Demistifying the astonishing success of mathematics: the case of gauge symmetry

Aldo Filomeno

Abstract

This paper argues against a strong philosophical interpretation of the leading role of mathematics in all of physics. To do so the paper focuses on a specific case study, that of the truly astonishing success of symmetry groups in modern particle physics. Specifically, I analyze the case of one local gauge symmetry, that of the strong nuclear interaction. I would say this is an especially pertinent case study, as gauge symmetry applies throughout most of our current best fundamental physics and the intimate relation with the physics it describes is particularly astonishing. The paper advocates for an understanding of mathematics only as an (especially appropriate) language which does nothing but describe patterns, a subset of which are instantiated in Nature. With such an understanding I argue that the effectiveness of mathematics is not unreasonable; on the contrary, it is to be expected. Such an explanation undermines the viewpoint that takes gauge symmetry principles as *a priori* reasonable or as some sort of necessary meta-laws. Likewise, such an explanation weakens the reasons to endorse a strong ontological commitment to the mathematical entities (as the diverse variants that suggest that the universe is fundamentally mathematical, like [Tegmark, 2014] or [French, 2014]).

Contents

1 Introduction

It is astonishing, some even said "unreasonable" [Wigner, 1960], the effectiveness of mathematics in physics. In the last decades, the role of a branch of mathematics —group theory— in the constitution of modern particle physics has brought even more enthusiasm. In section 2 I will try to briefly convey why this has been so. Then, the aim of this paper is to show that one should not be so enthusiastic—more precisely, not so enthusiastic *as to interpret symmetry principles as anything like a priori or necessary "superprinciples" or meta-laws* (in the line of [Weyl, 1952] or [Wigner, 1967]), *or as to take a too strong ontological commitment about the mathematical entities involved* (for instance in the line of [Tegmark, 2014]). Such a strong intrepretation of symmetry principles is motivated in section 2, while some representative examples of taking a strong ontological commitment of the mathematical entities are, for instance, Tegmark [2014] proposal that the world is fundamentally mathematical or the also wild-but-interesting

proposal of some structural realists that put the notion of structure in the first place, as the fundamental ontological category (e.g. French [2014]). Of course, there are other variants and the reasons for defending their positions do not reduce only to the mysterious effectiveness of the mathematics in physics. For instance, platonists in philosophy of mathematics advocate for the existence of mathematics—in this case in a platonic world. But they are not my target since they do not recur to the effectiveness of mathematics in support of their position. They recur to the *indispensability* of mathematics —arguably the only non-question begging reason in their support, as once said by [Field, 1980]. I will say something about indispensability after my proposed explanation.

In a nutshell, my main goal is to provide an explanation of the effectiveness of mathematics in physics according to which mathematics is understood just as an especially powerful, precise, and useful language for the description of the workings of Nature (or at least of our surrounding Nature). In other words, mathematical language is employed in the physical sciences to describe the spatial and temporal patterns of the world. Then, the complexity of the world that we are able to unveil directly depends on the richness of the language with which we are able to express it—and so certain advances in physics did necessarily require of previous advances in mathematics. Yet it is only a language, and it is not describing any *necessary* feature of the world or of its dynamics, nor it is itself constituting any *"part of physical reality"* (quoting a suggestion of this FQXI eassy contest).

Then, with the explanation proposed the effectiveness of mathematics cannot be regarded anymore as a reason to support 1) an interpretation of current symmetry principles as privileged in any sense (necessary or *a priori* reasonable) or 2) a strong ontological commitment to the mathematical entities.

Likewise, this essay also targets a widespread attitude among scientists and philosophers: that of those that, satisfied by the elegance and beauty of the mathematics that describe the dynamics of the world, seek no further explanation as to the existence of such laws. I argue that this attitude should change, as some have previously argued (e.g. [Wheeler, 1982], [Weinberg, 1981], or [Peirce, 1867]).

In section 2 I sketch how this enthusiasm towards the role of mathematics in physics is indeed well justified. Then, in section 3 I spell out my argument whose purpose is to attenuate such enthusiasm.

2   The "unreasonable" success of symmetry principles in physics

The history of modern physics has been invariably acknowledging what Galileo Galilei beautifully illustrated centuries ago:

> "Philosophy is written in this grand book, the universe [...] It is written in the language
> of mathematics, and its characters are triangles, circles, and other geometric figures; [...]"
> [Galilei, 1623, ch. VI]

a metaphor intuited much before, since Pythagoras of Samos, and that has been confirmed ever since. Regardless of the philosophical interpretation given to mathematics, it seemed that the laws of physics were inextricably formulated in mathematical terms. Nowadays, this long and successful love story is at its best, bolstered by the application of a new algebra, called Lie algebra, to the description of the elementary particles/fields and interactions of the Standard Model.

In particular, few decades ago symmetry principles could have been plausibly considered as a

sort of "superprinciples", in the sense of some kind of necessary *a priori* meta-laws. Foremost examples of such principles are the invariances under time and space translation. As is known, these two global continuous symmetries correspond to two principles of conservation, of energy and linear momentum respectively. Furthermore, the relation of symmetry principles with principles of conservation was generalized by Emmy Noether's theorem: for every continuous global symmetry of the Lagrangian there is a conservation law (and *vice versa*). If some principles at all could be assumed as necessary or *a priori*, these could have been reasonable candidates. One might feel reasonably satisfied with beautiful mathematical truths associated with the conservation of a basic property, ultimately grounding the existence of other, less fundamental, laws of nature.

Furthermore, the advent of more symmetries of a new type, local (also called 'internal') and following the so called gauge principle, has been taken as a sign of the "*elegance of nature*" [Wilczek, 2008, 63][1]. [Martin, 2003, 41] describes this point of view:

> "the 'gauge philosophy' is often elevated and local gauge symmetry principles enshrined. Gauge symmetry principles are regularly invoked in the context of justification, as deep physical principles, fundamental starting points in thinking about why physical theories are the way they are, so to speak. This finds expression, for example, in the prominent current view of symmetry as undergirding our physical worldview in some strong sense"

and [*ibidem*, p.52]:

> "gauge invariance is often invoked as a supremely powerful, beautiful, deeply physical, even undeniably necessary feature of current fundamental physical theory".

In fact, it also turns out that the gauge paradigm exhibits an appealing simplicity in that few inputs are required to specify full theories [Martin, 2003, 53], and "*the fact that all non-gravitational interactions fit into the gauge framework then lends this simplicity to a large part of fundamental physics*" [*ibidem*]. This leads to one of the most attractive features of this new physics: its unificatory role. All elementary interactions (though gravitation only in theory) are described in terms of local gauge symmetries. Thus, sophisticated "elegant" mathematics provide a foremost unified account of the fundamental interactions.

Last, but not least, physicists have proudly achieved so as a result of conceptual (mathematical) work much before the posterior solid experimental support: see e.g. [Bangu, 2008] for the study of impressive historical cases, like the prediction of the Ω^- boson (cf. [Weyl, 1952] and [Wigner, 1980] for elaborated reflections stemming from such astonishing successes).

In sum, all these have been central reasons for the enthusiasm towards this new physics and, more specifically, for the judgments of elegance and of (a vaguely stated) necessity[2].

Now, contrary to this positive attitude towards the symmetry principles I will argue that, in spite of their astonishing empirical and theoretical success, it is not unreasonable that such principles (and mathematics in general) are so effective in fundamental physics. On the opposite, it is what should be expected.

3   Elegant but contingent local gauge symmetry

3.1   The argument

Having outlined in the previous section some key points of the intimate relation of mathematics with the new physics, the crucial question can be formulated like that: Why is group theory so central to describing the physical world? And of course not only group theory but also Hilbert

spaces, riemannian geometry, differential calculus, and so on and so forth. In this essay I'll try to dissolve the wonder by sketching an explanation of why this is so. The conclusion, call it (C), is:

> *(C): It is not unreasonable to expect that the part of the world investigated by the physical sciences is going to be described by mathematics.*

I will put forward two premises (A) and (B) that aim to be sufficient to explain (C). Notably, the explanation proposed does not need to confer any privileged status neither to the dynamical principles of our current physics nor to mathematical entities in general. As such, it can be considered a better explanation than those that, too much nurtured by the effectiveness of mathematics postulate more abundant ontologies. Examples of the latter are the several variants that bestow existence to mathematics *in* the world, as [Tegmark, 2014] or [French, 2014].

First, premise (A). Scientists and engineers are aware of the utility of the 'divide and conquer' strategy, and so I am going to do via the premise (A): I will distinguish two different problems that should not be conflated. (A) consists in the acknowledgement of the following uncontroversial fact:

> *(A): The physical world displays stable spatiotemporal patterns.*

With this premise we assume the uncontroversial fact that the world is ordered, that it displays stable patterns of behaviour—what the laws of physics aim to describe. (A) is not only a necessary premise to justify (C), but it also helps to distinguish two different puzzling issues, namely (A) and (C) themselves. Thus, it hopefully contributes to face better both issues. That is, I am disentangling the puzzling fact that the world is ordered from the puzzling fact that mathematics seems unreasonably effective to describe the world. The fact that (A) is *distinguished* from (C) and *assumed* as a premise consists in the first of the two steps needed to explain (C). Of course (A), in this essay assumed, is prone of further investigation[3].

So, having disentangled these two different puzzles and assuming (A), now we will need a further premise (B) in order to explain (C). This second premise characterizes an essential property of mathematics. The underlying idea of (B) is that mathematics can be seen just as a language, the most appropriate language at our diposition to describe the degree of complexity of Nature's order—more exactly, the degree of complexity of the empirical data we have been able to extract from Nature[4]. Crucially, (B) defends in particular that it is a language full of non-actualized possibilities, and that the mathematics that is actually constituting our best physical theories is only one of the infinite possible mathematical descriptions of the regularities of a world. That is, among the infinite possibilities, the subset of maths that constitutes our actual physical theories is that which best fits with the patterns and order empirically found in Nature. In brief:

> *(B): Mathematics (or at least part of it) describes an extremely wide range of actualized as well as non-actualized structures.*

So, while the vast majority of patterns referred in (B) will not correspond to the *actual* structures/patterns of the world referred in (A), the acknowledgement of (A) and (B) allows the feasibility that *within this extremely wide space of possibilities a subset of those abstract patterns matches some of the actual patterns of Nature.* Hence, it should not strike us as unreasonable that mathematics is highly effective in physics, as the conclusion (C) states.

While the acceptance of (A) is hardly disputable, (B) should be justified, and this is what I am going to do in the next subsection. Before, let me consider a possible objection to the argument. An opponent could still be puzzled by such effectiveness, asking how is it that we *currently* have the proper mathematics for the current physical theories. However, this is a qualitatively different question than that which just asks for the unreasonable effectivenes of mathematics in physics *per se* and that my argument above aims to answer. This new question involves considerations regarding the maths we have discovered (or created, as you prefer) so far, and how is it that the existing maths is so successful with the existing physics. Interestingly, this new question allows me to add the observation that among the spatiotemporal patterns existing out there (as acknowledged in premise (A)), we are able to discern *only* those that our current mathematics—of a certain degree of expressivity σ— allows us.

In sum, (A) and (B) constitute a possible explanation of the effectiveness of mathematics in physics. Crucially, it is an explanation that does not need to postulate any sort of existence to the mathematical entities: as a language, those entities do not exist in the same sense as the terms of natural language do not exist.

The postulation of mathematics in the world (as in [Tegmark, 2014] and somehow also in [French, 2014]) or the privileged status of current symmetry principles (as portrayed in section 2) was motivated —among other reasons— by the unreasonable effectiveness of mathematics in physics. But my proposal explains such effectiveness in a way that does not need to make any strong philosophical interpretation of the dynamical principles or of the mathematical entities. And let me note that the alternative candidates do not have reasons to refute my premises (A) and (B), therefore they have at their own disposition an explanation with a more economic ontology. Thus, *caeteris paribus* this explanation is preferable.

Therefore, the mysterious effectiveness of mathematics ceases to be a valid reason to justify such strong positions (which obviously does not mean that such positions might not have other reasons in their support).

3.2 The case of the mathematical representation of the strong interaction

Is then my explanation compatible with the picture presented in section 2 regarding the mysterious effectiveness and astonishing success of gauge symmetries in particle physics? What about all the extraordinary virtues of the new physics? In this subsection I focus on highlighting how the symmetry groups constituting the Standard Model of particle physics are, in spite of their remarkable successes, not *a priori* reasonable nor necessary at all, but they are significantly *contingent* and have been chosen due to empirical adequacy *among a large space of possibilities*. In that way both premise (B) will be justified while also any sort of necessity or *a priori* reasonableness of the mathematics of our current physics will be explicitly refuted, contrary to what section 2 suggested.

Introducing SU(3) Let's take a look at one specific case: the color local gauge invariance of quarks, one of the several local gauge symmetries, hence part of the allegedly impeccable gauge paradigm. The color invariance is represented by the symmetry group SU(3), the Special Unitary group of degree 3[5]. Each gluon, to preserve the internal symmetry, carries one unit of color and one of anticolor (the "colors", 'red', 'green', and 'blue', name the charges of this interaction). Therefore, there are nine logically possible combinations of the 3 colors: $r\bar{r}, r\bar{g}, r\bar{b}, b\bar{r}, b\bar{g}, b\bar{b}, g\bar{r}, g\bar{b}, g\bar{g}$. Every symmetry group has the so called 'representations'. It is always the so called 'adjoint representation' that describes the force carriers, in this case the gluons[6]. The adjoint representation of SU(3) is not nine but eight dimensional. In this rep-

resentation the nine states are structured in an octet and the other state is a singlet element apart. The linearly independent base vectors that constitute the octet are:

$$
\begin{array}{llll}
|1\rangle &=& (r\bar{b}+b\bar{r})/\sqrt{2} & \quad |5\rangle &=& -i(r\bar{g}-g\bar{r})/\sqrt{2} \\
|2\rangle &=& -i(r\bar{b}-b\bar{r})/\sqrt{2} & \quad |6\rangle &=& (b\bar{g}+g\bar{b})/\sqrt{2} \\
|3\rangle &=& (r\bar{r}-b\bar{b})/\sqrt{2} & \quad |7\rangle &=& -i(b\bar{g}-g\bar{b})/\sqrt{2} \\
|4\rangle &=& (r\bar{g}+g\bar{r})/\sqrt{2} & \quad |8\rangle &=& (r\bar{r}+b\bar{b}-2g\bar{g})/\sqrt{6}
\end{array}
$$

and the singlet element is:

$|9\rangle \;=\; (r\bar{r}+b\bar{b}+g\bar{g})/\sqrt{3}$

The combination $r\bar{r}+b\bar{b}+g\bar{g}$ is not verified in experiment [Griffiths, 2008, 285].
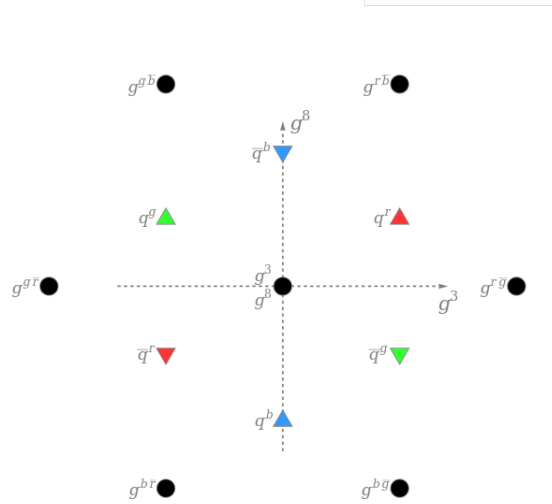

Figure 1: The pattern of strong charges for the three colors of quark, three antiquarks, and eight gluons (in black) with two of zero charge overlapping in the center. The vertical axis is strangeness and the horizontal is isospin.

Thus, the eight gluons that exist in Nature are described by the eight so called 'generators' that compose the octet, the set of linearly independent vectors above that form a vector base of the 8 dimensional group SU(3). Each generator aims to represent the color state of a certain type of gluon[7]. The situation is beautifully illustrated in figure 1. The octet of the figure illustrates indeed the existence of a tight pattern between the gluons (and also with the quarks).

However, to what extent should we "celebrate" the beautiful and unified pattern exhibited between the gluons? The next subsections argue that we should not celebrate too much, as the mathematical model is not so mysteriously successful or necessary as these aesthetic patterns (and all that has been said in section 2) might suggest.

The symmetry space First of all, consider the space of possibilities of symmetry groups. This space has been explored and classified in the 'Cartan classification'. The full classification of all possible 'simple' Lie algebras is divided in four types [Lederman and Hill, 2004, 315]:

1. Rotational symmetries of spheres that live in N real coordinate dimensions: O(2) = U(1), SO(3) = SU(2), SO(4), SO(5), ... , SO(N), ...

2. Rotational symmetries of spheres that live in N complex coordinate dimensions: U(1), SU(2), SU(3), SU(4), ... , SU(N), ...

3. Symplectic groups, which are the symmetries of N harmonic oscillators: Sp(2), Sp(4), ..., Sp(2N), ...

4. The 'exceptional' groups: G_2, F_4, E_6, E_7 , and E_8

As it appears, the resulting landscape is undoubtedly vast; indeed, it is infinite, as we see in the infinite order of the Lie groups. Thus this classification allows us to realize the first dimension of the contingency of the symmetry groups chosen: SU(3) is just one of the infinite groups at stake. Not to say that with this classification we are already assuming the subset of Lie groups, which is something hardly justifiable *a priori*.

Fermions There is a further layer of contingency if one focuses not on the bosons but on the fermions. As previously stated, for each group there are infinite possible representations. While for the bosons the representation chosen is always unique, namely the adjoint representation, for the fermions the physically interesting representations are the so called 'irreducible representations'. The states in the irreducible representation are those that possess the determinate properties measured in reality, like isospin and hypercharge for the case of SU(3)[8]. Thus, the connection of a symmetry group with physical reality —with empirical data— is made through the choice of an irreducible representation of the group [McKenzie, Forthcoming, 10]. It turns out, though, that there are infinite representations of this type for each group [*ibidem*]. So there is a connection mapping the irreducible representation with a physical interpretation of families of particles that would exist in the world. Therefore, there are *a priori* infinite possible classes of sets of particles allowed for *each* of the (in turn, infinite) symmetry groups.

The moral I want to draw is that, in the end, the particular final choice is made among an extremely vast space of possibilities.

But... the self-consistency is astonishingly constraining What remains, then, of the apparent inevitability and *a priori* reasonableness presented in section 2? Recall for instance the impressive historical cases of *theoretical* postulations of certain types of particles, empirically verified only much later. This preeminence of the theoretical research is due to the high degree of mathematical consistency and inter-dependence of the different parts of the theory of Quantum Chromo Dynamics (QCD). And this is an exceptional situation in the history of physics that is probably suggesting something. What could be suggesting? Wilczek [2008] talks about the approach to *perfection* of the theory of QCD. He does so by appealing to the notion of the *fragility* of a theory, which consists in pointing out the mathematical consistency between the different parts of the theory such that the possibilities that can be found *a posteriori* are highly constrained. Thus, the theory is fragile because it is very open to refutation: a slightly empirical inconsistency would imply that the whole theory is wrong! This characteristic is brilliantly illustrated by an analogy with a musical score: the score is *perfect* because if one displaces one note everything diminishes, if we displace a phrase, the whole structure falls [Wilczek, 2008, 135]. But then, it must mean something that such a fragile theory as QCD is obtaining such a precise and abundant empirical support.

My diagnosis of what does it mean is the following. Wilczek's analysis, interesting as it is, bolsters the plausibility that the theory is on the right track, i.e. that it is *true* (and as such it could be considered a novel argument for scientific realism). However, it is not an argument for any sort of necessity nor anything suggested in section 2, and it is not incompatible with the characterization of mathematics stated in (B).

Summary In conclusion, I want to underline that, *pace* section 2, it is a well-known fact that the mathematical description of the strong interaction is a contingent representation among a wide space of possibilities, and is clearly not *a priori* nor necessary in spite of its elegance, of its

fragility, of the unified account with the rest of interactions, and of the theoretical predictions much before any empirical evidence.

[Martin, 2003, 52] shares the same diagnosis "against" gauge invariance. He remarks other factors that have to be taken into account when a gauge-invariant term is added into the lagrangian. These other requirements are Lorentz invariance, simplicity, and renormalizability [Martin, 2003, 44]. His main upshot is to highlight the heuristic character of such symmetries, showing how *"the gauge fields are put in by hand to large extent"* [Martin, 2003, 45].

Recapitulating, this review has been carried out to show the different dimensions of contingency of an especially beautiful and celebrated part of mathematical physics, while at the same time justifying the characterization of mathematics stated in (B).

3.3 Corollary: should we be satisfied with such laws?

There is another moral that stems from the partial characterization of mathematics presented in 3.2. If symmetries are neither necessary nor *a priori*, they can be hardly considered as undisputable irreducible primitives of an ontology. Hence, some sort of explanation of symmetry principles is desirable.

All the candidate fundamental physical theories, even the most natural versions, share such local gauge symmetries as an essential constituent, thereby becoming subject of the present analysis[9]. To advocate for this need of explanation of symmetries—and in general of the fundamental laws of nature—is a secondary goal of this essay. It is not a trivial goal because nowadays the widespread attitude in physics and philosophy of physics is to assume bigger symmetry groups in higher energy scales, without any worry as to their explanation. In general, the worries revolve around the process of (spontaneous) symmetry breaking towards lower energies, whereas the other way around, towards higher energies, symmetries are just assumed to be restored. In fact, in the compendium [Brading and Castellani, 2003], in the encyclopedia entry [Brading and Castellani, 2013], or in the handbook chapter [Bangu, 2013] any demand of explanation of the highly symmetric picture assumed at high energies is barely cited.

4 Concluding remarks: it was not so unreasonable

The goal of this paper has been to discard the astonishing effectiveness of mathematics as a reason to endorse a strong interpretation of mathematics in the line of [Tegmark, 2014], [French, 2014] or [Weyl, 1952], given the alternative explanation that has been sketched.

Let me frame my analysis among other philosophical discussions and review whether it is independent of them. The characterization of mathematics carried out in premise (B) and defended in the previous section 3 can be naturally framed along with a nominalist version of the so called 'structuralism' in philosophy of mathematics (see e.g. Shapiro [1997]), dating back to Richard Dedekind. Still, I think that other philosophical approaches to mathematics are compatible with the undemanding claim stated in (B)—it is hard to see any incompatibility of (B) with logicism, formalism, or intuitionism.

There are other discussions related to the nature of mathematical entities: do mathematicians discover or create such mathematical objects? I would say this question is orthogonal to my argument, i.e. there is no need to commit to a specific answer.

Another discussion regards the alleged *indispensability* of genuinely mathematical terms in the explanation of physical phenomena. Maths seems to be *"indispensable to our best theories of the world"* [Bueno, 2014, 2.2]. Then, some defended an entailment that goes from the indispensability of the entities in our best scientific theories to the existence of such entities. Just the opposite of what I defend here.

The indispensability argument is independent from my proposed explanation of the effectiveness

of mathematics. Even so, let me mention what can be said about it from the viewpoint here proposed. Accepting the characterization of mathematics presented, *there is not any reason to accept an inference that goes from indispensability to existence.* Not to say that the indispensability is even disputable, as defended e.g. by [Field, 1980]. More specifically: assuming that mathematics is a language, even if accepting its indispensability to the expression of our best theories of the world, mathematical entities do not exist in the same sense as the terms of natural language do not exist[10].

All in all, if we assume (whether we are convinced or not by the previous subsection 3.2) the characterization of mathematics as stated in (B) and we distinguish as a separated issue the fact that the world displays patterns and order—as stated in (A)—, then (C) is justified. That is: it is not unreasonable to expect that the physical world is going to be described by mathematics. In fact, it should not appear unreasonable that there is a correspondence between a subset of all the possible abstract patterns that mathematics offers to us and the actual patterns displayed by the physical world. Within this picture it is, in fact, what should be expected[11].

Notes

[1] The gauge principle specifies a procedure for obtaining an interaction term in the Lagrangian which is symmetric with respect to a continuous symmetry. The results of localizing (or 'gauging') the global symmetry group involves the introduction of additional fields so that the Lagrangian is extended to a new one that is covariant with respect to the group of local transformations. Remarkably, it turns out that nowadays all the fundamental interactions of the Standard Model can be described according to this procedure.

[2] I am assuming an ontological interpretation of local gauge symmetries, not representing a mathematical redundancy in our description of the world. This means that, paraphrasing Wigner, local gauge symmetries do have an ontological "active" role and do provide physically significant claims about the carvings of Nature. While my assumption is widespread, the issue is nevertheless unsettled and the alternative interpretations exist.

[3] The acknowledgment of (A) motivates questions like 'Why there are laws?' or similarly the study of what has been dubbed as the emergence of "order from chaos", investigated for instance by Prigogine and Stengers [1984] or Peirce [1867]. Let me also cite myself, as my doctoral dissertation reflects upon the notion of law of nature and studies specifically the plausibility of formation of stable behaviour from a lawless fundamental level Filomeno [2014].

[4] I am hoping that in this brief essay it will suffice to stick with a pre-theoretical/intuitive understanding of what a language is, given that a full characterization of mathematics as a language is not necessary for my argument. Either way, a pertinent definition of 'language' is when it is understood as a grammar, where a grammar is defined as a system that contains representations and rules that relate such representations.

[5] The story goes roughly like this: the mathematical description SU(3) has been chosen following empirical adequacy and consistency with the rest of models of particle physics, constituting a highly unified model of the fundamental interactions. Unified mainly because the same principle, the local gauge principle, is shared by every model of each type of field/particle. (More precisely, some global models have solid empirical support but are not so unified as to, for instance, incorporate gravity, while others are more unified but lack empirical evidence. The Standard Model and Supersymmetry are the classic respective examples). Thus, the theory of Quantum Chromo Dynamics (QCD) successfully describes the color strong interaction: the property of color is conserved due to a certain type of bosons (force carrier particles) called gluons, exchanged in the interactions (whether I refer to fields or particles is irrelevant for our purposes). Each of them carries one unit of color and one unit of anticolor. Thus they guarantee the conservation of the initial color that changes in the quark in a 'strong' interaction with another quark. Formally, SU(3) is a real group in complex dimensions of degree N=3 of the classical Lie groups. The dimension of SU(N) groups, as real manifolds, is $N^2 - 1$. Therefore for SU(3) the dimension is 8. To preserve the color eight gluons do the work. The structural representation of (the properties of) those eight gluons is the symmetry group SU(3). The gauge fields/particles are associated with a set of vectors that are the so called 'generators' of the group. All this representation is translated into a new term in the lagrangian so that the lagrangian becomes invariant under the operations of the group.

[6] An adjoint representation of a Lie group G is one of the ways of representing the elements of the group as linear transformations of the group's Lie algebra, where the elements constitute a vector space (this is what a representation is in general). Specifically, the adjoint is the representation in which the structure constants themselves form a representation of the group.

[7] The states are added in linear combinations according to the principle of superposition of Quantum Mechanics. The numerical parameters are required for normalization.

[8] Informally, the irreducible representations of a group are the representations of the smallest possible order, i.e. those that cannot be further reduced. More technically, they are said to have no nontrivial invariant subspaces. Let me also note that the force-carriers correspond, in general, to the eigenvectors of the generators while the eigenvalues of those eigenvectors are the physically measurable charges (color, in this case).

[9] The prospect of a future unification is what might resolve the tension, because a unification of the several interactions would be obviously simpler, more natural and therefore allegedly more *a priori* reasonable. However, this hope has been seriously undermined in studies like [Maudlin, 1996] or [Morrison, 2013]. In the same line, the most recent experiments at the LHC are strongly suggesting the abandonment of supersymmetry in its most natural and simpler versions, since these are not finding the expected empirical support.

[10] There is abundant philosophical literature dealing with the puzzling ontological status of abstract entities, like linguistic entities (terms, propositions), concepts, or fictional characters. Some seek to grant a certain type of existence to such entities. However, it is crucial to note that the type of existence they grant is always different from the usual sense of the term 'existence' (roughly understood as concrete spatiotemporal location). Therefore, my claim that linguistic terms do not exist, given that I am employing the usual sense of the term, is not threatened by those discussions.

[11] Let me thank here the valuable suggestions of my smart colleagues of the LOGOS Graduate Reading Group (especially David Rey, Mairym Llorens, Romina Zuppone and Roger Deulofeu).

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