The Hole at the Center of Creation

by George Gantz[1]

"Consequently he who wishes to attain to human perfection, must therefore first study Logic, next the various branches of Mathematics in their proper order, then Physics, and lastly Metaphysics." - Maimonides[i]

Introduction:

Since the dawn of recorded history, and likely before, humans strived to understand the world into which they are born and from which they ultimately pass. The human desire to understand flows from the capacity for abstraction and reflection, married to the human impulses to survive, to procreate and to belong. These capacities and impulses arose through the evolution of the human species in the crucible of life, and they have been fruitfully applied to yield vast increases in our knowledge and control of the world we inhabit.

Among the key questions that have pre-occupied the human desire to understand, two stand out. What is the stuff of which the world is made, and how does it work? From the inquiries these questions invited, natural philosophy was born, emerging in full flower in ancient Greece more than two millennia ago. Key among the early insights was the recognition that the world operates according to specific regularities, and these regularities obey logical rules. The study of these regularities, as manifest in the stuff of which the world is made, is physics. Abstracted from the substance of the world, these rules are the basis for geometry and arithmetic, the earliest studies in mathematical order.

Labeling these inquiries does not, however, put to rest the re-formulated fundamental questions: what is the source of the regularities and why do they follow the rules as they do? These questions continue to baffle modern thinkers as much as they did the ancients. We are left with puzzles, including the one Eugene Wigner explored in his 1960 paper "The Unreasonable Effectiveness of Mathematics in the Natural Sciences".[ii] As he stated: "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve." This puzzle is at the heart of the current FQXi essay contest "Trick or Truth".[iii]

The Truth is that there is a hole at the center of creation, afflicting both mathematics and physics - an infinite void made visible to us in the form of ineluctable paradoxes. The hole is most elegantly revealed in Gödel's incompleteness theorems[iv]. Assuming that the world is logically consistent, there are truths about the world that cannot be proven from within the world. More tangible manifestations of the hole can be found in physics: The double-slit experiments[v] show that particles paradoxically behave as waves; Heisenberg's Uncertainty Principle[vi] reveals hard limits to what we can know at the Planck scale.

[1] Many thanks to those who read and commented on my draft essay: Wenda Gantz, Justin Junge, Sylvia Shaw, David Hurst and Aram Yardumian.

The Trick is that in pursuing fundamental questions on the nature of creation, of logical order, and of consciousness, we are led inexorably to an infinite void, a barrier to our ability to know, one that we cannot cross without reaching for a transcendent metaphysical explanation. Yet, precisely because it is transcendent, we have no empirical evidence for such an explanation and can only make intuitive judgments in choosing between alternatives.

In a sense, this leaves each of us with the task of writing our own creation story to fill the void. For many, the stories in the religious texts with which we were raised or that we have embraced are perfectly adequate. As allegory, nearly all can be understood as consistent with the findings of empirical science, although some reinterpretation of the ancient words or narratives may be required. Others seek new mythologies. Some deny that there is any explanation, or any need for one, being content to believe only what they can see and touch (metaphorically speaking).

This essay explores our failure to grasp the hole at the center of creation and explains its metaphysical genesis. I will offer a creation myth that respectfully echoes the ancient sacred text of the Book of Genesis, while being grounded in what we now know about mathematics and physics. My goal is to provide a coherent and consistent explanation of the Hole at the Center of Creation, one that also serves as the key to the Whole that Encompasses Creation.

The Greeks Started It:

Pythagoras (570 – c. 495 BC) is often credited with the integration of geometry and arithmetic into what we now know as mathematics.[vii] The Pythagoreans were thought to revere mathematics as the fundamental basis for understanding the world, but struggled with the apparent paradox that certain geometrical features were incommensurate with the ratio of any two natural numbers. One can imagine the fear and loathing that the proof of incommensurability might have created in the heart of a true believer --- which is why legends say that its discoverer, Hippasus, was thrown overboard by his fellow mathematicians (or killed by the Gods).[viii] The solution for incommensurability was, of course, the later discovery of "irrational numbers".[2]

The relationship of the physical world to the mathematical also raised paradoxical questions for the Greeks. Of great interest was the question of whether the world (space and time) was discrete or continuous. Zeno[ix] (c. 490–430 BC) receives much of the credit for formulating the paradoxes by which he countered prevailing notions of space and time through proofs by contradiction, supporting a conclusion that movement was an illusion. Most famous among these, perhaps, is the paradox of Achilles and the tortoise - as Achilles chases the tortoise, he will always reach the point where the tortoise was, but the tortoise will have moved forward. How can Achilles ever win the race? Aristotle (384 BC−322 BC) offered a solution by noting that as a moving object covers a smaller distance, the time required to cover the distance gets smaller as well --- leading to the notion that space and time must both be infinitely divisible.

[2] While technically accurate, the name "irrational" seems ironic for a mathematical quantity, one that has been followed by a host of interesting names for other categories of numbers including: real, transcendental, imaginary, transfinite and hyperreal. This suggests that mathematicians may struggle with the metaphysical implications of new concepts when they are first explored.

Thus, Greek mathematics and physics converged on a common metaphysic, that abstract numbers and the space they represent are continuous and infinitely divisible, ultimately setting the stage for Newtonian calculus and classical physics. That orderly metaphysical framework became conventional wisdom for nearly two millennia - until relativity and quantum mechanics tore it to shreds.

The Greeks also argued about the relationship between abstract mathematics and the world. Plato (c.428 - 348 BC) is credited for authoring the concept that what is real is an abstract world of perfect forms, knowable a priori through the mind, with the physical world being a mere shadow. Aristotle took a more realist perspective, suggesting that the forms are discovered through investigations of the real world. Even today, there are those who argue that the practice of mathematics is the discovery of a priori form, and those who believe that math is created by humans for the empirical investigation of the world. This debate is as alive today as it was more than 2,000 years ago.[x]

Aristotle also laid an important foundation for later human cognitive enterprise in his principles of logic.[xi] In addition to his syllogistic method of analysis, he promoted the search for first principles. For most of the past two millennia these included the law of identity, the law of negation and the law of the excluded middle. To paraphrase: A statement implies itself; If a statement is true, it's negation cannot also be true; A statement is either true or not true --- there is no "in between". The clarity of these black and white rules of logic and the elegant perfection of mathematical form are the basis for reductionist thinking: Any of the world's mysteries can be measured, parsed and analyzed logically to yield the correct rational answer. "Logos" is master of the universe.

The Reductionist Agenda:

The Greek modes of thought continued to set the agenda for centuries to follow, but in the European enlightenment, led by giants such as Copernicus, Galileo and Newton, advances in mathematics and physics (including astronomy) drove the metaphysical framework firmly in new directions. The Ptolemaic conception of fixed heavenly spheres was supplanted by the Copernican model of a spinning Earth orbiting the Sun. The invention of calculus fundamentally reaffirmed the notion of space and time as infinitely divisible continua. Both set the stage for Newton's Laws of Motion and the subsequent transformational leap for empirical science.

The Newtonian worldview imagines the universe as a set of interacting mechanical objects, whose states and positions are perfectly calculable.[xii] The simplicity and power of this narrative appeals to human psychology. It became the story of what the universe is and how it works. Fundamental units interact in deterministic patterns all of which can be measured and predicted. Moreover, any mechanism or process can simply be reduced to its component parts in order to yield precise understanding.

In such a formulation, there is no hole at the center. If things are not yet clear, or if anomalies arise between one's observations and one's theory, it's because there is a deeper layer, a finer grained existence underneath the one we are investigating. Peel each layer of the onion back and you get another layer - an orderly process of reduction, until the ultimate graining is reached.

In the centuries to follow, this reductionist agenda dominated most of science, and much of Western culture as well. At the same time, scientific progress and the technological engine of economic growth created a buoyant optimism that such progress would continue and that ultimate knowledge was just a matter of time. This optimism was reflected in physics, which was seen as the ultimate science that would eventually explain all phenomena in terms of fundamental substances and forces, related in precise mathematical laws. As A.A. Michelson put it in 1903: "The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote. … our future discoveries must be looked for in the sixth place of decimals."[xiii]

We see evidence of reductionism in many fields throughout the 19th and 20th century: physics, chemistry, biology, evolution, genetics and psychology. One of the capstones of reductionism, the Standard Model of Particle Physics, was finally completed to the satisfaction of most physicists in the 21st century, with the verification of the Higg's Boson.[xiv] Another, the orderly structure and consistency in the periodic table of chemistry, has also been essentially completed with discoveries and synthetic creations through element 118.[xv] Beyond the hard sciences, reductionist approaches can be found in art, music, and literature as well as social and political science.

Perhaps the ultimate in reductionist trends can be found at the intersection of philosophy and mathematics. Driven by many great minds including Cantor, Frege, Russell, Whitehead, and others, mathematics focused increasingly on formalized mathematical methods that served to ground the various fields on explicit axioms and proofs. This process led to significant advances in the late 19th century in formal logic and set theory. These developments found an ultimate expression in the early 20th century philosophical movement known as logical positivism (LP). LP made claims for the reducibility of knowledge to formal truth statements and logical proofs.[xvi] Its companion, computing theory, postulated that all such proofs were derivable from mechanical algorithms that could be programmed into machines.[xvii]

The Emergent Black Hole:

The optimism of so many in physics, mathematics, philosophy and other fields that carried humanity so triumphantly into the 20th century, began to falter as the new frontiers of knowledge were being explored. In physics, the concept of unchanging, predictable Newtonian space and time became twisted as Einstein's relativity theories took hold. Relativity integrated time and space but also undermined the intuitive comprehensibility of the physical world. Time is now an "illusion", a function of both motion and position.[xviii] There is no absolute reference frame.

The early findings of quantum mechanics included wave-particle duality and Heisenberg's uncertainty principle. The probabilistic features of ultimate reality they revealed unraveled confidence in our ability to know and predict. Quantum physics also discovered puzzles in the relationship between observer and observation, leading to speculation that consciousness is integral to reality. In cosmology, steady state theories failed in light of findings confirming the universe began in a Big Bang. Black Holes, infinitely dense and impenetrable discontinuities in

the fabric of space and time were first theorized and then identified. Findings in chaos and complexity theory confirmed that there are processes we cannot model, trajectories we cannot predict and details we will never know about the world. The scientific optimism of the century before faded in light of the paradoxes and the limits that our 20th century investigations revealed.

In mathematics, the strange properties of infinity and curious logical paradoxes, such as "This Statement is False", led to several key discoveries. Gödel's incompleteness theorems[xix] proved that in any formal logical system, consistency (the principle that a statement and its negation could not both be true) implied incompleteness (there are undecidable but true statements). If we accept that the universe we are in is consistent (a strongly held metaphysical belief - Aristotle's second principal), then there are truths we cannot prove. Add to this Turing's computability dilemma[xx]: While all math and logic can be coded in machine language and all problems solved through machine algorithms, it is impossible to know whether the solution can be achieved in a finite amount of time. If that were not sufficient, in Cantor's paradise of multiple infinities, it is impossible to conceive of the largest infinity.[xxi]

Indeed, these frontier discoveries in the fields of physics and mathematics revealed what many western thinkers had ignored for centuries --- that there is an impenetrable void, a metaphysical Black Hole, at the center of creation. Reductionism peels away the layers of the onion of reality only to find, at its center, that which is unknowable.

The twin failures of physics and mathematics are different and the differences are revealing. The limitations in physics are limitations in our understanding of the world. Some (but not all) of these may ultimately be resolved with deeper theories that better reconcile or explain things that we do not understand today. Perhaps a new Aristotle, Copernicus or Einstein will build bridges across some of the anomalies that currently go begging. In contrast, the limits in mathematics are limitations in the structure of thought itself. They are integral to any world we can imagine as well as to the world in which we live (and in which we conduct physics).

The Various Responses:

Individuals and groups have different responses to the hole at the center of creation. Many mystics, in any of the varieties of mysticism that weave through the history of human philosophical and religious traditions, seem to have found answers for themselves. However, the nature of mysticism as a private experience precludes validation and confirmation by those who have not had the experience. Many theists offer rational explanations that fill the hole by their definition of the infinite Creator God. Yet faith in God is not universally shared, and some consider either mysticism or theism to be unsupported and untenable. Beyond these, a variety of responses are common.

One response, perhaps the default, is ignorance. This can take the form of fundamentalism, unacknowledged ignorance through adherence to a belief set so strong that the question of a hole at the center of creation is never posed. It can also take the form of a materialistic worldview --- unexamined ignorance, a "no belief" mindset that simply accepts that what is immediately tangible is all that matters.

Another reaction to the hole at the center of creation can be depression and alienation. One can make the case that the threads of modern existential or nihilist philosophies can be traced to the consequential outcomes of the mechanistic worldview. The modern world is characterized by the dehumanization of the industrial enterprise, the tragedy of horrific wars and the loss of meaning from a deterministic and valueless conception of the world and life. The loss of meaning can be a particularly subtle malaise.[3]

But perhaps the most common response to the hole at the center of creation, among active practitioners in science, mathematics and philosophy, seems to have been avoidance in the form of ducking the question. This tactic has its roots in Greece: Zeno notably claimed to prove the illusion of motion through the Achilles-Tortoise paradox, a metaphysical claim that ducks the question by denying the reality of our own senses. The echoes of Zeno's claim can be heard in the attempts to rationalize relativity by claiming that "time is an illusion." The Copenhagen Interpretation[xxii] in quantum mechanics is another modern example of ducking the question. To paraphrase: Let's all agree just to assume that the waveform embodies duplicate realities, and don't ask too many questions. Schrodinger's cat, [xxiii]of course, is an example of one such prohibited question.

Similarly, the multiverse theory of an expanding and potentially infinite set of alternative universes, each triggered by the various superposed states at the point of waveform collapse, is a facile response to the question of why the universe we inhabit seems to be so finely tuned. We can avoid having to explain why the universal constants are precisely what they are --- by postulating that we just happen to be (presumably randomly) in the one universe where these constants are as we measure them. Tegmark[xxiv] has taken this one step further by suggesting that there are universes that fill out each and every infinite variety of possible mathematical structure. We can avoid having to answer Wigner's question as to why math works so well in explaining physics --- by postulating that math and physics are isomorphic realities. We just happen to be (presumably randomly) in the one universe where physics maps to mathematics in the peculiar way it does. Both theories rest on the metaphysical premise that the specific nature of the physical reality we experience is unintended. In light of the apparently unique properties of our world, these theories allow us to avoid introducing the concepts of intention, choice, value or purpose in the grand scheme of creation.

Perhaps the ultimate example of ducking the question can be found in philosophy, in a famous work by Ludwig Wittgenstein. The Tractatus Logico-Philosophicus[xxv] begins with a proposition "1 - The World is all that is the Case", and then fills out the propositions by which the world maps to language and hence to logic. This mapping and the logic that guides knowledge of truth was the ultimate quest of logical positivism, and proponents argued that it provided the only correct pathway to truth. The work ends in a final proposition: "7 - That whereof we cannot speak, we must remain silent." This is an absolutely brilliant way to avoid any questions that one cannot answer by declaring them "out of bounds."

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[3] I have wondered whether the deep metaphysical implications of their work may have had some influence on the tragic circumstances surrounding the end of life for the three great mathematicians noted previously: Georg Cantor, Alan Turing and Kurt Gödel.

A similar method has been explored for expunging paradox from mathematics. This attempt involves building axiomatic systems that outlaw self-referential statements; thereby avoiding the features exploited by Gödel in the proof of his incompleteness theorems. While this is interesting from a technical perspective, it offers little hope for the questions we are seeking to address, as reflection and recursion are fundamental features of the universe and our place in it.

In fact, the hole at the center of creation ultimately lies in our reflective contemplation of the distinction between something and nothing, one and the Void, being and nothingness.

Postulating a Metaphysics of Creation:

These arguments have brought us to the point we noted in the Introduction. We have the opportunity to write our own creation story in a way that incorporates the realities of what we know in mathematics and physics, and what we know that we cannot know, into a coherent narrative mythology. My religious tradition is Christian, and I find the language of the Old Testament (King James translation) to be well suited for this purpose. With deep respect to the sacred text that informs this narrative, I offer the following allegorical creation story and brief annotation in two Chapters.

Chapter Alpha:
1 In the beginning was the Void, and the Void was without form and silent.
2 Then a Voice said, "Let there be One", and there was One. The One was a set apart from the Void, and it was good.
3 The Voice spoke to the One and said, "Thou shall be married to Necessity. Be fruitful and multiply, as thy progeny shall fill all the corners of the Void with Form, and potentiality shall be thy blessing."
4 Potentiality flowed from the One and Necessity and it was well formed, and it was good.
5 But all was silent within the Form, so the Voice said, "let there be a sound, and it shall bring forth the firmaments," and it was so.
6 Blessed were the structures that emerged and evolved, following pathways through the potentiality of Form, as the Cosmos came into being.
7 Blessed also was the intentionality that gave purpose to the Cosmos, and to the beauty, the order, and the life that emerged, and it was very good.

This narrative outlines a metaphysical framework that presupposes a two-stage process for creation. Of course, the process is entirely outside of time and space, and therefore is not sequential. Yet the two steps are conceptually distinct reflections of the nature of math and physics. The first stage of the process is the separation of One {1} from the Void {0} --- a first metaphysical distinction. This distinction subsequently gives rise through Necessity (set theoretic constructions and logical operations) to the infinite logical space of Form --- mathematics in its purest sense, the potential state-space for the universe-to-be.

But Form is itself motionless and silent. It is potential but has no substance. A second stage in creation is needed; the process of coming-into-being. This requires a second metaphysical distinction, a physical one. Sound, a physical movement through space, is the analogical symbol

for this step. With sound, we have space and time, energy and motion, and the emergent realities of physics, chemistry and biology. This physical reality begins to flow along a precise set of potential pathways available in the Form.

However, key to the process of becoming is its inherent intentionality --- the willing of the merely possible into the physically real. In this narrative that willing is accomplished through the volitional expression, the words, of the Voice. The chapter concludes with the acknowledgement that the structures and life that emerge and evolve are beautiful, orderly and "very good."

Chapter Omega:
1 And it came to pass that life emerged and brought forth a human species that ate freely of the fruit of the tree of knowledge, and who had dominion over the fish of the sea, over the birds of the air, over all the cattle, over all the earth.
2 Humans explored the structures of the Cosmos and studied the Form that came from the marriage of the One with Necessity. The day arrived when humans began to perceive limits to Cosmos and Form.
3 In that day, many saw the Void but knew it not. Some cast themselves into despair and were consumed. Others hid in caves and tunnels, unwilling to look upon the face of the Void.
4 Are there any among you that can embrace this Truth? The Void casts its timeless shadow across the Form and Cosmos of creation, but there is a Voice that is both its master and its reflection.
5 Blessed are those who know the Void and hear the Voice, that their lives may be full.

Verse 3 of Chapter Omega is a characterization of our present state in comprehending the hole at the center of creation. The late 19th and 20th century brought us to the edge of the Void in mathematics and physics. As noted in the previous section of the essay, many people seem happy to ignore or deny it, while others are caught in the despair of depression or alienation. Many have been ducking the question --- essentially building caves and tunnels to avoid its pressing reality.

The final verses of Chapter Omega put the answer squarely. Yes, the Void, the dark place in our understanding, the knowledge space where our empirical and logical tools cannot probe, is real and casts its shadow across both the physical Cosmos and the potential Form. However, we are here. The Void cannot bootstrap itself --- hence the need, in both the narrative and in our conceptual framework, for a Voice. Call it what you will, something makes a first distinction of a set apart from the Void, bringing Form, and something wills the physical Cosmos into being.

Verse 4 ends with the acknowledgement that what is master to the Void is also its reflection. Zero and infinity are integrally connected in the history of mathematics, opposing poles in Riemannian complex geometry, and critical yet paradoxical aspects of mathematical theory from Zeno to Cantor. [xxvi] Their mutual self-reflection, in the mythical Void and Voice, are an essential starting point for Creation.

Verse 5 finishes the narrative with a final blessing, one that deserves a note of explanation. Those who understand the Hole at the Center of Creation recognize that they have a choice about

how to perceive life. It can be viewed as a Void, as essentially purposeless or random. Or it can be viewed as a Voice with intention, an infinite source from which we can find purpose and meaning of limitless capacity. Indeed, if we so choose, our "lives may be full."

The Whole at the Center of Creation:

It may help to use a simple analogy to explain the metaphysics of creation postulated above. Just as electricity in a circuit is defined by a voltage potential and the flow of an actual current, so too is creation defined by a potential, mathematical Form, and the actual flow of the physical Cosmos. Significantly, for electricity to flow the circuit needs to be complete - this recursive "closing of the loop" allows the electricity to flow and to do work. The work, of course, is the purpose of the electricity flowing in the circuit.

In much the same way the circuit of creation, including both the potential of Form and the flow of Cosmos, needs to be completed by a recursive closing of the metaphysical loop, in this case through a reciprocal, reflective relationship between Void and Voice. The resulting flow is the basis by which the work and purpose of creation, the emergence of the physical universe, of life and of consciousness, is accomplished.

At the individual level, our conscious awareness of our relationship to our own existence (our "having been created") closes the loop and provides the vehicle for intentionality, the will to achieve a purpose and to act in accordance with that willing. At the level of creation, the reflective dialogue between Void and Voice closes the loop, and from this creation proceeds. How we describe this dialogue is a matter of choice. That such a dialogue sits at the heart of creation is, to me, indisputable.

The creation myth that we each construct for ourselves is not subject to empirical validation or logical proof. But in this myth we can seek to validate all that we know, and all that we have learned, including the relationship between mathematics and physics. We can also choose to invest it with purpose. This requires one to accept the intentionality of creation, the infinite Voice that is master to the Void. By so doing, we can find the Whole that Encompasses Creation.

The Whole that Encompasses Creation is that which we believe in yet cannot know - it is what gives each of us purpose.

"The more deeply human wisdom commits itself to the investigation of the divine or infinite essence, the more deeply it is involved in a labyrinth. if the philosophers must inquire, by relation and proportion, into the essence of the infinite from the essence of the finite, the infinite will necessarily appear null in relation to the finite, or else the finite null in relation to the infinite…" p.10f

"We conclude therefore… that the infinite is the cause of the finite…" p.17

Emanuel Swedenborg, The Infinite, and The Final Cause of Creation (1734)
tr. James John Garth Wilkinson. London 1847. [xxvii]

End Notes and references:

[i] Moses Maimonides, Guide for the Perplexed (c.1190), tr. Friedlander (1904) Ch XXXIV, from http://www.sacred-texts.com/jud/gfp/gfp001.htm (last downloaded 2-24-15).
[ii] Eugene Wigner ,"The Unreasonable Effectiveness of Mathematics in the Natural Sciences, in Communications in Pure and Applied Mathematics, vol. 13, No.1 (February 1960).
[iii] FQXi Essay Contest, Trick or Truth: the Mysterious Connection Between Physics and Mathematics. http://fqxi.org/community/essay/rules (downloaded February 22, 2015.)
[iv] An excellent resource on Gödel is Ernest Nagel and James R. Newman, Gödel's Proof; Revised Edition, by Ernest Nagel and James R. Newman. New York University Press 2001.
[v] For one explanation, see Bruce Rosenblum and Fred Kuttner, Quantum Enigma: Physics Encounters Consciousness, Oxford University Press, 2011. Chapter 7.
[vi] Ibid. Chapter 10.
[vii] Pythagoras is also credited with the name "mathematics." see: http://www.storyofmathematics.com/greek.html (last downloaded 1-6-15), and http://en.wikipedia.org/wiki/Greek_mathematics (last downloaded 1-6-15).
[viii] Ibid.
[ix] see, e.g. (http://en.wikipedia.org/wiki/Zeno's_paradoxes) (last downloaded 2-22-2015).
[x] see for example, James Franklin, "What is mathematics about?" in Aeon online April 7, 2014 (http://aeon.co/magazine/science/what-is-left-for-mathematics-to-be-about/ (last downloaded 1/3/15), or Ferenc Csatári "Some Remarks on the Physicalist Account of Mathematics" in the Open Journal of Philosophy, 2012. Vol.2, No.2, 165-170.
[xi] see: "Three Laws of Logic" http://en.wikipedia.org/wiki/Law_of_thought#The_three_traditional_laws (last downloaded 2-22-2015), and "Aristotle's Logic", Stanford Encyclopedia of Philosophy, http://plato.stanford.edu/entries/aristotle-logic/ (last downloaded 2-22-2015).
[xii] see, e.g. Rosenblum, op cit, Chapter 3.
[xiii] A.A. Michelson, Light Waves and Their Uses (1903), 23-4.
[xiv] see for example, http://www.newscientist.com/topic/higgs-boson (last downloaded 2-22-2015).
[xv] National Geographic News online, October 28, 2010. "Element 118 Created, This Time for Real, Scientists Say" http://news.nationalgeographic.com/news/2006/10/061020-new-element.html, (last downloaded 2-23-2015).
[xvi] see for example, http://en.wikipedia.org/wiki/Logical_positivism, (last downloaded 2-22-2015).
[xvii] see Melanie Mitchell, Complexity: A Guided Tour, Oxford University Press 2009, Chapter 4.
[xviii] see Brian Greene The Elegant Universe, Vintage Books (2000), Chapter 2.
[xix] Nagel op. cit.
[xx] Mitchell op.cit.
[xxi] Rudy Rucker, Infinity and the Mind, Princeton University Press (1999), page 44ff.
[xxii] Rosenblum, op cit., Chapter 10.
[xxiii] Ibid. Chapter 11.
[xxiv] Max Tegmark, Our Mathematical Universe: My Quest for the Ultimate Nature of Reality, published by Alfred A. Knopf (2014).
[xxv] Ludwig Wittgenstein, Tractatus Logico-Philosophicus (1921), tr. D. F Pears and B.F. McGuinness, Routledge Classics, 2001 publication.
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[xxvii] accessed at https://archive.org/stream/outlinesofphilos00swed#page/6/mode/2up (last downloaded 2-24-2015)