*, we arrive at the basic idea of the

¹ In particular, a not-so-small number of possible non-commensurate interpretations of quantum mechanics appears to be a part of the price we are paying for that.

² Each natural number n has a unique 'successor' $s(n)$, and all natural numbers are thus inductively constructed starting from 0.

evolving transformations system (ETS) formalism [6], developed by us during the last twenty years. The latter, accordingly, should be viewed as an *event-based structural generalization* of the numeric representation. However, as we shall see, despite the numeric point of departure, *the resulting relational/informational form of object representation* embodies the radical departure from the historically entrenched spatial form of object representation.

2.2 The proposed informational structure of Nature

I now briefly address the issue of *the class of objects*³ *as the basic organizational unit in the informational structure of Nature*, with the expectation that it is this structure that is responsible for guiding the physical instantiation of objects in the universe. The following three related postulates should eventually be verified.

As we know, objects in nature do not pop up out of nowhere but always take some time to appear, and in each case, the way an object appears is similar to the way some other, ‘similar’, objects appear. In other words, as far as we know, *there is no object in nature that does not belong to some class* of closely related objects, be it an atom, a cloud, a star, a black hole, a stone, a worm, a protein, or a stop sign. Since we view objects as processes, we have

Postulate 1: the universe is a family of evolving and interactive classes of (irreversible) processes.

Moreover, in view of the close connection between an object and its class we also have

Postulate 2: for each class there exists—and stored in some form—its *class representation*, which is responsible for the class integrity by guiding the generation of its elements (and which *evolves* together with them).

It is useful to keep in mind that the last postulate can be seen as a continuation of a very remarkable line of thought going back to Aristotle, Duns Scotus, and Francis Bacon, among others. Our last postulate deals with the nature of object representation.

Postulate 3: similar to a biological organism, with its developmental ‘program’, any object in nature also coexists with its actual *formative/generative history* (which, of course, must be consistent with the above class representation).

In light of this, the ‘similarity’ of objects should be understood as the ‘similarity’ of their formative histories, and hence *this formative history (or object’s structure) must be captured in the object’s formal representation*.

Finally, in the proposed informational view, *events become the basic units of representation*. Regarding events in general, for example, references [7, pp. 290–96], [8, Chapter 11] are of more interest to physicists, but events became also a popular currency in philosophy and psychology (e.g. [9], [10]).

As a prelude to the next section, here are several quotes from James Jeans [7, pp. 293–95, my emphasis]:

Thus the ‘world-line’ of a particle is . . . not a line at all, but a . . . curved region, and must logically be separated into small curved spots—the particle resolves itself into events. Most of these events are unobservable; it is only when two particles meet or come near to one another that we have an observable

³ To facilitate initial reading, ‘object’ is used instead of ‘process’.

event which can affect our senses. We have no knowledge of the existence of the particle between . . . [events], so that *observation only warrants us in regarding its existence as a succession of isolated events*.

. . . *Thus the events must be treated as the fundamental objective constituents* and we must no longer think of the universe as consisting of solid pieces of matter which persist in time, and move about in space.

. . . we now begin to suspect that *events and not particles constitute the true objective reality* . . .

And here is Lee Smolin on events: “From this new point of view, the universe consists of a large number of *events*. An event may be thought of as a smallest part of the process, a smallest unit of change. . . . The universe of events is a *relational universe*. That is all its properties are described in term of relationship between the events.” [4, p. 53]

2.3 The initial ETS concepts

Consistent with the first postulate, the concept of class pervades all levels of consideration in ETS. For example, even the most basic concept of primitive event is defined by means of the *classes* of primal processes (Fig. 1).

I restrict myself to informal descriptions of several basic concepts, while the formal definitions can be found in [6], Parts II and III, which is the principal exposition of the formalism (see also [11] – [14]). The important point to keep in mind is that, in the formalism, each ‘object’ is viewed and represented as a (temporal) structural process, ‘struct’ (see Fig. 2), which is a stream of interconnected structured events.

2.3.1 Primitive events

The most basic concept is that of a **primitive event**, or **primitive transformation**, or simply **primitive**, pictorial depictions of which are shown in Fig. 1 (see also Fig. 2). Each **primitive** stands for a *fixed kind* of micro-event—a site of interaction of several processes—which is responsible for transforming several **initial primal processes** (the lines just above an event in Fig. 2) into several **terminal primal processes** (the lines just below an event in Fig. 2). *The formal structure⁴ of a*

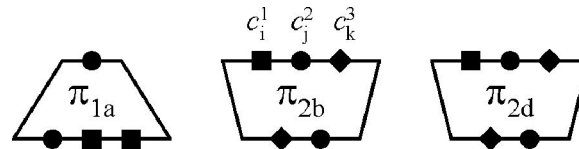


Figure 1: Pictorial depiction of three primitives. The first subscript in the primitive’s name stands for the class of primitives sharing the same structure (and hence the same *overall shape*), e.g. π_{2b} and π_{2d} . **Initial classes** of processes are shown as *small solid shapes* on the top and **terminal classes** are those on the bottom of each primitive. The only labels of the individual processes, i.e. of the elements of these classes, shown in the figure are c_t^s ($s = 1, 2, 3$) — the t^{th} process in the initial class C_s for primitive π_{2b} , where $b = \langle c_i^1, c_j^2, c_k^3 \rangle$.

primitive event depends mainly on the two fixed tuples: of initial and terminal *classes* (of processes). A *concrete* primitive depends, in addition, on the *concrete* initial and terminal processes that must come from the corresponding classes of processes. Since we discuss throughout, except for Section

⁴ This structure is called the corresponding *abstract primitive* and looks schematically exactly like any of those in Fig. 1, except its label does not have the second subscript, e.g. π_2 . It actually stands for the class of the corresponding *concrete* primitives.

2.3.5, only *the initial stage of representation*, the structure of all initial and terminal processes is suppressed (hence the name ‘primal’), as is the internal structure of the event itself (hence the name ‘primitive’), and what is being captured by the formal structure is the ‘external’ structure of the event.

The concept of primitive is *relatively* intricate, but in the more important sense, it is simpler than practically any formal concept: it carries *identical semantic and syntactic loads*, i.e. it functions syntactically/formally exactly according to its semantic content. This is a unique feature of ETS representational formalism [15], since *in any spoken language or any formalism in science the syntax is not related to the semantics*.

As nature is comprised of various temporal processes composed of events, examples of the above events are all around us (and depend, of course, on the chosen initial stage of representation): all events in particle physics (the initial and terminal processes can easily be read off from the corresponding Feynman diagrams); formation of a two-cell blastula from a single cell (initial process is the original cell and the terminal processes are the resulting two cells); etc. The crucial point is that *the proposed structure of the event is universal*.

2.3.2 Structs

The second basic ETS concept is that of a **struct**⁵, which is a (temporal) stream of interconnected primitives, Fig. 2.

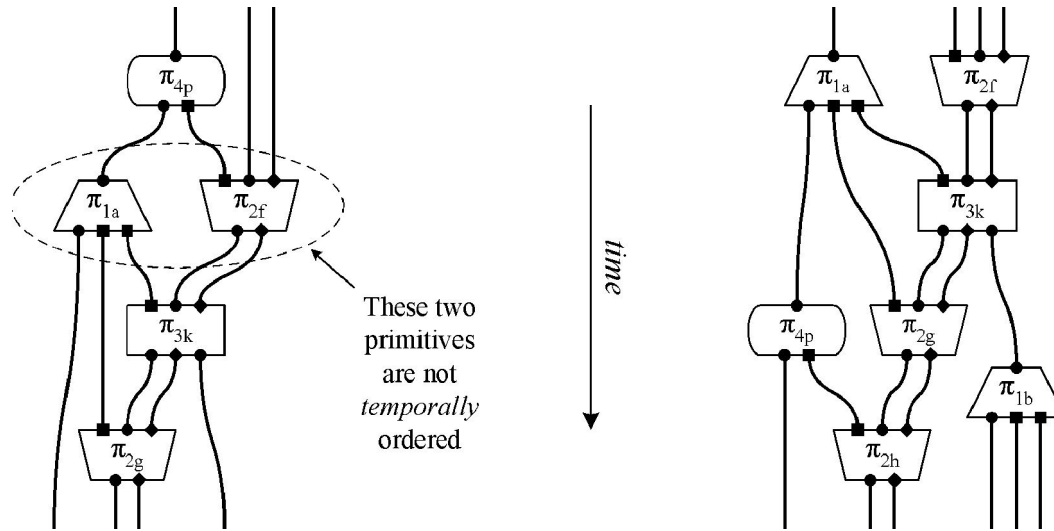


Figure 2: Pictorial depiction of two (short) structs.

Comparing figures 2 and 3, it is easy to see how the Peano construction of natural numbers (Fig. 3) was generalized to the construction of structs: *the single rudimentary structured unit out of which a number is built is replaced by several structural ones*, i.e. by ETS primitives. The decisive consequence of the structuredness of the units is that we can now see *which* unit is attached to *which* and ‘*when*’.⁶ Hence, the struct, as the resulting object representation—and also as the output of the *structural measurement process*⁷—for the first time, embodies both relational and temporal information in the form of the *formative, or generative, object history* recorded as the stream of the

⁵ Or, more accurately, **level 0 struct**, implying that *even at the initial stage of representation* there are higher level structs (which are hierarchical partitions of the former).

⁶ The “flow of information among events” emphasized in [4, p. 210] becomes tractable now.

⁷ See [16] (which is now outdated).

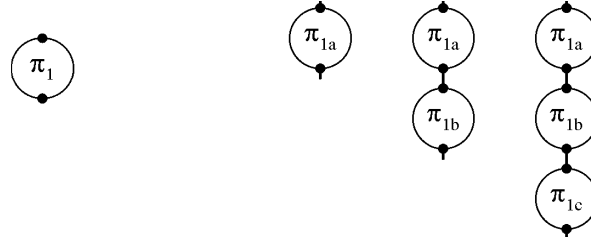


Figure 3: The single primitive for the ETS representation of natural numbers (left) and the three structs representing numbers 1, 2, 3.

corresponding (structured) events. Thus, the concept of struct can justifiably be viewed as *the natural structural generalization of the numeric representation*. At the same time, the struct embodies a *fundamentally new, ‘local’ and ‘non-linear’, kind of temporality*, which can be called structural or relational and *which should clarify the nature of temporality in general*.

Finally, *the struct*, as capturing both *forms* of object representation, ensures their agreement: one kind of struct is constructed by an agent *during its interaction with the object* (the agent’s representation, based on the agent’s primitives), while the other kind is constructed by Nature during the entire process of the object’s formation (based on the complete set of primitives, Postulate 3).

2.3.3 Temporal and spatial representations

The independent nature of ETS representation suggests the primacy of this—temporal, or informational, or structural—representation over the conventional mathematical spatial representation (see Fig. 4). Moreover, it seems quite possible—given some local parameter(s)—to instantiate the latter on the basis of the former.⁸ Within the context of computer science, we *illustrated* this in [6], Section 8, and in the master’s thesis [17].

2.3.4 Level 0 classes

The third concept is that of a (level 0) **constraint**, which is a more involved concept and is not discussed here; see [6], Section 5. It is a formal *specification of a family of structs* sharing a common structural ‘backbone’ and composed of similar structural components.

The fourth basic concept is that of a **class of structs** (which can possibly be multileveled even at the initial stage of representation). A single-level (or level 0) class is defined via a single-level **class generating system**, which specifies a *stepwise mode of construction* of the class elements. Each (non-deterministic) **step** is specified by the corresponding *set* of (level 0) constraints, which restrict the kinds of *struct segments* admissible at this step during the construction of a class element. Such step follows (possible) interim steps by the ‘environment’, i.e. by several *other generating systems whose events may ‘intervene’ in the construction process*. Thus, the constraints must be flexible enough for the construction process to proceed under the *allowable* interactions, in which case the intervening events become a part of the resulting class element, i.e. of the corresponding struct.

2.3.5 Transition to the next representational stage

To clarify the nature of the most basic ETS concept, the primitive, a few words about the concept of **the next stage of representation** are in order. Transition to the next representational stage (Fig. 5) is associated with a representational compression, in which certain recurring global patterns of process interactions, called **transformations**—which is the fifth basic concept—are *compressed*

⁸ The primitives might be instantiated as (structural) ‘pulsations’.

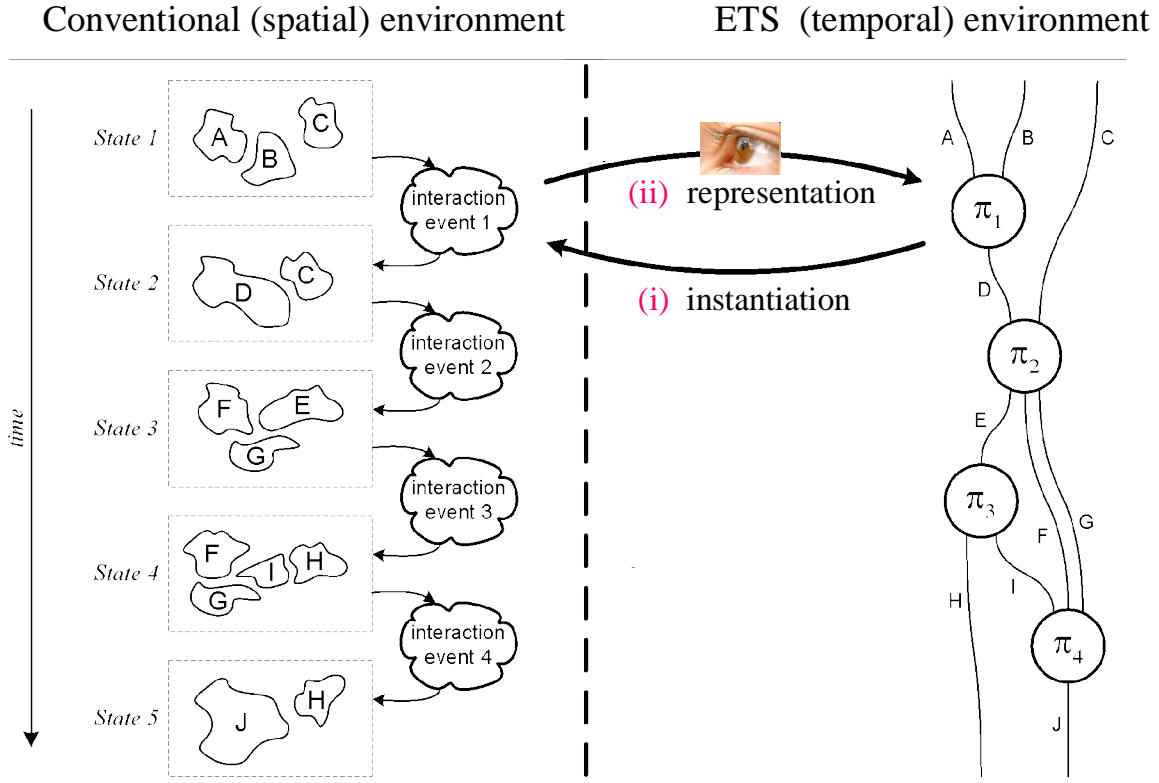


Figure 4: On the left, objects A and B in *State 1*, as a result of the first event (denoted by primitive π_1 on the right), merge to form D in *State 2*. Three subsequent state changes are also illustrated: D and C change into E, F, and G; E divides into I and H; finally F, G, and I merge to form J.

- (i) This is the direction of precedence of temporal encoding over the spatial one *in nature*: the spatial representation can be instantiated on the basis of the temporal one.
- (ii) This is the direction in which *an agent's sensory representation* appears to operate.

into the (new) primitives for the next stage: each of the interacting processes is *compressed* into a primal process, and the segment in which the processes interact is compressed into the **next stage primitive** event. Thus, ETS offers a *seamless integration of representational stages* within a single formalism.

3. Some immediate implications for physics

The new, ‘structural’, mathematics—including some analogues of topology and algebra—is still to be developed, but even now, at the very beginning, one can already intuit how *the immediate features* of the ETS representation may help to explain, for example, the previously baffling features of quantum processes. In particular, the quantum nature of processes appears to be adequately explicated by the proposed event-based representation: quanta vs. instantiated events. The ‘particle-wave duality’ can probably be explained by the possible forms of interaction between a quantum process and its environment: as mentioned in Section 2.3.4, *the resulting* quantum process *always* depends on the external events that are allowed to intervene in its construction during its generation. Similar, and in light of ETS *natural*, explanation may apply to ‘the most profound mystery’ of quantum mechanics, quantum entanglement. Indeed, if several quantum processes are from the same class—i.e. their instantiations are being guided by the same generating process—and

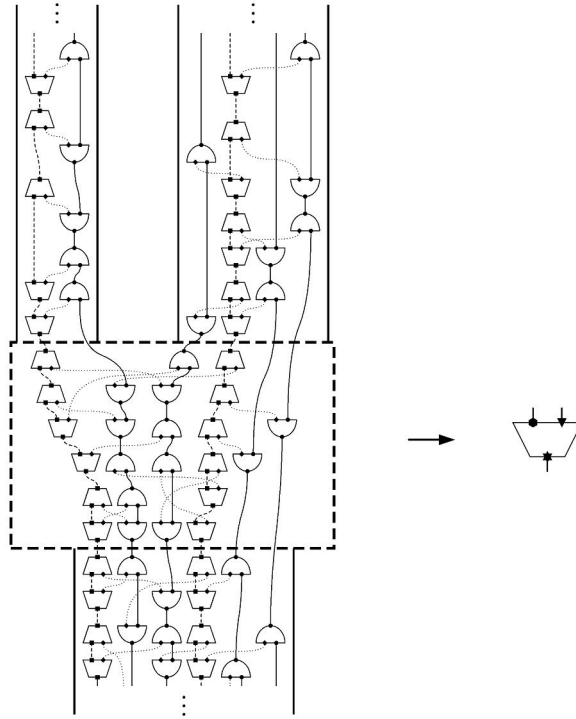


Figure 5: A transformation (left) and the corresponding next-stage primitive (right). An illustration of a transformation corresponding to a (highly stylized) *formation of a lithium hydride molecule* (terminal process) from hydrogen (left) and lithium (right) initial processes. The four primitives deal with the QED only and represent emission/absorption of a photon by electron (semi-circles) or nucleus (trapezoids). The body of the transform (heavy dashed line) depicts an imaginary restructuring of the two initial processes into the terminal one.

we are modifying one of such processes, we might be modifying the generating process itself (Postulate 2), which should then modify all the processes it generates.⁹

The observation that, in contrast to much larger objects, for example, all electrons are ‘identical’ finds now a very natural explanation: the electron processes are much closer to, if not the, stage 0 processes, as compared to the much larger processes, whose larger structs allow for a *much greater variation within the same class*.

Moving on to the *conventional concept of time*, we see that this concept is simply obviated, since ETS representation, *the struct*, embodies a more general, *structural*, form of temporality. Finally, a critical role of the concept of the shortest path in physics can possibly be explained by the role which ETS places on the formative histories.

4. Conclusion

I view the *limitations of physics* in light of the following, quite natural, scientific philosophy. When we observe that the existing *fundamental* formal structures become less effective—than we expected them to be—in capturing new phenomena, instead of trying to obfuscate these *formal* structures, by

⁹ Compare with the view expressed by John Bell: “For me, it is *so reasonable to assume* that the photons in those experiments carry with them programs that have been correlated in advance, telling them how to behave.” [18, p. 84, my emphasis]

‘hybridizing’ or ‘fuzzifying’ them¹⁰, we should be seeking *fundamentally* new formal schemes and structures that would be *more effective*. One should always remember that the quest of science is the quest for ‘meaning’, and the best scientific way we have found to get at the meaning is via the fundamental formal structures¹¹ within the chosen *representational formalism*, which has been fixed so far. However, there is no guarantee that we may never need to change *the underlying representational formalism itself*. Following this wisdom and having understood the *inherent* limitations of the numeric representations ([6], [19]), we proposed the ETS formalism for structural representation. The next stage is to begin its elaboration and examination in various applications.

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¹⁰ This has been an increasing trend during the last hundred years, including much greater emphasis on probabilistic models in physics and artificial intelligence, which is to me another tell-tail sign that we are reaching the useful limits of the numeric representation.

¹¹ Incidentally, statistics (in contrast to mathematics) is not concerned with the development of such structures.

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