

Is the moon there when nobody looks?

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I. ABSTRACT

Einstein asked: whether I really believed that the moon exists only when I look at it. In this essay, we partially try to answer this question. We considered some realistic non-local hidden variable models which simulate the quantum correlation function (singlet state). We have derived inequalities which are based on these models and show that these inequalities are violated by quantum predictions. It prompts revisiting such models from logical perspective. These results raise some questions: Can quantum predictions are simulated by nonlocal realistic models? Can this approach be extended to general cases? In this essay, We will try to answer these questions.

II. MOTIVATION

The development of quantum mechanics in the early twentieth century obliged physicists to radically change some of the concepts they employed to describe the world. It challenged some of the accepted assumptions physicists were using for a long time. Uncertainty principle and quantum entanglement are at the heart of quantum physics, both for its conceptual foundations and for applications in quantum information. Entanglement is a property unique to quantum systems. Two systems are said to be quantum entangled if they are described by a joint wave function that cannot be written as a product of wave functions of each of the subsystems.

Quantum entanglement was first viewed as a source of paradoxes, most noticeably the Einstein-Podolsky-Rosen paradox (EPR) [1], which explicitly suggested that any physical theory must be both local and realistic. Two main assumptions of Einsteins realism involve separability and locality principles.

Einstein has described his key point of reality as follows: "This [simultaneous predictability] makes the reality of P , and q , depend upon the process of measurement carried out on the first system which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this." Einstein-Podolsky-Rosen paradox simply concludes that objective reality is incompatible with the assumption that quantum mechanics is complete. This conclusion has not affected subsequent developments in physics and it is doubtful that it ever will. As Pais tells us [2]: "We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it". Moreover, Planck wrote "Is there an external world?" Plancks and Einsteins question lies in the background behind the famous question: Can quantum mechanical description of reality be considered complete?

As we read at Bells paper [3], the EPR conclusion serves to present "an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality". Moreover, Bell showed that the correlations among the measurement outputs of space-like separated parties on some quantum states cannot be reproduced by a local theory. Bell's theorem is the first place where the locality assumption is quantified.

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These conditions then manifested themselves in the so-called Bell inequality [3], where locality is a crucial assumption but is violated by quantum mechanical predictions. In the Bell's terminology, quantum mechanics is not a locally causal theory. In what has become standard terminology, this fact is often referred to as quantum non-locality and has been recognized as the most intriguing quantum feature. Bell's inequality has been derived in different ways [4, 5]. Over the past thirty years a very large number of experiments have been conducted with the aim of testing the predictions of quantum mechanics against those of local hidden-variable theories [6]. All of them give strong indications against local hidden variable theories. Non-locality is a fascinating chapter of physics and has attracted much attention since its discovery because it relates two fundamental aspects of nature, special relativity and quantum mechanics.

Recently, the Bell's original work has been extended to some classes which can be organized into three different categories:

1- Nonlocal Hidden-Variable Theories. In 2003, Leggett proposed an alternative model for non-local correlations which is incompatible with quantum predictions [7]. The Leggett's model (LM) is based on the following assumptions: (i) all measurement outcomes are determined by pre-existing properties of particles, independently of the measurement (realism); (ii) physical states are statistical mixtures of subensembles with definite polarization, where (iii) polarization is defined such that expectation values taken for each subensemble obeys the Malus law [7, 8].

Afterwards, Gröblacher *et al.* have theoretically and experimentally shown that LM is incompatible with the experimentally observable quantum correlations [8–10]. Moreover, Branciard *et al.* [11] and independently Colbeck and Renner [12], have gone beyond LM and have considered general hidden variable models which have both local and non-local parts. They have shown the existence of quantum correlations that are incompatible with any hidden variable model having a non-trivial local part, such as LM. Conversely, others have shown that this definitions of local part and trivial are not useful for addressing non-classical quantum correlations [13]. Furthermore, they have considered contextual models in which the results of parties' outputs depend explicitly and nontrivially on the local hidden variables [13, 14].

2- Simulating quantum correlation function. Some authors have extended Bell's approach, by considering realistic interpretation of QM and using shared random variables augmented by classical communication or nonlocal effects and simulated the quantum correlation function (singlet state) [13–29]. Afterwards, this approach has been extended to quantum predictions for all of product of Pauli operators on n-qubit GHZ state [30] and graph states [31].

3- Breaking the Bell barrier. Independently of the above developments, Popescu and Rohrlich [32] have raised a question: can there be stronger correlations than the quantum mechanical correlations that remain causal (i.e., that do not allow signaling)? They answered by exhibiting an abstract non-local box wherein instantaneous communication remains impossible. This non-local box is such that the CHSH inequality is violated by the algebraic maximum value of 4, while quantum correlations achieve at most $2\sqrt{2}$ [32–34]. There is a question of interest: Considering that perfect non-local boxes would not violate causality, why do the laws of quantum mechanics only allow us to implement non-local boxes better than anything classically possible, yet not perfectly [35]? Recently, van Dam and Cleve considered communication complexity as physical principle to distinguish physical theories from non-physical ones. They proved that the availability of perfect non-local boxes makes the communication complexity of all Boolean functions trivial [36]. Afterwards, Brassard *et al.* [35] showed that in any world in which communication complexity is nontrivial, there is a bound on how much nature can be nonlocal. Besides, Pawłowski *et al.* [37] defined information causality as a candidate for one of the foundational assumptions of quantum theory which distinguishes physical theories from non-physical ones. In addition of this approach, there is another link between the uncertainty principle and non-locality. This indicates that quantum mechanics cannot be more

nonlocal with measurements that respect the uncertainty principle and the link between uncertainty and non-locality holds for all physical theories [38]. In other words, the degree of non-locality of any theory is determined by two factors: the strength of the *uncertainty principle* and the strength of a property called *steering*, which determines which states can be prepared at one location given a measurement at another.

In the following, I briefly describe my recent works and future research plans.

My Research Statements

III. MY WORKS ON LEGGETT'S MODELS AND RELATED PAPERS

Although it seems assumptions of LM are “natural”, but they are in conflict with quantum predictions. Does it mean one of these assumptions is in conflict with QM? Can Leggett's inequality be extended to non-local hidden-variable models such as LC's model? In these part of work, we have tried to answer these questions. Explicitly, we have demonstrated that the incompatibility of non-local hidden variable models [7, 8, 11, 12] can be argued differently, without need to invoke violation of an inequality. I have proved that validity of Malus law leads to an incompatible results with quantum correlation functions. Afterwards, I have considered a new type of probability distribution function $F(\hat{u}, \hat{v}) = F(\hat{u})\delta(\hat{u} + \hat{v})$ [39] and have shown that this incompatibility is valid for new Leggett's probability distribution function. Furthermore, I try to extend LM to more general cases with minor assumptions (such as counterfactual definiteness) [14].

IV. THE BASIC PRINCIPLES AND SIMULATING QUANTUM CORRELATION FUNCTION

Although, a common sense doesn't exist about violation of Bell's inequality and scientists believed that Bell's inequality is based on different assumptions [40], however, we can find two common assumptions at them, locality and reality.

The Bell-inequality violations for the vast majority of the quantum-foundations community is that it signals nature to be non-local. But non-locality is only one of two possible explanations for the violation. The other is that quantum measurement results do not preexist in any logically determined way before the act of measurement. In this part, we have gone to beyond of present approaches. Here, we consider models which neither have trivial communication complexity [35, 36] nor break information causality principles [37]. For example, I have considered the aforementioned principles and Boole's inequality [41] and proved that it is impossible to construct consistent shared random variable theories augmented by classical communications [15]. Furthermore, we have reviewed all recent works on the simulation of singlet quantum correlation function [13–29]. We have derived inequalities which are based on these models. **These inequalities are violated by quantum predictions.**

To clarify our approach, here, we derive inequality which based on Toner and Bacon protocol [15] and show that it is violated by quantum correlation function.

In the Toner and Bacon protocol Alice and Bob share two independent random variables $\hat{\lambda}_1$ and $\hat{\lambda}_2$ which are real three dimensional unit vectors. They are independently chosen and uniformly distributed over the unit sphere (infinite communication at this stage). Alice measures along \hat{a} , Bob measures along \hat{b} . They obtain $\alpha \in \{+1, -1\}$ and $\beta \in \{+1, -1\}$ respectively, where α and β indicate whether the spin is pointing along (+1) or opposite (−1) directions with respect to chooses to measure. TB protocol proceeds as follows: (1) Alice's outputs are $\alpha \equiv A = -\text{sgn}(\hat{a} \cdot \hat{\lambda}_1)$. (2) Alice sends a single bit $c \in \{-1, +1\}$ to Bob where $c = \text{sgn}(\hat{a} \cdot \hat{\lambda}_1)\text{sgn}(\hat{a} \cdot \hat{\lambda}_2)$. (3) Bob's outputs are $\beta \equiv B = \text{sgn}[\hat{b} \cdot (\hat{\lambda}_1 + c\hat{\lambda}_2)] =$

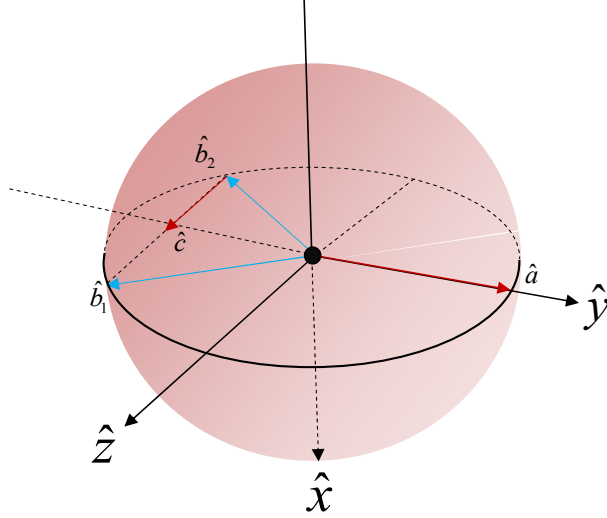


FIG. 1: (color online) Alice and Bob measurement settings are presented by \hat{a} and \hat{b}_i , $i = 1, 2$ respectively which lie in the $y - z$ plane so that the angles \hat{a} and \hat{b}_i , $i = 1, 2$ being evenly spread as $\Delta = \hat{b}_i - \hat{a} = \pi - \alpha$. In this figure, $-\hat{a}$ divided angle between \hat{b}_1 and \hat{b}_2 to two equal parts.

$\frac{1+c}{2} \text{sgn}[\hat{b} \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)] + \frac{1-c}{2} \text{sgn}[\hat{b} \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)]$, where sgn function is defined by $\text{sgn}(x) = +1$ if $x \geq 0$ and $\text{sgn}(x) = -1$ if $x < 0$. The joint expectation value $\langle AB \rangle$ is given by equation (1) in [15].

In the TB model, we consider two sets of measurement settings (\hat{a}, \hat{b}_1) and (\hat{a}, \hat{b}_2) in the $x - y$ plane so that $-\hat{a}$ divided angle between \hat{b}_1 and \hat{b}_2 to two equal parts ($2\alpha = \angle(b_1 - b_2)$) as show Fig. (1). We take upper bound of the LM for these measurement settings and obtain

$$|A(\hat{a}, \hat{\lambda}_1) - B(\hat{a}, \hat{b}_i, \hat{\lambda}_1, \hat{\lambda}_2)| \leq 1 - A(\hat{a}, \hat{\lambda}_1)B(\hat{a}, \hat{b}_i, \hat{\lambda}_1, \hat{\lambda}_2), \quad i = 1, 2. \quad (1)$$

For simplicity, we represent the parties outputs by $B(\hat{a}, \hat{b}_i, \hat{\lambda}_1, \hat{\lambda}_2) \equiv B_i$. According to TB outputs, the above inequalities transformed to

$$|A - B_i| \leq 1 - AB_i, \quad i = 1, 2. \quad (2)$$

Summing up two inequalities together, one obtains:

$$|B_1 - B_2| \leq |A - B_1| + |A - B_2| \leq 2 - AB_1 - AB_2. \quad (3)$$

$$\begin{aligned} & \left| \frac{1+c}{2} \left\{ \text{sgn}[\hat{b}_1 \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)] - \text{sgn}[\hat{b}_2 \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)] \right\} \right. \\ & \quad \left. + \frac{1-c}{2} \left\{ \text{sgn}[\hat{b}_1 \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)] - \text{sgn}[\hat{b}_2 \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)] \right\} \right| \leq 2 - AB_1 - AB_2 \end{aligned} \quad (4)$$

Integrating over the constant probability distribution $F(\hat{u}, \hat{v}) = \frac{1}{(4\pi)^2}$, we have

$$\begin{aligned} & \frac{1}{(4\pi)^2} \int d\hat{\lambda}_1 \int d\hat{\lambda}_2 \left| \frac{1+c}{2} \left\{ \text{sgn}[\hat{b}_1 \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)] - \text{sgn}[\hat{b}_2 \cdot (\hat{\lambda}_1 + \hat{\lambda}_2)] \right\} \right. \\ & \quad \left. + \frac{1-c}{2} \left\{ \text{sgn}[\hat{b}_1 \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)] - \text{sgn}[\hat{b}_2 \cdot (\hat{\lambda}_1 - \hat{\lambda}_2)] \right\} \right| \leq 2 - \langle AB_1 \rangle - \langle AB_2 \rangle, \end{aligned} \quad (5)$$

The l.h.s. can be calculate directly, however, according to Fig. (2), we can calculate l.h.s. which

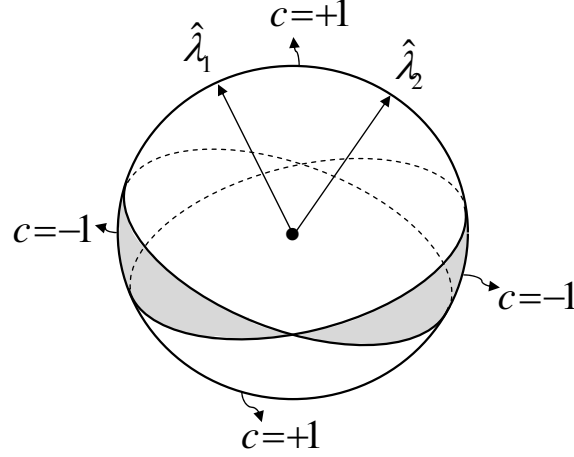


FIG. 2: The shared unit vectors $\hat{\lambda}_1$ and $\hat{\lambda}_2$ described in the text divide the Bloch sphere into four quadrants, as shown. Alice's and Bob's actions depend on which quadrant their respective measurement axes lie in, and in Bob's case, the bit he receives from Alice. If \hat{a} lies in the shaded region, Alice sends $c = -1$ and if her measurement axis lies unshaded region she sends $c = +1$ to Bob. Toner and Bacon deduce that Bob obtains no information about Alice's output from the communication.

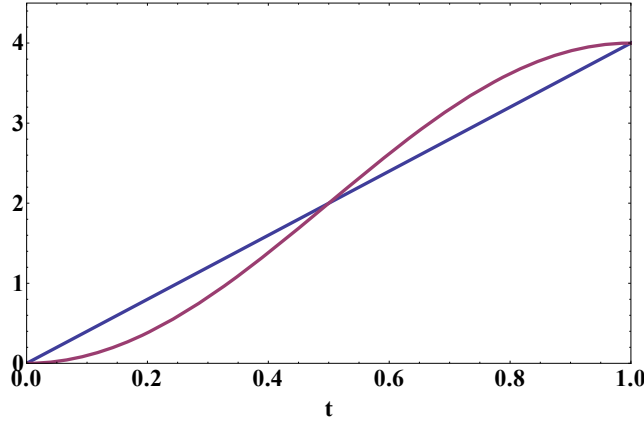


FIG. 3: (color online) In this plot, the blue and the red lines show l.h.s. and r.h.s. of inequality (6). The inequality is violated for $0 < \alpha < \pi/2$ interval, where, $\alpha = t\pi$, and $0 \leq t \leq 1$.

is equal to one $\frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin \theta d\theta = 1$ by taking \hat{a} in the \hat{y} direction. In the second integral, we take $\hat{b}_1 - \hat{b}_2 = 2(1 - \cos 2\alpha)\hat{c}$, $\hat{c} = \hat{z}$ and $\hat{a} = \hat{y}$ directions, therefor, integral is equal to $\frac{1}{2\pi} 2(1 - \cos 2\alpha) \int_0^{\pi} d\varphi \int_0^{\pi} \sin \theta |\cos \theta| d\theta = 1 - \cos 2\alpha$. Therefore, Eq. (5) becomes

$$\frac{4\alpha}{\pi} \leq 2 - \langle AB_1 \rangle - \langle AB_2 \rangle, \quad (6)$$

With using quantum correlation function $\langle A_{2i-1} B_{2i} \rangle = -\cos(a_{2i-1} - b_{2i}) = \cos \delta$, the above inequality is transformed to $1 - \cos 2\alpha \leq 2 - 2\cos \delta$ which is clearly violated over finite range of $0 < \alpha < \pi/2$.

V. QUESTIONS AND OUTLOOKS

These results raise some questions: Can quantum predictions are simulated by nonlocal realistic models? On the other hand, can this approach be extended to general cases? Recently, an operational definition of

contextuality are introduced. In an operational interpretation of a physical theory, the primitive elements are preparation procedures, transformation procedures, and measurement procedures [43–46]. In the next step, we try to extend our approach to these contextual models and answer to these questions.

In other hand, recently, researchers take their attention to find a complete set of natural and information-theoretic principles that clarify relation between non-locality and quantum mechanics. This new approach aims to derive connections between quantum mechanics (its application in quantum information and quantum computing) and some basic principles which are independent from quantum theory and its mathematical structures. These attempts have been based on the new principles which comes from computer science or information theory. The communication complexity [36] the information casuality [37] provided us with some rationale for why limits on quantum theory may exist. But evidence suggests that many of these attempts provide only partial answers.

I have asked a question: Is there some “natural” principles that restricts the degree non-locality of nature? Here, we take a very different approach and get very interesting primary results. My Principles are angular momentum quantization and conservation laws. My initial results have shown some of non-local models which simulate quantum correlation functions are not consistence with this principles and we have extended my approach to some simple types of non-local boxes.

These arguments indicate that we must have a deeper understanding of the notions of **non-locality**, **reality and entanglement**. Therefore, this research is expected to provide insights in many other areas of fundamental physics and to contribute to the development of a new informational view of nature.

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