Meaning of the Wave Function

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Abstract

According to the standard probability interpretation, the wave function is a probability amplitude, and its modulus square gives the probability density of finding particles in certain positions in space. In this essay, we show that this central assumption of quantum mechanics may have an ontological extension. It is argued that microscopic particles such as electrons are indeed particles, but their motion is not continuous but discontinuous and random. On this view, the modulus square of the wave function not only gives the probability density of the particles being *found* in certain locations, but also gives the probability density of the particles *being* there. In other words, the wave function can be regarded as a representation of the state of random discontinuous motion of particles, and at a deeper level, it may represent the dispositional property of the particles that determines their random discontinuous motion.

The wavefunction gives not the density of stuff, but gives rather (on squaring its modulus) the density of probability. Probability of what, exactly? Not of the electron being there, but of the electron being found there, if its position is 'measured'. Why this aversion to 'being' and insistence on 'finding'? The founding fathers were unable to form a clear picture of things on the remote atomic scale. (Bell 1990)

1 Introduction

The physical meaning of the wave function is an important interpretative problem of quantum mechanics. The standard assumption is that the wave function is a probability amplitude, and its modulus square gives the probability density of finding particles in certain locations at a given instant. This is usually called the probability interpretation of the wave function. Notwithstanding its great success, the probability interpretation is not wholly satisfactory because of resorting to the vague concept of measurement (see, e.g. Bell 1990).

Recently a new penetrating analysis shows that the wave function not only gives the probability of getting different outcomes, but also may offer a faithful

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representation of reality (Pusey, Barrett and Rudolph 2012). This analysis confirms the earlier result obtained based on protective measurements (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993), and shows that the standard assumption is ripe for rethinking. In fact, the realistic view of the wave function is already a common assumption in the main alternatives to quantum mechanics such as the de Broglie-Bohm theory and the many-worlds interpretation. Unfortunately, however, the precise meaning of the wave function is still an unresolved issue in these theories.

What, then, does the wave function truly represent? In this essay, we will try to answer this fundamental question through a new analysis of protective measurements and the mass and charge distributions of a quantum system[1]. The answer may help to understand the deep nature of quantum reality.

2 Measuring the state of a quantum system

The meaning of the wave function is often analyzed in the context of conventional (impulse) measurements, for which the coupling interaction between the measured system and the measuring device is of short duration and strong. As a result, even though the wave function of a quantum system is in general extended over space, an ideal position measurement can only detect the system in a random position in space[2]. Then it is unsurprising that the wave function is assumed to be related to the probability of the random measurement result by the standard probability interpretation. This also indicates that conventional measurements cannot obtain enough information about a single quantum system to determine what physical state its wave function represents.

Fortunately, it has been known that the physical state of a single quantum system can be protectively measured (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2009)[3]. A general method is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction (in some situations the protection is provided by the measured system itself), and then make the measurement adiabatically so that the state of the system neither collapses nor becomes entangled with the measuring device appreciably. In general, the measured state needs to be known beforehand in order to arrange a proper protection. In this way, such protective measurements can measure the expectation values of observables on a single quantum system, and in particular, the mass and charge distributions of a quantum system as one part of its physical state, as well as its wave function, can be measured as expectation values of certain observables. Since the principle of protective measurement is independent of the controversial collapse postulate and only based on the linear Schrödinger evolution (for microscopic systems such as electrons) and the

[1]For a more detailed analysis see Gao (2011a, 2011b, 2011c).

[2]In this essay we only consider the spatial wave functions of quantum systems.

[3]Note that the earlier objections to the validity and meaning of protective measurements have been answered (Aharonov, Anandan and Vaidman 1996; Dass and Qureshi 1999). Uffink's (1999) objection seems to be the unique exception. Although Vaidman (2009) regarded this objection as a misunderstanding, he gave no concrete rebuttal. Recently we have argued in detail that Uffink's objection is invalid due to several errors in his arguments (Gao 2011d).

Born rule[4], which are two established parts of quantum mechanics, its result as predicted by quantum mechanics can be used to investigate the meaning of the wave function[5].

According to protective measurement, the charge of a charged quantum system such as an electron is distributed throughout space, and the charge density in each position is proportional to the modulus square of the wave function of the system there[6]. Historically, the charge density interpretation for electrons was originally suggested by Schrödinger when he introduced the wave function and founded wave mechanics (Schrödinger 1926). Schrödinger clearly realized that the charge density cannot be classical because his equation does not include the usual classical interaction between the densities. Presumably since people thought that the charge density could not be measured and also lacked a consistent physical picture, this initial interpretation of the wave function was soon rejected and replaced by Born's probability interpretation (Born 1926). Now protective measurement re-endows the charge distribution of an electron with reality by a more convincing argument. The question then is how to find a consistent physical explanation for it[7]. Our following analysis can be regarded as a further development of Schrödinger's original idea to some extent. The twist is: that the charge distribution is not classical does not imply its non-existence; rather, its existence may point to a new, non-classical picture of quantum reality hiding behind the mathematical wave function.

3 Electrons are particles

The key to unveil the meaning of the wave function is to find the physical origin of the charge distribution. The charge distribution of a quantum system such as an electron has two possible existent forms: it is either real or effective. The distribution is real means that it exists throughout space at the same time, e.g. there are different charges in different positions at any instant. The distribution is effective means that there is only a localized particle with the total charge of the system in one position at every instant, and the time average of its motion (during an infinitesimal time interval) forms the effective distribution in the whole space. Moreover, since the integral of the formed charge density in any region is required to be equal to the average value of the total charge in the region, the motion of the particle is ergodic.

These two existent forms of the charge distribution of a quantum system have different physical effects, and thus they can be distinguished. Experiments show that different charges in different positions at a given instant have electrostatic interaction, while a charge at one instant has no electrostatic interaction with the charge at another instant. Therefore, if the charge distribution is effective, then there will exist no electrostatic self-interaction of the effective distribution, while if the charge distribution is real, then there will exist electrostatic self-

[4]It is worth noting that the possible existence of very slow collapse of the wave function for microscopic systems does not influence the principle of protective measurement.

[5]It can be expected that protective measurements will be realized in the near future with the rapid development of quantum technologies (cf. Lundeen et al. 2011).

[6]See the Appendix for an introduction of this important result.

[7]The proponents of protective measurement did not analyze the origin of the charge distribution. According to them, this type of measurement implies that the wave function of a single quantum system is a real physical wave (Aharonov, Anandan and Vaidman 1993).

interaction of the real distribution. In short, the first form entails the existence of electrostatic self-interaction of the charge distribution of a quantum system, while the second form does not.

Since the existence of electrostatic self-interaction is inconsistent with the superposition principle of quantum mechanics, and especially, the existence of such electrostatic self-interaction for individual electrons already contradicts experimental observations (e.g. the results of the double-slit experiments with single electrons)[8], the charge distribution of a quantum system such as an electron must be effective. This means that at every instant there is only a localized particle with the total mass and charge of the system, and during an infinitesimal time interval the time average of the ergodic motion of the particle forms the effective mass and charge distributions of the system. In short, electrons are particles, and their charge distributions in space, which are measureable by protective measurements, are formed by the ergodic motion of these particles.

4 Particles move in a discontinuous and random way

The next question is which sort of ergodic motion the particles undergo. If the ergodic motion of a particle is continuous, then it can only form the effective mass and charge distributions during a finite time interval. But the effective mass and charge distributions of a quantum system at each instant, which is proportional to the modulus square of the wave function of the system at the instant, is required to be formed during an infinitesimal time interval near the instant. Thus it seems that the ergodic motion of the particle cannot be continuous.

We can also reach this conclusion by analyzing a concrete example. Consider an electron in a superposition of two energy eigenstates in two separated boxes $\psi_1(x) + \psi_2(x)$. In this example, even if one assumes that the electron as a localized particle can move with infinite velocity, it cannot continuously move from one box to another due to the restriction of box walls. Therefore, any sort of continuous motion cannot generate the effective charge density $e|\psi_1(x) + \psi_2(x)|^2$. One may object that this is merely an artifact of the idealization of infinite potential. However, even in this ideal situation, the model should also be able to generate the effective charge distribution by means of some sort of ergodic motion of the electron; otherwise it will be inconsistent with quantum mechanics[9].

On the other hand, if the motion of a particle is discontinuous, then the particle can readily move throughout all regions where the wave function is

[8] As another example, consider the electron in the hydrogen atom. Since the potential of the electrostatic self-interaction is of the same order as the Coulomb potential produced by the nucleus, the energy levels of hydrogen atoms will be remarkably different from those predicted by quantum mechanics and confirmed by experiments if there exists such electrostatic self-interaction for individual electrons. For a detailed analysis see Gao (2011c).

[9] It is very common in quantum optics experiments that a single-photon wave packet is split into two branches moving along two well separated paths in space. The wave function of the photon disappears outside the two paths for all practical purposes. Moreover, the experimental results are not influenced by the environment and setup between the two paths of the photon. Thus it is very difficult to imagine that the photon performs a continuous ergodic motion back and forth in the space between its two paths.

nonzero during an arbitrarily short time interval at a given instant. Furthermore, if the probability density of the particle appearing in each position is proportional to the modulus square of its wave function there at every instant, the discontinuous motion can also generate the right effective mass and charge distributions. This may solve the problems plagued by the classical ergodic models. The discontinuous ergodic motion requires no existence of a finite ergodic time. A particle undergoing discontinuous motion can also move from one region to another spatially separated region, no matter whether there is an infinite potential wall between them, and such discontinuous motion is not influenced by the environment and setup between these regions either.

In conclusion, we have argued that the mass and charge distributions of a quantum system such as an electron are formed by the discontinuous motion of a localized particle with the total mass and charge of the system, and the probability density of the particle appearing in each position is proportional to the modulus square of its wave function there.

5 Meaning of the wave function

According to the above analysis, microscopic particles such as electrons are indeed particles. Here the concept of particle is used in its usual sense. A particle is a small localized object with mass and charge, and it is only in one position in space at an instant. Moreover, the motion of these particles is not continuous but discontinuous in nature. We may say that an electron is a quantum particle in the sense that its motion is not continuous motion described by classical mechanics, but discontinuous motion described by quantum mechanics.

From a logical point of view, for the discontinuous motion of a quantum particle, there should exist a probabilistic instantaneous condition that determines the probability density of the particle appearing in every position in space, otherwise it would not "know" how frequently they should appear in each position in space. In other words, the particle should have an instantaneous property that determines its motion in a probabilistic way. This property is usually called indeterministic disposition or propensity in the literature[10]. As a result, the position of the particle at every instant is random, and its trajectory formed by the random position series is also discontinuous. In short, the motion of the particle is essentially discontinuous and random.

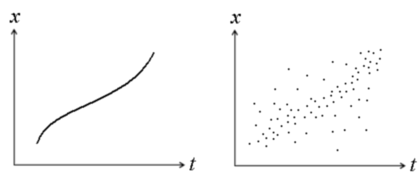

Figure 1. Continuous motion vs. discontinuous motion

Unlike the deterministic continuous motion, the trajectory function $x(t)$ can no longer provide a useful description for random discontinuous motion. For

[10]It is worth stressing that the propensities possessed by the particles relate to their objective motion, not to the measurements on them as in the existing propensity interpretations of quantum mechanics (cf. Suárez 2007).

a quantum particle, there is no continuous trajectory at all. Rather, the random discontinuous motion of the particle forms a particle "cloud" extending throughout space (in an infinitesimal time interval), and the state of motion of the particle is represented by the density and flux density of the cloud, denoted by $\rho(x,t)$ and $j(x,t)$, respectively. This is similar to the description of a classical fluid in hydrodynamics. But their physical meanings are different. The particle cloud is formed by the random discontinuous motion of a single particle, and the density of the cloud, $\rho(x,t)$, represents the objective probability density of the particle appearing in position x at instant t. By assuming that the nonrelativistic equation of motion is the Schrödinger equation in quantum mechanics[11], the complex wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for a constant phase factor):

$$\psi(x,t) = \sqrt{\rho(x,t)}e^{im\int_{-\infty}^{x}\frac{j(x',t)}{\rho(x',t)}dx'/\hbar}. \tag{1}$$

In this way, the wave function $\psi(x,t)$ also provides a complete description of the state of random discontinuous motion of a particle.

The description of the motion of a single particle can be extended to the motion of many particles. At each instant the quantum system of N particles can be represented by a point in a 3N-dimensional configuration space, and the motion of these particles forms a cloud in the configuration space. Then, similar to the single particle case, the state of the system is represented by the density and flux density of the cloud in the configuration space, $\rho(x_1, x_2, ...x_N, t)$ and $j(x_1, x_2, ...x_N, t)$, where the density $\rho(x_1, x_2, ...x_N, t)$ represents the probability density of particle 1 appearing in position x_1 and particle 2 appearing in position x_2, ..., and particle N appearing in position x_N. Since these two quantities are defined not in the real three-dimensional space, but in the 3N-dimensional configuration space, the many-particle wave function, which is composed of these two quantities, is also defined in the 3N-dimensional configuration space.

One important point needs to be stressed here. Since the wave function in quantum mechanics is defined at a given instant, not during an infinitesimal time interval, it should be regarded not simply as a description of the state of motion of particles, but more suitably as a description of the dispositional property of the particles that determines their random discontinuous motion at a deeper level[12]. In particular, the modulus square of the wave function determines the probability density of the particles appearing in certain positions in space. By contrast, the density and flux density of the particle cloud, which are defined during an infinitesimal time interval at a given instant, are only a description of the state of the resulting random discontinuous motion of particles, and they are determined by the wave function. In this sense, we may say that the motion of particles is "guided" by their wave function in a probabilistic way.

6 Conclusions

In this essay, we have argued that quantum mechanics may have already spelled out the meaning of the wave function. There are three main steps to reach this conclusion.

[11] For a derivation of the free Schrödinger equation see Gao (2011c).

[12] For a many-particle system in an entangled state, this dispositional property is possessed by the whole system.

First of all, protective measurement, whose principle is based on the established parts of quantum mechanics, shows that the charge of a charged quantum system such as an electron is distributed throughout space, and the charge density in each position is proportional to the modulus square of its wave function there. Next, the superposition principle of quantum mechanics requires that the charge distribution is effective, that is, it is formed by the ergodic motion of a localized particle with the total charge of the system. Lastly, the consistency of the formed distribution with that predicted by quantum mechanics requires that the ergodic motion of the particle is discontinuous, and the probability density of the particle appearing in every position is equal to the modulus square of its wave function there.

Therefore, quantum mechanics seems to imply that the wave function describes the state of random discontinuous motion of particles, and at a deeper level, it may represent the dispositional property of the particles that determines their random discontinuous motion. In particular, the modulus square of the wave function not only gives the probability density of the particles being *found* in certain locations as the standard probability interpretation assumes, but also gives the probability density of the particles *being* there. It will be interesting to see how this new interpretation of the wave function can be extended to quantum field theory and what it implies for the solutions to the measurement problem.

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Appendix: Protective measurement of the charge distribution of a charged quantum system

Since the existence of the charge distribution of a charged quantum system is the basis of our analysis of the meaning of the wave function, we will briefly illustrate this important result here. For a more detailed analysis see Aharonov and Vaidman (1993), Aharonov, Anandan and Vaidman (1993, 1996), and Gao (2011c).

Consider the spatial wave function of a single quantum system with negative charge Q (e.g. $Q = -e$):

$$\psi(x,t) = a\psi_1(x,t) + b\psi_2(x,t), \tag{2}$$

where $\psi_1(x,t)$ and $\psi_2(x,t)$ are two normalized wave functions respectively localized in their ground states in two small identical boxes 1 and 2, and $|a|^2+|b|^2 = 1$. An electron, which initial state is a small localized wave packet, is shot along a straight line near box 1 and perpendicular to the line of separation between the boxes. The electron is detected on a screen after passing by box 1. Suppose the separation between the boxes is large enough so that a charge Q in box 2 has no observable influence on the electron. Then if the system were in box 2, namely $|a|^2 = 0$, the trajectory of the electron wave packet would be a straight line as indicated by position "0" in Figure 2. By contrast, if the system were in box 1, namely $|a|^2 = 1$, the trajectory of the electron wave packet would be deviated by the electric field of the system by a maximum amount as indicated by position "Q" in Figure 2.

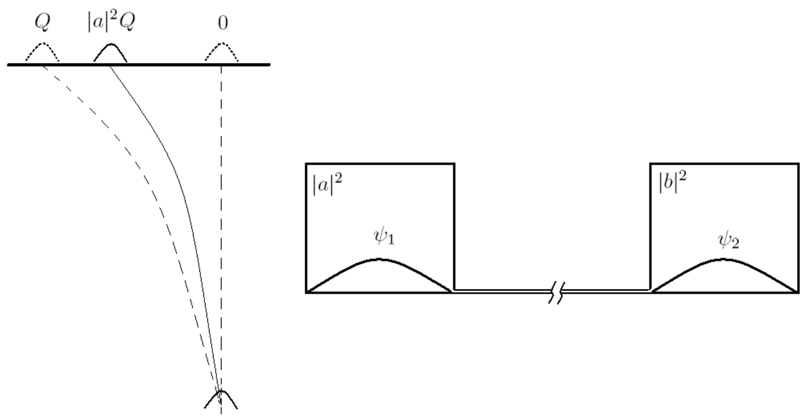

Figure 2. Scheme of a protective measurement of the charge density of a charged quantum system

We first suppose that $\psi(x,t)$ is unprotected, then the wave function of the combined system after interaction will be

$$\psi(x,x',t) = a\varphi_1(x',t)\psi_1(x,t) + b\varphi_2(x',t)\psi_2(x,t), \tag{3}$$

where $\varphi_1(x',t)$ and $\varphi_2(x',t)$ are the wave functions of the electron influenced by the electric fields of the system in box 1 and box 2, respectively, the trajectory of $\varphi_1(x',t)$ is deviated by a maximum amount, and the trajectory of $\varphi_2(x',t)$ is not deviated and still a straight line. When the electron is detected on the screen, the above wave function will collapse to $\varphi_1(x', t)\psi_1(x, t)$ or $\varphi_2(x', t)\psi_2(x, t)$. As a result, the detected position of the electron will be either "Q" or "0" on the screen, indicating that the system is in box 1 or 2 *after* the detection. This is a conventional impulse measurement of the projection operator on the spatial region of box 1, denoted by A_1. A_1 has two eigenstates corresponding to the system being in box 1 and 2, respectively, and the corresponding eigenvalues are 1 and 0, respectively. Since the measurement is accomplished through the electrostatic interaction between two charges, the measured observable A_1, when multiplied by the charge Q, is actually the observable for the charge of the system in box 1, and its eigenvalues are Q and 0, corresponding to the charge Q being in box 1 and 2, respectively. Such a measurement cannot tell us the charge distribution of the system in each box *before* the measurement.

Now let's make a protective measurement of A_1. Since $\psi(x, t)$ is degenerate with its orthogonal state $\psi^{'}(x, t) = b^*\psi_1(x, t) - a^*\psi_2(x, t)$, we need an artificial protection procedure to remove the degeneracy, e.g. joining the two boxes with a long tube whose diameter is small compared to the size of the box. By this protection $\psi(x, t)$ will be a nondegenerate energy eigenstate. The adiabaticity condition and the weakly interacting condition, which are required for a protective measurement, can be further satisfied when assuming that (1) the measuring time of the electron is long compared to $\hbar/\Delta E$, where ΔE is the smallest of the energy differences between $\psi(x, t)$ and the other energy eigenstates, and (2) at all times the potential energy of interaction between the electron and the system is small compared to ΔE. Then the measurement of A_1 by means of the electron trajectory is a protective measurement, and the trajectory of the electron is determined by the expectation value of the charge of the system in box 1. In particular, when the size of box 1 can be ignored compared with the separation between it and the electron wave packet, the trajectory of the center of the electron wave packet, $\vec{r_c}(t)$, will satisfy the following equation:

$$m_e \frac{d^2\vec{r_c}}{dt^2} = -k \frac{e \cdot |a|^2 Q}{|\vec{r_c} - \vec{r_1}|(\vec{r_c} - \vec{r_1})}. \tag{4}$$

where m_e is the mass of electron, k is the Coulomb constant, $\vec{r_1}$ is the position of the center of box 1. Then the electron wave packet will reach the position "$|a|^2Q$" between "0" and "Q" on the screen as denoted in Figure 2, where $|a|^2Q$ is the expectation value of the charge Q in the state $\psi_1(x, t)$ in box 1, namely the integral of the charge density $Q|\psi(x)|^2$ in the region of box 1. This result of protective measurement indicates that there exists a charge $|a|^2Q$ in box 1.

In conclusion, protective measurement shows that the charge of a charged quantum system is distributed throughout space, and the charge density in each position is proportional to the modulus square of its wave function there.