Instants, inertia and quantum motion[1]

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We present a new historical and logical analysis of motion. The analysis may help to reveal the structure of time as well as the meaning of quantum theory. It is argued that, if time is composed of instants as commonly assumed, the phenomenon of inertial motion may imply that motion has no cause and thus is spontaneous. The spontaneity of motion logically requires that motion is essentially random and discontinuous. We first analyze the random discontinuous motion in continuous space-time. It is shown that the wave function in quantum mechanics can be taken as a mathematical complex that describes the motion, and the equation of motion may assume the same form as the Schrödinger equation. However, the randomness of motion cannot emerge in continuous space-time. This is unnatural in logic and also contradicts experience. We then analyze motion in discrete space-time. It is argued that the discreteness of time can release the randomness of motion as experience reveals; the equation of motion in discrete space-time is a revised Schrödinger equation containing an extra stochastic nonlinear term, which results from the discreteness of time and will result in the dynamical collapse of the wave function. Moreover, the collapse can also lead to the appearance of continuous motion in the macroscopic world. Thus, in all probability, time is essentially discrete and the actual motion is the random discontinuous motion in discrete space-time, which provides a unified picture of motion in the microscopic and macroscopic worlds.

Happy is he who can recognize the causes of things.
—Virgil

1. Introduction

Motion is the eternal subject of human enquiry. It is an experiential fact that objects can move, and macroscopic objects appear to move continuously. However, how objects move in reality is still a great puzzle. We cannot simply regard the appearance of continuous motion as the real picture of motion. Moreover, the debate about the interpretation of quantum mechanics, which is regarded as the most fundamental theory of nature, makes the puzzle of motion more complex. In this essay, we will try to find the actual picture of motion through a new historical and logical analysis[2]. The analysis may also help to reveal the structure of time and the meaning of quantum theory.

2. Zeno's paradoxes and the 'at-at' theory

The puzzle of motion was first pondered over by Zeno of Elea seriously. He conceived many paradoxes of motion, one of which is the arrow paradox [1]. As we will see, the solution of the paradox is the first step towards the actual picture of motion.

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[1] This essay is dedicated to my ancestors who wrote the great *Classic of Changes*. A more detailed version is available on my website www.quantummotion.org.

[2] In this essay, we only consider the motion of the mass center of an object or a particle, which can be described by a material point. For simplicity, we always say the motion of an object or a particle.

Imagine an arrow in flight. At every instant in time, it is located at a specific position. If the instant is durationless, then the arrow does not have time to move and is at rest during that instant. Now, during the subsequent instants, it must also be motionless for the same reason. Since the entire period of motion consists only of instants, all of which contain the arrow at rest, Zeno concludes, the flying arrow is always at rest and cannot be moving. This conclusion still holds true when the instant has finite duration, which is the smallest part of time. Suppose that the arrow actually moved during such an instant. It would be at different locations at the start and end of the instant, which implies that the instant has a 'start' and an 'end', which in turn implies that it contains at least two parts, and thus is not the smallest part of time at all. This leads to a contradiction. So, the flying arrow cannot be moving even when the instant has finite duration.

The standard solution of the paradox is as follows[3]. It is fallacious to conclude from the fact that the arrow doesn't travel any distance in an instant that it is at rest. Motion has nothing at all to do with what happens during instants; it has instead to do with what happens between instants. In short, motion is merely a feature of being in different locations at different times, and that is that. If an object has the same location at the instants immediately neighbouring, then we say it is at rest; otherwise it is in motion. This view is called the 'at-at' theory of motion [3]. Therefore, since the arrow in flight has different positions at different instants, it is surely moving.

The solution of Zeno's arrow paradox reveals that motion is merely being in different positions at different times. However, it is still unknown how the transition from one position to another is accomplished. Is the transition continuous as classical mechanics assumes or discontinuous as quantum mechanics implies?

3. The origin of motion

Motion is probably much stranger than we usually think. We are only familiar with the apparent motion of macroscopic objects after all. How about the motion of microscopic particles? It seems that the double-slit experiment cannot be explained in terms of the picture of continuous motion, and thus the motion of microscopic particles, which cannot be directly observed by our naked eyes, may be not continuous but discontinuous. However, there have been hot debates about the explanation of the double-slit experiment and the meaning of quantum theory describing the experiment [4-5]. In order to move on before experience can help us, we had better come back to the origin of motion. Motion involves change in position. If we can find the actual cause of the change, we may discover how objects move.

Aristotle was the first to seek the origin of motion fundamentally. He held that external force is the cause of motion. Especially, motion requires a force to sustain it; moving objects only continue to move so long as there is an external force inducing them to do so. This is intelligible, as it accords with the principle of causality, which states that there is no effect without a cause. However, Aristotle's theory is inconsistent with experience. For instance, an arrow keeps flying after the bowstring is no longer pushing on it. In order to avoid the difficulty for explaining such continued motion, the idea of impetus was presented notably by Jean Buridan [6]. He argued that a projectile continues in motion because of the force transmitted to it by the agent that launched it. This force internal to objects is called impetus. So, according to Buridan, impetus is the cause of motion. The motion that occurs without an external force is sustained by an internal motive force, the impetus, which can be transferred from an external propelling agent that initializes the motion.

The impetus belief was very popular in the pre-Newton times. In fact, Newton was also its adherent at one time. He called it "the force of a body". More surprisingly, many contemporary students still have such a

[3] Another solution is the so-called gunk view, according to which time and space are infinitely divisible like gunk. The gunk concept can be traced back to Aristotle, and has also been discussed recently [2]. However, this solution encounters serious problems in both mathematics and physics.

conviction.[4] The popularity of the impetus belief implies that it has some reasonable elements. Firstly, this belief might be derived from a lifetime of kinesthetic experience, and seems consistent with everyday experience. Secondly, the belief is natural and intelligible. Motion involves change of position. A change requires a cause according to the principle of causality. So motion must have a cause. Since continued motion (e.g. the flight of an arrow) occurs without an external force in the direction of motion, there must exist an internal motive force that moves the object in this direction. It is just the impetus. Because the change of position is primary for motion, this line of reasoning seems quite justifiable.

The impetus theory, however, is not consistent with experience either. In addition, the theory cannot account for the relativity of motion or the equivalence between motion and rest. It is just the consideration of the equivalence that made Newton finally transform impetus into inertia, and further founded classical mechanics [8].

In Newton's world, motion and rest are equivalent. An object can sustain its motion just as it can sustain its rest. No force is needed to sustain the motion of an object, and the object in a state of motion possesses an "inertia" that causes it to remain in that state of motion. This postulate is well summarized in Newton's first law of motion or the law of inertia. This law perfectly accords with our macroscopic experience, and it also appears logical and even self-evident. A freely moving object should hold its velocity, as there is no cause resulting in the change of its velocity. Thus the object must continuously move in a straight line with a constant speed, as the law of inertia requires[5]. But has the law of inertia disclosed all secrets about the origin of motion?

4. Motion has no cause

According to Newton's view, force is not the cause of motion but the cause of the change of motion. Motion itself needs no causal explanation; a free object moves because it has inertia and thus can sustain its motion. Yet there is a big trap here. As we have known, motion is essentially being in different positions at different times, and there is no motion at each instant. So no motion is available for an object to sustain at every instant in reality; motion and its velocity simply don't exist at instants[6]. Therefore, Newton's view is not wholly valid. In a sense, the concept of inertia diverts our attention from position change, which is *the* essential characteristic of motion, to velocity change, which is merely an apparent characteristic of motion.[7] As a consequence, it evades the question about the cause of motion, and veils the real picture of motion[8].

Although Newton's explanation of inertial motion is not right, his discovery can actually lead to the correct answer. According to Newton, neither external force nor internal force is the cause of motion. So there is only one left possibility, i.e., that motion has no cause. No cause determines how the position of an object changes. This is an important result of this essay. In fact, the relativity of motion requires that motion has no cause. Suppose an object is initially at rest relative to an observer, and no force causes its motion. When the observer moves with a constant velocity relative to the object, he will observe that the object also moves relative to him. Since no influence is added to act upon the object during this process, the motion of the object must have no cause.

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[4] Ref. [7] showed that nearly 80% of a group of engineering freshmen had the impetus belief. They thought the force from hand pushes up on the tossed coin when it is on the way up.

[5] It should be noted that pure inertial motion does not occur in nature. According to the existing physical theory, it can only occur at an infinite distance from all sources of gravity.

[6] Note that the standard definition of velocity is that the instantaneous velocity for an object is the limit of the object's average velocity as the time-interval around the point in question tends to zero. As thus, the standard velocity is local but temporally extrinsic [9], and it cannot determine the change of the instantaneous state such as position.

[7] As we will see later, velocity is only an approximate and average display of motion, and it does not exist in a strict sense for the motion of objects, which is random and discontinuous in nature.

[8] This fact has been neglected after Newton. The orthodox view is "we need only explain changes in motion, not motion itself."

Now we find that objects are not inertial but active in essence[9]. An object can spontaneously change its position without a cause, while inertial motion is only its apparent display. In reality, there are more direct evidences of the spontaneity of motion in the microscopic world. For example, the alpha particles can spontaneously move out from the radioactive isotopes without any external influence. Such phenomenon is well known as radioactivity or spontaneous decay, which generally exists in the microscopic world. During the spontaneous decay process, the decay time of each radioactive atom in the substance is completely random.

We note that the spontaneity of motion is also a consequence of the existence of instants. The argument is as follows. First, the properties defined at instants cannot determine the position change of an object. An object's instantaneous state should not imply, in virtue of logic and definition alone, any constraints on its instantaneous states at other times [2,12]. Secondly, the properties defined during infinitesimal time intervals such as velocity cannot determine the position change of an object either. In fact, these properties are determined by the position change of the object during infinitesimal time intervals according to the definition. Consequently, during the inertial motion of an object, no properties or causes determine the position change of the object, and it must change its position spontaneously.

5. Motion is random and discontinuous

Motion involves change in position, and such a change has no cause. This turns out to be the key to open the door to the real motion.

We first consider a freely moving object. The object is in one position at an instant, and spontaneously appears in another position at next instant. The position of the object is constantly changing, but no cause determines the position change of the object. So the object doesn't "know" how to move at each instant, and can only move in a purely random way. Indeed, nothing causes it to move in a special way. This argument is logical. If a change happens without a cause, then the change must be random. By comparison, if a cause results in a change, then the change will be determined by the cause in a lawful way. Therefore, the position change of a freely moving object must be essentially random.

The random change of position between instants means an object is in one position at an instant, and at the instant immediately neighbouring it randomly appears in another position, which is probably not in the neighborhood of the previous position. As thus, the trajectory of the object must be not continuous but discontinuous. Since the change of position is random at all times, the trajectory must be discontinuous everywhere. In a word, the motion of free objects is essentially discontinuous and random.

We then consider the motion of an object influenced by an external force or interaction. Can the external force determine the position of the object at every instant and change the random discontinuous motion to deterministic continuous motion? The answer is negative. The reason lies in that a purely random process cannot be changed to a deterministic process in essence. If the external force is not random, then it is obvious that the motion of an object under its influence will still be random. If the external force is also random, then since the combination of two independent random processes still leads to a random process, the motion of an object under such an influence will also be random. So the motion of an object under the influence of an external force is still discontinuous and random.

To sum up, the actual motion of objects is essentially discontinuous and random[10]. During the course of motion, the transition from one position to another is not continuous and deterministic but discontinuous and random.

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[9] It can be seen that the spontaneous motion obviously contradicts the principle of causality. However, the existence of spontaneous motion without cause may have a deeper logical basis, and this basis will unify the principle of causality and indeterminism into a generalized principle of causality. According to the principle, there are two kinds of causes: concrete causes and universal causes, and accordingly there are two kinds of events: lawful events and random events [10-11].

[10] Note that two ancient thinkers Epicurus and Al-Nazzam had ever proposed similar ideas. Epicurus's atomic swerve is random but without explicit discontinuity [13], while Al-Nazzam's leap motion is discontinuous but without explicit randomness [4]. In a

6. A theory of random discontinuous motion (RDM)

A definition of RDM can be given using three presuppositions about the relation between the physical motion and the mathematical point set. It is:

(1). Space-time is continuous.

(2). A moving particle is represented by one point in space-time.

(3). The RDM of a particle is represented by a random discontinuous point set in space-time[11].

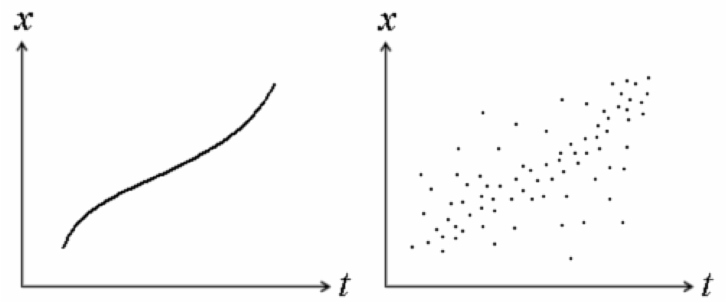

Fig.1 Continuous motion and random discontinuous motion

The picture of RDM is as follows. A particle is in one position at an instant, and at the instant immediately neighboring it randomly appears in another position. Its trajectory is not continuous but discontinuous and random everywhere, and the particle moves throughout the whole space with a certain position density distribution during an infinitesimal time interval near each instant.

The mathematical description of RDM can be obtained by using the theories of point set and measure. Two quantities are needed for a direct description. The first one, which is called position measure density and denoted by $\rho(x,t)$, provides a description of the local point distribution situation for the discontinuous point set, and it represents the relative frequency of the particle appearing in an infinitesimal space interval dx near position x during an infinitesimal interval dt near instant t. $\rho(x,t)$ satisfies the normalization relation $\int_{-\infty}^{+\infty}\rho(x,t)dx=1$. The second one, which is called position measure flux density and denoted by $j(x,t)$, provides a description of the change of the local point distribution situation with time. $\rho(x,t)$ and $j(x,t)$ satisfy the measure conservation equation $\frac{\partial\rho(x,t)}{\partial t}+\frac{\partial j(x,t)}{\partial x}=0$. The position measure density $\rho(x,t)$ and the position measure flux density $j(x,t)$ provide a complete description of RDM for a single particle[12].

With respect to the RDM of a particle, the motion of the particle is completely discontinuous and random. The probability for the particle to appear in position x at instant t is the position measure density $\rho(x,t)$.

sense, random discontinuous motion can be regarded as an integration of Epicurus's random swerve and Al-Nazzam's discontinuous leap. Besides, it can be shown that random discontinuous motion is more natural and logical than continuous motion from a mathematical point of view by considering the theories of point set and measure [11].

[11] Here the discontinuous point set denotes the set of the points (t,x) in continuous space-time, for which the function $x(t)$ is discontinuous at all instants.

[12] It is very natural to extend the description of the motion of a particle to the motion of many particles. For the latter, the density functions will be defined in multi-dimensional configuration space as the wave function in quantum mechanics [10].

There is no law for the instantaneous state of a particle, and the trajectory function $x(t)$ is purely random and discontinuous at all instants. However, the discontinuity of motion is absorbed into the motion state of the particle, which is defined during an infinitesimal time interval in continuous space-time, by the descriptive quantities: position measure density $\rho(x,t)$ and position measure flux density $j(x,t)$. As a result, the evolution law for the motion state of a particle may exist and be a continuous and deterministic equation.

A primary analysis has shown that the simplest non-relativistic evolution equation for RDM may assume the same form as the continuous Schrödinger equation in quantum mechanics [10]. The wave function in quantum mechanics can be taken as a mathematical complex that describes RDM[13]. Concretely speaking, the wave function $\psi(x,t)$ can be expressed by the position measure density $\rho(x,t)$ and the position measure flux density $j(x,t)$:

$$\psi(x,t) = \rho(x,t)^{1/2} e^{iS(x,t)/\hbar} \tag{1}$$

where $S(x,t) = m\int_{-\infty}^{x} \frac{j(x',t)}{\rho(x',t)}dx'$, m is the mass of particle, $\hbar$ is Planck constant divided by 2π. Since the latter provide a complete description of RDM, the wave function also provides a complete description of RDM. These results imply what quantum mechanics describes is probably RDM.

7. Moving in discrete space-time

Although motion is discontinuous and random between instants, the discontinuity and randomness are absorbed into the motion state defined during an infinitesimal time interval in continuous space-time. As a result, the motion state (e.g. the wave function) and its evolution equation (e.g. the Schrödinger equation) are continuous and deterministic. Then where does the randomness of motion go? How can the spontaneous motion present itself?

The sticking point lies in the assumed continuity of space-time. As we know, the continuity of space-time cannot be confirmed in essence; we can never measure infinitely small space-time regions, let alone durationless instants and positions. To put it in more objective words, a durationless instant cannot present itself. So, it is natural that the randomness of motion, which exists at individual durationless instants in continuous space-time, cannot emerge through observable physical effects. We can only measure finite time intervals, but during which there is no randomness at all.

Thus, if space-time is continuous, then the inherent randomness of motion cannot be released. This result not only contradicts experience, but also is very unnatural in logic. As a consequence, space-time should be discrete in nature. Since instants as finite intervals can have physical effects and be measured in principle, the inherent randomness of motion, which exists at instants, can emerge in discrete space-time. Concretely speaking, when a particle stochastically stays in a position during a finite instant, its finite stay may have a tiny random effect on the evolution of the motion state. Then during a much longer time interval, such tiny effects will continually accumulate to generate the observable random phenomena.

[13] Note that the wave function, if taken as a description of the objective indefinite state of motion, also leads to RDM, as the indefinite position state can only exist in an infinitesimal time interval, during which all possible positions are randomly distributed.

A basic definition of discrete space-time can be given as follows. In discrete space-time, there exist a minimum time interval T_U and a minimum space interval L_U, which is called time unit and space unit, respectively. Due to the discreteness of space-time, any physical being can only exist in a space region not smaller than a space unit, and any physical becoming can only happen during a time interval not shorter than a time unit. Modern physics implies that the values of time unit and space unit are $T_U = 2T_P$ and $L_U = 2L_P$, where $T_P = (G\hbar / c^5)^{1/2}$ and $L_P = (G\hbar / c^3)^{1/2}$ are Planck time and Planck length[14]. These values are extremely small, so we never directly "see" the discreteness of space-time, and have been thinking space-time is continuous.

We then give a description of RDM in discrete space-time. In discrete space-time, the particle is no longer in one position at one instant (as in continuous space-time), but limited to a space unit during a time unit. This defines the existent form of a particle in discrete space-time. During a finite time interval much larger than a time unit, the particle moves randomly and discontinuously throughout the whole space with a certain position density distribution. This defines the average motion state of a particle in discrete space-time. The proper description of the motion state is the (time-averaged) position measure density and the (time-averaged) position measure flux density. A primary analysis shows that the evolution of RDM in discrete space-time satisfies a revised Schrödinger equation containing an extra stochastic nonlinear term, which results from the discreteness of space-time [10,15]. This term will lead to the dynamical collapse of the wave function or random localization, during which the randomness of motion is released. The equation can be formally written in a discrete form as follows:

$$\psi(x, t + T_U) - \psi(x, t) = \frac{1}{i\hbar} H\psi(x,t) T_U + S\psi(x,t) \quad (2)$$

where H is the corresponding Hamiltonian, and S is the stochastic nonlinear operator. A new testable prediction of the theory is that a superposition of two energy eigenstates will randomly collapse to one of the states after the following collapse time:

$$\tau_c \approx \left(\frac{E_P}{\Delta E}\right)^2 T_U \quad (3)$$

where $E_P \equiv h / T_P$ is the Planck energy, ΔE is the energy uncertainty of the state.

In conclusion, RDM requires the discreteness of space-time to release its randomness; on the other hand, the discreteness of space-time can indeed release the inherent randomness of motion through a stochastic collapse process[15]. Thus the actual motion is the random discontinuous motion in discrete space-time in all probability.

[14] Note that there are indeed some clues of the discreteness of space-time in modern physics. For instance, the appearance of infinity in quantum field theory and singularity in general relativity has implied that space-time may be not continuous but discrete. Besides, it has been widely argued that the proper combination of quantum theory and general relativity, two results of which are the formula of black hole entropy and the generalized uncertainty principle, may inevitably result in the discreteness of space-time (see, e.g. [14]).

[15] It is worth noting that the discreteness of space-time also has more restrictions on the possible forms of motion, and it can give a new support of the existence of RDM [10-11,16].

8. The appearance of continuous motion

If the motion of objects is essentially random and discontinuous, then why does the motion of macroscopic objects appear continuous? In order to make RDM acceptable, we must solve this puzzle. Here we will present a general analysis.

The crux of the matter lies in that RDM often happens in extremely short space intervals for macroscopic objects. As thus, a large number of minute discontinuous motions may generate the average display of continuous motion. Concretely speaking, the evolution of RDM is dominated by the random localizing process for a macroscopic object. This localizing process proceeds very frequently, and thus the position density distribution of the object will always concentrate in a very tiny local region. So, a macroscopic object will be limited in a local position at each instant, and can but be approximately still or move continuously in appearance. This then generates the apparent continuous motion of objects in the macroscopic world. In addition, it can be shown that the equations of continuous motion (i.e. Newton’s equations of motion) can also be derived from the above equation of motion as an approximation [10]. Thus classical mechanics is an almost exact description of the motion of macroscopic objects.

9. Conclusions

The main ideas and results presented in this essay are summarized as follows.

(1). The structure of time may determine whether or not the world is deterministic. If time is composed of instants (0-sized or finite-sized) as usually assumed, then, based on a new analysis of inertial motion, motion will have no cause and thus will be random and discontinuous in nature. As thus, the world is probably essentially indeterministic.

(2). The equation of random discontinuous motion (RDM) in continuous space-time may assume the same form as the Schrödinger equation in quantum mechanics. This indicates what quantum mechanics describes may be RDM of particles.

(3). The existence of RDM further requires the discreteness of time in order that the randomness of motion can be released as quantum experiments reveal. Thus time is essentially discrete (i.e. composed of finite-sized instants) and the actual motion is RDM in discrete space-time in all probability.

(4). The equation of RDM in discrete space-time is a revised Schrödinger equation containing an extra stochastic nonlinear term, which results from the discreteness of time and will result in the dynamical collapse of the wave function. The collapse can lead to the appearance of continuous motion in the macroscopic world.

(5). The dynamical collapse of the wave function will generate an arrow of time. In some sense, the discreteness of time results in the arrow of time.

Before concluding, I want to list three unsolved problems relevant to my ideas, and hope they can attract the attention of both experts and ordinary readers.

1. The strict proof of (2) is still wanting. It may closely relate to the peculiar structure of RDM (e.g. intrinsic symmetry).
2. Further theoretical and experimental researches of (4) are needed in order to test the ideas.
3. Understanding RDM in terms of instants, for example, how a particle “knows” which position it will be at next instant in the sense of probability. This may relate to Aristotle’s concept of motion, i.e. that motion is the actualization of potentiality.

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