Undecidability as the Framework for Quantum Theory and Spacetime[1]

Baruch Garcia[2][3]

The idea of a correspondence between quantum theory and undecidability has been around for nearly half a century, starting with John Archibald Wheeler. For decades, Wheeler filled up thousands of pages of notebooks searching for a connection. Now other physicists and even organizations such as FQXi are following suit. For Wheeler, this connection was key to understanding the foundations of physics. [Whe98] [Whe89] [Whe11][Gef14]

Parallels

There are several similarities between undecidability and quantum theory. The parallels briefly mentioned here will be investigated more thoroughly throughout this paper.

Parallel 1: Both have a self-referential nature.[4]

Parallel 2: Both provide epistemological limits.

Parallel 3: Both have been used in philosophy and misused in pseudoscience, yet have been rigorously proven and applied in various parts of math, physics, and computer science [Poo14][Sip13].

Parallel 4: Both belong to the foundations of their respective fields.

Parallel 5: Both have a quasi-paradoxical nature.

Parallel 6: Both have dualities - the particle-wave duality and the computable-consistent (or complete-consistent) duality.

Parallel 7: Both describe the topology of 4-dimensional spacetime at the Planck scale.

Parallel 8: Both have been used by physicists as limitations on what a unified (final) theory of physics can look like.

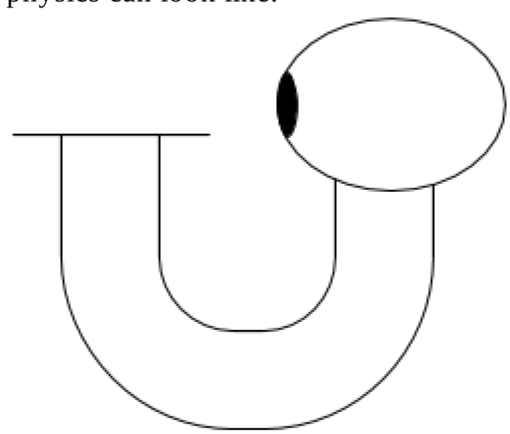

Figure 1: John Wheeler's "U-diagram" of the universe coming into being through quantum mechanics. Here, we see the observer as part of the quantum mechanical system. The eye is the observer or measuring device registering the quantum phenomena which make up the observer in turn. This diagram has been recently discussed within the scientific community by physicists such as Witten and Maldacena. Much like the proofs for undecidability, this picture of quantum mechanics relies on the concept of self-reference. [Whe98][Whe11b][Wol17][Mal15]

We have a tried and tested set of mathematics at our disposal which can account for the unintuitive nature of quantum mechanics. This can introduce new vistas and applications to unresolved problems in fundamental physics.

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[2] baruchgarcia@utexas.edu

[3] The author thanks S. Aaronson, T. Banks, R. Geroch, T. Jacobson, L. Smolin, and others for helpful conversations.

[4] Self-reference should be distinguished from solipsism; there is nothing solipsistic about the proofs for undecidability.

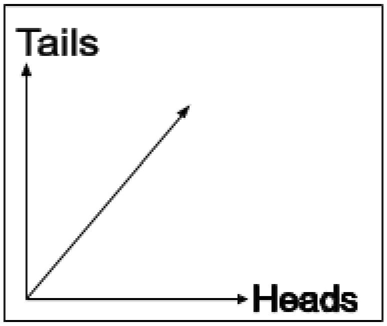

Figure 2a: Simplified - heads & tails - version of quantum mechanics [Wei86].

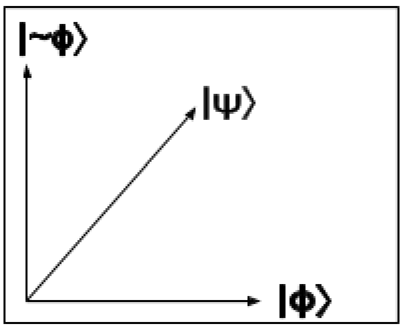

Figure 2b: Simplified version of quantum mechanics with labels. The wave function |ψ⟩ eigenstate |ϕ⟩ and orthogonal eigenstate |~ϕ⟩. Mathematically, this is a two-dimensional positive real-valued Hilbert space.

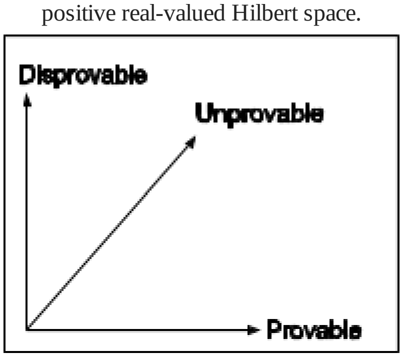

Figure 2c: Outline of correspondence between undecidability and quantum mechanical Hilbert space.

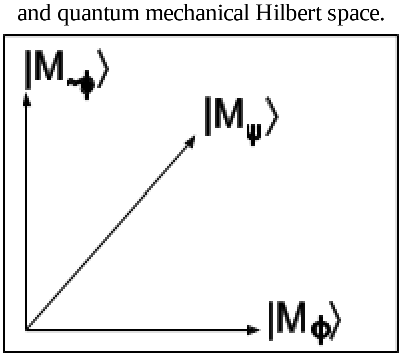

Figure 2d: Specific example using undecidability of the equivalence problem (EQ_{TM}) which asks $M_\psi \stackrel{?}{=} M_\phi$.

The Correspondence in a Nutshell

First, we will look at the basics of quantum mechanics (Figures 2a & 2b), then we will see the general idea of how undecidability provides a logical framework (Figure 2c) and look at a specific example (Figure 2d). Then we will look at an application of this new framework to quantum gravity. We will see how this resolves the “causal diamond lemma”, which tells us that the geometry of spacetime itself is subject to undecidability. Finally, we will address a common argument which pits undecidability against the existence of a unified theory of physics.

Quantum

Let us look at a simplified version of quantum mechanics adapted from Weinberg [Wei 86] in which we consider flipping a coin. We have two orthogonal (distinct) states - heads and tails. These states can be any two physically distinct states - here and there, up and down, clockwise and counterclockwise, etc. The arrow in the middle is called the state vector or wave function. When we measure the wave function, it collapses to either heads or tails randomly, with a probability given by a rule called the Born rule. However when we do not measure the wavefunction it is in a superposition of heads and tails - it is *both* heads and tails. Some like to compare a superposition to a spinning coin in mid-air, which works on a superficial level, but actual quantum mechanical superposition has no classical (non-quantum) counterpart [Dir30]. One cannot measure a superposition, by definition. If some state *is* measured, it just gets labelled as an orthogonal (distinct) state. The arrow can rotate through the graph deterministically, or stay stationary (also deterministically), as long as it has a length of 1. This is a special case of what we call unitarity.

Let us call the wavefunction $|\psi\rangle$, the heads "eigenstate" $|\phi\rangle$, and the tails "orthogonal eigenstate" $|{\sim}\phi\rangle$. This special state space is called a Hilbert space. So either $|\psi\rangle$ collapses to state $|\phi\rangle$ or $|{\sim}\phi\rangle$ nondeterministically when measured, or $|\psi\rangle$ is in superposition between $|\phi\rangle$ and $|{\sim}\phi\rangle$, rotating (or stationary), deterministically when not measured. The two process by which $|\psi\rangle$ can change were called Process 1 and Process 2 by von Neumann [von32]:

Process 1: **Measurable/Not Deterministic**. The results follow a probability distribution dictated by the Born rule. Also known as the collapse of the wave function or the particle description of quantum mechanics.
Process 2: **Deterministic/Not Measurable.** The Schrödinger equation (or path integral) is used to calculate the evolution of the wave function. Also known as superposition or the wave description of quantum mechanics.

Box 1: The duality of quantum theory.

If quantum mechanics describes the physics of everything, then is the world deterministic or non-deterministic? Is it unitary or nonunitary? Is it continuous or discontinuous? This, ever so briefly explained, is what some call the "measurement problem" or "measurement paradox", or even "particle-wave" (or more commonly "wave-particle") duality.[5]

Let's say instead of flipping the coin, we just lay it on the table and cover it then uncover it. We will know that the coin is heads 100% of the time or tails 100% of the time. Technically, this is called measuring a quantum system in an eigenstate of the observable chosen. One example is shining a vertically polarized photon through a vertical polarizer with the same exact orientation. This is a kind of trivial case.

We also have what is called entanglement, if we give two people - Alice and Bob- coins, and Alice and Bob flip their coins, we get a random outcome, yet they are always either correlated or anticorrelated.

Instead of coins, we deal with polarization of photons or the spin of electrons, etc. This correlation cannot be explained by any classical means or classical logic. Many have tried to find ways around this non-intuitive result, but without success. Bell's theorem was the first and most famous theorem to confirm the correctness of the quantum picture.

Undecidability

Undecidability tells us that for certain problems, generalized algorithms do not exist[von53]. For a generalized theorem to exist, every output would have to be computable *and* consistent. While Gödel did not use the term "computable" in his original 1931 paper, he did clarify later that his terms

[5] It is important to note that measurement provides the logical framework for decoherence, not vice versa. Physicists do not guess or divine the Lindblad equation, they look at measurements to define it[Sch03].

of “complete” and “provable” meant a mechanical proof, by a machine, following the work of Turing [Davis77]. Like the imperfect, but useful example of the coin, let us begin with a diagonalization argument for Grelling’s Paradox. If we define the word “heterological” to mean “not describing itself”, then “French” is heterological because it is written in English and “polysyllabic” is not heterological because it does have many syllables. The question then if “heterological” is heterological gives no straight answer. If it is, then it isn’t and if it isn’t then it is.

	“written”	“poly-syllabic”	“French”	“hetero-logical”
written	**<u>yes</u>**	yes	yes	yes
poly-syllabic	yes	**<u>yes</u>**	no	yes
French	no	no	**<u>no</u>**	no
hetero-logical	**<u>no</u>**	**<u>no</u>**	**<u>yes</u>**	**<u>?</u>**

Table 1: Grelling’s paradox in diagonal form.

$$? \begin{cases} Yes/No \\ Blank\ Space \end{cases}$$

Let us look at a diagonalization proof for the acceptance problem (A_{TM}) adapted from Sipser [Sip13]. M stands for some Universal Turing Machine (UTM) and <M> stands for the description, encoding, or Gödel number, of the machine. This is equivalent to writing quotation marks around M (see technical endnote 2 for more detail). UTM D asks if some UTM M accepts its own description (e.g. a compiler for the language Python written in Python). If M does accept its own description, then D rejects <M>. Conversely, if M rejects its own description, then D accepts <M>. In other words D computes the opposite of the diagonal entries. What happens if we run D on itself? If D accepts <D>, then D rejects <D>, and if D rejects <D>, then D accepts <D>. Therefore no generalized algorithm can exist, because <D> cannot both be computed by D and be consistent; the acceptance problem is undecidable.[6]

	$<M_1>$	$<M_2>$	$<M_3>$	$<D>$
M_1	**<u>Accept</u>**	Accept	Accept	Accept
M_2	Accept	**<u>Accept</u>**	Reject	Accept
M_3	Reject	Reject	**<u>Reject</u>**	Reject
D	**<u>Reject</u>**	**<u>Reject</u>**	**<u>Accept</u>**	**<u>?</u>**

Table 2: The acceptance problem A_{TM} adapted from Sipser [Sip12]

$$? \begin{cases} Accept/Reject \\ Blank\ Space \end{cases}$$

Rice’s theorem tells us that this notion of undecidability - the non-existence of generalized algorithms - applies to *every* non-trivial semantic (i.e. functional) property. Semantic means a property of the entire program. Trivial property means you will always get either S or ~S. This is the same as restricting row D of table 2 to just accepting or just rejecting, so there is no need for a question mark. In other words computations are completely optional, trivial; the outcome is already decided. Non-trivial means the computations are necessary.

Let us look at the two processes, borrowing terminology from von Neumann, where the question mark in the diagonal argument either

[6] Gödel’s theorem can be derived from these arguments [Sip13][Aar13].

becomes two contradictory entries or is left blank.

Process 1: Computable/Inconsistent: "Computable" means proven by a physical, mechanical process done by a machine, or an algorithm carried out by a human with ink and paper following an algorithm. The diagonal entry is filled in by contradictory statements. We can both *prove and disprove* a statement simultaneously; it is inconsistent.

Process 2: Consistent/Uncomputable: No contradictions occur, however we cannot run D on itself. The diagonal entry is left blank. It is *unprovable* by a physical, mechanical process; it is uncomputable.

Box 2: The duality of undecidability. It may be more common to hear how consistency and completeness are incompatible. I have chosen the word "computable" since the word "complete" usually refers to an entire formal system, and since "computable" emphasizes physical proof.

An oracle Turing machine can tell us whether a UTM has some property S or ~S. The oracle $M^{A_{TM}}$ decides the acceptance problem A_{TM}. With an oracle, we just move the row which negates the diagonal entries down by one, which allows D to either accept or reject its own description, making it decidable. The oracle cannot say whether it will accept its own description.

	$<M_3>$	$<D>$	$<M^{A_{TM}}>$
M_3	**Reject**	Reject	Accept
D	Accept	**Accept/ Reject**	Accept
$M^{A_{TM}}$	**Accept**	**Reject/ Accept**	**?**

Table 3: An oracle can decide whether D accepts or rejects its own description, but the question of whether the oracle accepts its own description is undecidable.

Correspondence with Quantum Theory

Process 1 of quantum mechanics is a physical measurement, yet the outcome is inconsistent. For example, if an electron is prepared in the same initial state, sometimes it is measured in the spin-up state, and sometimes it is measured in the spin-down state. In other words, we are proving that the electron is in the spin-up state then disproving that the electron is in the spin-up state.

This is *fundamentally* different from classical indeterminism. Diaconis et al. have made a coin-flipping machine which uses precisely the same initial conditions. We do not have an uncoordinated human flipper flipping each time. Note that we cannot make a quantum analog of the coin-flipping machine which can predict each quantum outcome.

With classical probability, the initial conditions are different, e.g. tossing a coin at different angles, leading to different final conditions. With quantum probability, the initial conditions are exactly the same but lead nevertheless to different final conditions. This is what is meant by saying that quantum measurements are "inconsistent".[7]

A mathematical proof or mechanical computation - which are equal, as described by the Church-Turing thesis - can be seen as writing ink symbols on a piece of paper (either by a person or a Turing machine).

[7] Does this mean quantum mechanics is an inconsistent theory? No! If the probabilistic distributions were inconsistent with each other, then we would rule quantum mechanics out as a scientific theory. If one experimenter measures a 50/50 distribution of spin-up and spin-down, then any other experimenter will measure the same (consistent) probability distribution. "Inconsistency" refers to the inconsistency of individual measurements given the same initial conditions.

Likewise an observation of a quantum phenomenon may be seen as a crystal of silver bromide darkening on a screen. In both cases we have a tangible physical change informing us of a new piece of information.

So if "computable" refers to Process 1, then "consistent" must refer to Process 2. Process 2 is a deterministic evolution. The Schrödinger equation evolves precisely the same way each time given the same initial conditions. Yet we cannot directly measure the Schrödinger equation. Why? Because ψ is evolving through various superpositions, and superpositions are, in principle, not measurable.

What about the infinite dimensional case of the Dirac delta with a continuous spectrum of eigenvalues? Easy! Choose some eigenvector to correspond with provability then every orthogonal eigenvector corresponds with disprovability.

Okay, but what about when we measure a system in an eigenstate of an observable chosen? There is no uncertainty there. In undecidability, this corresponds to trivial properties of Rice's theorem, where the outcome is known ahead of time.

In fact it is these trivial properties which allow for the computation in quantum computation. Quantum computers make use of results such as interference patterns which are the same every time.

Oracles provide a framework for entanglement, as we will see.

The Equivalence Problem

Let us look at a concrete example of undecidability called the equivalence problem and how it relates to quantum theory (see Figure 2d). The Acceptance Problem reduces to the Emptiness Problem, which asks whether a Turing machine is the empty set $M \stackrel{?}{=} \varnothing$ [Sip13].The Emptiness problem reduces to the Equivalence Problem which asks whether two Turing machines are equal. $M_1 \stackrel{?}{=} M_2$ by setting M_2 to either the empty language or a non-empty language, then asking $M_1 \stackrel{?}{=} \varnothing$. Now let us change the subscripts.

$$M_\psi \stackrel{?}{=} M_\phi$$

Either it is computable and inconsistent or consistent but uncomputable. A measurement of a system measured in an eigenstate of the observable chosen just means M_ψ is some trivial property of an empty or non-empty language

Now let's look at an oracle for the equivalence problem.

$$M_B \stackrel{?}{=} M_A \quad \text{(decidable)}$$

We can say whether M_A and M_B are equal, but then we cannot say whether M_C is equal to the other Turing machines. M_C is the oracle.

$$M_C \stackrel{?}{=} M_B \text{ (undecidable)}$$

Now let's relabel M_B and M_A so they represent eigenvectors of Bob and Alice, and let M_C represent a wave function ψ.

$$M_{0\,(Bob)} \stackrel{?}{=} M_{0\,(Alice)} \text{ (Decidable)}$$

$$M_{\psi} \stackrel{?}{=} M_{0\,(Bob)} \text{ (Undecidable)}$$

The question of whether Alice observing 0 and Bob observing 0 is equivalent is decidable. If $M_{0\,(Alice)} = M_{0\,(Bob)}$, we have correlation, and if $M_{0\,(Alice)} \neq M_{0\,(Bob)}$, we have anti-correlation. The oracle for the equivalence problem, $M_{\psi} \stackrel{?}{=} M_{0\,(Alice)}$, is therefore undecidable. Once the undecidable oracle M_{ψ} is added into the equation, we go from dealing with classical bits to qubits.[8]

Spacetime

A donut is topologically equivalent or "homeomorphic" to a coffee mug, because you can morph one into another without cutting or gluing. Perelman proved this equivalence problem for topology is decidable up to 3 dimensions.

A.A. Markov Jr. proved that there is no generalized algorithm which tells us whether two compact 4-dimensional manifolds are equivalent [Mar58]. We say that the topological equivalence problem or homeomorphism problem is undecidable. Markov also showed that this was a consequence of Rice's Theorem, so like Rice's theorem, we have trivial topologies [Zom05].

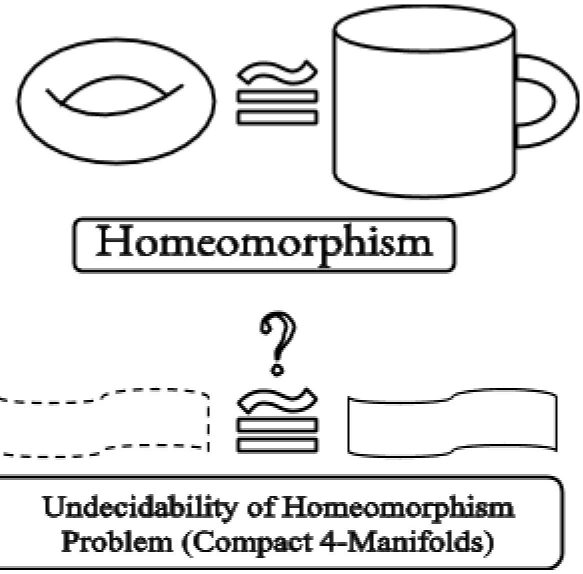

Figure 3: The undecidability of the homeomorphism (topological equivalence) problem.

At first, this does not seem to apply to spacetime, because many times, we consider spacetime to be a non-compact manifold without a boundary, extending to infinity. Even if spacetime was compact, it would be conceivably very difficult to empirically test this, especially if part of our universe is outside the observable cosmological horizon. But then we have to consider causal diamonds, the patches that make up the quilt we call spacetime.

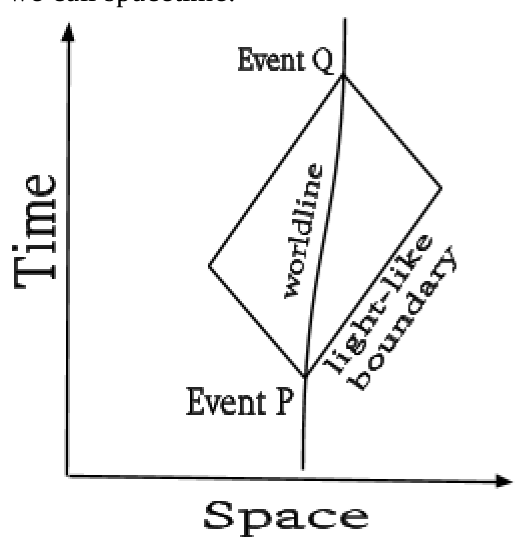

Figure 4: A causal diamond/compact 4-manifold.

[8] Although these models only deal with a positive real-valued Hilbert space, they suffice as a logical structure for quantum theory. For more on the full quantum formalism, see Technical Endnote 1.

There is a well known theorem in Lorentzian geometry, which says that a metric is completely determined in terms of properties of its collection of causal diamonds.
- Tom Banks [Ban18]

Quantum mechanics is fundamentally about one subsystem of nature probing the rest. The most elementary act of observation a subsystem of the universe can make is to send a probe out into the world at one event and receive a response back at a future event. The act of probing the world is represented by a causal diamond, making them primary structures. -Lee Smolin [Smo18]

Causal diamonds are determined by two events on a worldline, P and Q. The future light-cone of P intersects with the past light-cone of Q to make a causal diamond. Anything within that diamond can be affected by P and can affect Q. The boundaries (a.k.a. "null" or "light-like" boundaries) represent the speed of light, and these boundaries are what make causal diamonds compact. As compact 4-manifolds, causal diamonds are subject to the undecidability of the homeomorphism problem.

> *Causal Diamond Lemma (of the Homeomorphism Problem)*:
>
> Given suitable causality constraints (global hyperbolicity), if we do not restrict the topology of the causal diamond $\mathscr{D}$ by hand then there is no generalized algorithm which tells us whether the causal diamond $\mathscr{D}$ is homeomorphic or not homeomorphic to some other compact 4-manifold $\mathscr{M}$. [Gar20]

The only way around this lemma is to artificially restrict the topology by hand. The topology of spacetime is not specified by general relativity, since the Einstein equations are local, and topology is global. Any restriction of the topology seems to be given by physicists, and not by nature.

There is a cosmic censorship theorem by Friedman, Schleich and Witt which tells us that exotic topologies cannot be probed without collapsing into a singularity. However, because energy conditions which are crucial to the theorem can be violated at the Planck scale (Heisenberg's uncertainty principle), these topological fluctuations are possible only at the Planck scale.

And note that while a change in the metric does not necessarily imply a change in topology - a donut and coffee mug have a different metric but same topology - a change in topology does imply a change in the metric; we also have metric fluctuations.

What if one tries to get around the causal diamond lemma by considering a region of spacetime outside all compact causal diamonds? Then the region of spacetime is not observable, by definition, because information cannot be transmitted faster than the speed of light.

Penrose, Geroch, Hartle have all suggested a connection between quantum gravity and undecidability[Ger86][Pen89], and here we finally have a concrete example that does not depend on the overall topology of the universe.

How do we explain this undecidability of spacetime at the Planck scale? By topological fluctuations in spacetime we call quantum foam![9]

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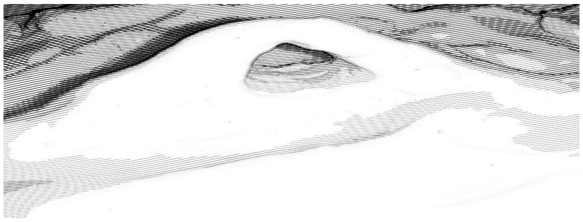
[9] While cumulative LIV (Lorentz-Invariance Violation) models of foam have been experimentally excluded, non-cumulative LIV models such as Wheeler's / this current model have not. [Per14]

(I)



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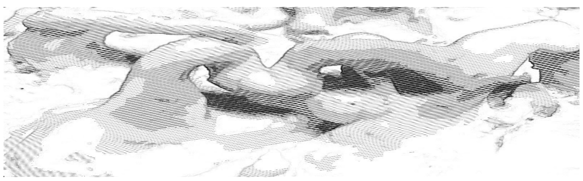

Figure 5: Quantum foam or spacetime foam. Spacetime topological fluctuations at $10^{-35}m$. Example I is not homeomorphic to example II. This illustration is adapted from Thorne [Tho94]. There is no left and right, before and after at this scale. The very concepts of space and time lose meaning [Whe98].

We might ask whether a causal diamond $\mathscr{D}$ is homeomorphic to example II, which has a single wormhole. This is undecidable.

While some may say quantum foam is just a hypothesis, we cannot say the same thing about the causal diamond lemma. And while the causal diamond lemma is obviously not a full-fledged quantum theory of gravity, it does demonstrate how undecidability provides a framework for spacetime which must be taken into account in a successful quantum theory of gravity.[10]

[10] There may be no generalized algorithm which tells us whether $\mathscr{M}$ and $\mathscr{N}$ are homeomorphic, but if we attach them, giving us the connected sum $\mathscr{M}\#\mathscr{N}$, then we can say the new manifold $\mathscr{M}\#\mathscr{N}$, is homeomorphic to itself, but there is no generalized algorithm that can say if it is homeomorphic to $\mathscr{L}$. This acts like a special case of an oracle for the homeomorphism problem. Since oracles give a framework for entanglement, this leads us to the conclusion that topology and entanglement are connected, a proposition which is being studied by Susskind, Maldacena and others.

Reconciling Undecidability and Unification through Quantum Theory

…[W]e are not angels, who view the universe from the outside. Instead, we and our models are both part of the universe we are describing. Thus a physical theory is self referencing, like in Gödel’s theorem. One might therefore expect it to be either inconsistent, or incomplete.
- Stephen Hawking [Haw03]

Little astonishment there should be... if the description of nature carries one in the end to logic the ethereal eyrie at the center of mathematics. If, as one believes, all mathematics reduces to the mathematics of logic, and all physics reduces to mathematics, what alternative is there but for all physics to reduce to the mathematics of logic? Logic is the only branch of mathematics that can "think about itself".... "An issue of logic having nothing to do with physics" is one's natural first assessment of the starling limitation on logic discovered by Gödel. -John A. Wheeler [Mis73]

There is an epistemological/linguistic argument used by some physicists which states that undecidability and the existence of a fundamental theory are incompatible. Yet, now that we know there is a connection between undecidability and quantum theory, and that a fundamental theory should be quantum [Wei93], the conflict is no more. Quantum theory - this generalized quantum theory which includes Planck scale fluctuations - bridges the gap between undecidability and unification. In future papers[11], we will discuss quantum interpretations, quantum cosmology, and quantum biology. At the foundation of all of these is the simple concept of self-reference.[12]

Gödel's insight might be the key thing. To me, quantum theory is the great mystery that we will someday unravel and understand "How come?" And the answer to that question will at the same time be the answer to the question "How come existence?" I can't believe that they are separate questions.
- John Archibald Wheeler [Whe96][Whe11]

[11] To be posted in jawarchive.wordpress.com.

[12] See Wheeler’s U-diagram (Figure 1).

Technical Endnote 1: Quantum Formalism

Peculiarities which separate the quantum formalism from the classical formalism imply a duality which is unique to quantum theory.

$$\textit{Quantum Formalism} \Rightarrow \textit{Quantum Duality}$$

To the right we have a table deciphering how various aspects of the quantum formalism encode this duality, with some accompanying notes below.

• Born Rule

We know from basic trigonometry, that an infinite sine wave whose domain extends from $-\infty$ to ∞ can be encoded in a unit circle, where the y or x coordinate is plotted against θ. The equation for a unit circle can be extended to a unit sphere $x^2+y^2+z^2=1$ and so on, to any number of dimensions, so we are not limited by the dimensionality of Hilbert Space. Taking the square of the absolute value of a complex number a+bi gives us a^2+b^2 which just doubles the dimensionality from an N-sphere to a 2N-sphere.

• Complex Numbers

There is the trivial case of $\mathbb{R}^2$(e.g. linear polarization) which can be enough for quantum computation. Non-trivial cases (e.g. circular, elliptical polarization) require complex numbers.

Since the quantum duality has been shown to correspond with undecidability, we can see how undecidability provides a framework for the peculiarities of the quantum formalism.

Name	**Example**	**Particle (Process 1)**	**Wave (Process 2)**
Planck's Constant (set h=1)	E=hv p=hk	Energy or momentum of particle	Frequency or inverse wavelength of wave
Heisenberg Uncertainty Principle	$[\hat{x},\hat{p}] = i\hbar$	$\widehat{xp} \neq \widehat{px}$ (inconsistent particle trajectories)	ψ (consistent evolution)
Born Rule Example for $\mathbb{R}^2$ e.g. linear polarization. Can be generalized to $\mathbb{C}^N$	Probability of ϕ = |Amplitude of $\phi\|^2$	$P_x + P_y = 1$ (probability of particle measured in state)	$x^2+y^2=1$ (rotation around unit circle)
Complex Amplitudes	$\mathbb{C}^N$	N dimensional Hilbert space (number of possible state for particle)	Multiplying by complex number rotates vectors in complex plane.
Hermitian and Unitary Operators	$\hat{U} = e^{i\hat{H}}$	Hermitian operators have orthogonal eigenvectors (possible states for particles)	Unitary Operators rotate a state vector through Hilbert space.
Linearity/ Additivity (Scaling without additivity yields trivial global phase shifts.)	L(u+v) = Lu + Lv L is a linear operator, u and v are vectors.	u or v (possible states for a particle)	u+v (super-position)
(Dis) Continuity	Collapse (eg unit step function) & Evolution (e.g. sine wave)	If x=0 ± 0.0...01 Then y = 0 or 1 (not consistent)	If x=0 ± 0.0...01 Then y = 0 ± 0.0...01 (consistent)
Density Matrix	ρ	Diagonals (Trace)	Off-Diagonals

Technical Endnotes 2 (Undecidability): Name vs Object and Self Reference

Boston is populous.
"Boston" is disyllabic.
-WVO Quine, Mathematical Logic [Qui40]

In mathematical logic, we must distinguish between the object being discussed vs the name of the object[13]. Some new to the subject may see this as philosophical pedantry[14], but it is the keystone of the proofs for undecidability[15] [Qui87][Qui40].

Turing and Gödel defined proof or computation to means provable by mechanical means [Dav77]. By "mechanical" we mean some structured physical device. The structure follows some set of rules, but there must also be a physical component. This may also refer to a human "computer" following an algorithm and physically recording everything; ink on paper or chalk on a chalkboard. In other words, proofs demand numerals, not just numbers, names, not objects. Proof requires naming.

True "Snow is white." ≡ Snow is white.

Tarski's disquotational T-schema. [Tar33][Qui92].

Truth is defined as disquotation. This is the opposite of when we put untrue sentences in scare quotes. Take for example:

Al is drinking tea. Al is drinking "tea".

The two sentences are inconsistent with each other. In the second sentence, it is implied Alfred is drinking something distinct from tea (perhaps beer). Disquotation is the removal of inconsistency. Consistency does not always imply truth, but truth implies consistency.

Undecidability proofs require self-reference[16]. The decidability of the reals is precisely because we cannot replicate the same self-referential argument. Undecidable languages can be decided by oracles. But when you self-apply the oracles, they become undecidable themselves. Or take Zeno Turing machines which do an infinite number of steps in a finite time, by letting the first step take ½ a second, the next step ¼, the third, ⅛, etc. This decides the halting problem. Yet when you apply the Zeno machine to itself, we get undecidability[Pot08]. So it is not so important whether we discuss the halting problem, an oracle machine, or a Zeno machine. There is nothing intrinsic to their rules and axioms which tell us whether or not they are undecidable. What is important is whether those formal systems/rules can be self-referential and *are* self-referential. If a statement proves itself, proof requires naming ie. quotations. Consistency requires disquotation. The two are incompatible.

Measurement requires a physical change in a detection screen or geiger counter. Naming requires a physical change like ink on paper, chalk on a chalkboard, or even a stylus impression on clay. Measurement is naming. Self-reference/ self-naming is the keystone of the quantum-undecidability correspondence.

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[13] If one is uncomfortable using quotation marks to distinguish between an object and its name, as some philosophers of language are, one may just utilize an overbar in the place of quotation symbols.
[14] Telling a program to print "x and y" vs "x" and "y", gets you two very different results.
[15] Gödel numbering refers to the names of the symbols, i.e. the symbols in quotation marks [Göd31]. Likewise the description of UTM D written as <D> can simply be seen as its Gödel number. <D> is analogous to "D" where the brackets act as quotation symbols.

[16] As opposed to independence from axioms e.g the CH.

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