Mountains on the Moon:
The Multiverse and the Limits of Physics

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Abstract

Can we make sense of a multiverse? I argue that many multiverse models can be meaningfully discussed, and confidently albeit not definitively evaluated using conventional theoretical and observational techniques. Further, I suggest that the residual uncertainty in our conclusions about any multiverse model is a novel manifestation of a routine phenomenon in modern cosmology: *extreme cosmic variance*.

"There is a mountain 10,000 feet high on the other side of the moon." Philosopher Alfred Ayer offered this proposition in 1934 not as a foray into amateur selenography, but as an example of a statement which, while well-posed and *verifiable* in principle, might remain forever untested in practice [1]. Ayer himself knew that only "practical disabilities" stood in the way of testing claims about the far side of the moon, and indeed, his example was indeed rendered obsolete only 25 years later when the Soviet probe Luna 3 snapped images of the the far side of the moon. We can now state with certainty that the highlands on the lunar far side rise by more than 10,000 feet [2].

Of course, it is unlikely Ayer lost any sleep worrying about lunar topography; his example was purely for the sake of argument.* When physicists make similarly speculative claims it is because we care about the answers; what keeps us up at night is precisely the worry that these claims may never be settled. Take, for example, the proposition "Our visible universe is just one pocket in the 'multiverse', a vast ensemble of 'universes'." What may make this claim untestable is not our inability to construct suitably sensitive instruments, but rather the finite speed of light, which ensures that distant pockets in the multiverse are forever unobservable. Conversely, the great leap forward in rocketry that led to moon missions was purely technological, and did not require rewriting the foundational laws of physics.

The fixed and finite speed of light is a bedrock principle of modern physics: finding an exception to this rule would immediately overturn relativity, a long-established paradigm.† Paradoxically, as we will see below,

*In discussing Ayer's *example*, I am not adopting his broader philosophical position.

†General relativity may offer loopholes in the form of wormholes and "warp drive" spacetimes, but there is no reason to believe that these exotic scenarios can be exploited in the physical universe, and I will not consider them further.

the possible existence of the multiverse follows from arguments that grow out of quantum mechanics and general relativity. If signals *can* travel at arbitrary velocities, general relativity would need major revisions or outright replacement – and there is no guarantee that arguments which point to the existence of a multiverse would survive this process.

The prohibition on faster-than-light signals limits us to a finite subvolume of a potentially infinite universe, so we cannot experimentally determine whether the *visible universe* is a representative sample of the universe as a whole. More dramatically, if our universe is merely one of a potentially infinite ensemble of "pocket universes" in a multiverse, the *visible* universe is a single subvolume of a single pocket. In typical multiverse scenarios, pockets are separated by regions of exponentially expanding spacetime, so photons (or any other physical signal) can never travel *between* pockets. To attempt such a trip is analogous to running on a racetrack which is stretched as the race progresses. If the stretching is slow enough, the runner can make it to the finish line, but if the length of the track regularly doubles (as it would with exponential expansion), the finish line forever recedes from the hapless athlete.

Pockets other than our own are thus isolated from any imaginable experimental apparatus, so the existence – or non-existence – of these pockets has no measurable consequences. This is why, unlike Ayer's mountain, claims about the multiverse might be untestable *even in principle*, lying beyond the purview of science. Further, in many multiverse scenarios, particularly those based on the string landscape [3], the apparent "laws" of particle physics differ between pockets. Thus, even if we construct an "ultimate theory of nature," it need not make distinctive predictions, in which case it is apparently untestable and, ironically, thus outside the realm of science. If one accepts that multiverse models are untestable, we are face to face with the limits of what is ultimately possible in physics.

Fortunately, the situation may be more subtle than the argument sketched in the previous paragraph would suggest. In particular, in this essay I will develop the following argument:

- The multiverse is not a "theory" *per se*; its existence is a *prediction* of many well-posed cosmological models.
- Each cosmological model that predicts a multiverse predicts a *specific* multiverse whose properties are characteristic of that theory.
- Even though we cannot disprove the existence of a *generic* multiverse, we can make meaningful statements about the likelihood that our visible universe exists with a given multiverse model.
- In fact, experimental cosmologists have already made such statements, although phrased in ways that made no mention of a multiverse.

- Statements as to whether we reside within any specific multiverse will always be probabilistic rather than definitive, and this residual uncertainty can be understood as an extreme manifestation of *cosmic variance*.

Our ability to *test* the multiverse or reach firm conclusions about a candidate "theory of everything" is thus more nuanced than it might have seemed. Some multiverse scenarios may well be essentially untestable. However, a genuine scientific discussion of the multiverse is at least possible, using only tools already employed by theoretical physics and cosmology.

Predicting the Multiverse

To put this discussion on a more quantitative footing, we should review the overall features of *inflation* [4], an era immediately after the big bang during which the universe expands at near-exponential speeds. This rapid stretching ensures that the universe is smooth and uniform on large scales; as such, inflation is a key component of almost all fundamental cosmological scenarios. Not only does inflation explain the overall uniformity of the universe, but quantum fluctuations during inflation plant the seeds that grow into the galaxies and clusters of galaxies that exist today. In most implementations, inflation occurs when the universe is dominated by the vacuum energy of a spinless (or scalar) field, ϕ, described by a potential $V(\phi)$, and the field slowly "rolls" downhill towards the minimum of the potential. This motion is classical, but the quantum fluctuations are just as likely to send ϕ uphill as downhill.

In the simple models, $V(\phi)$ has a straightforward algebraic form, e.g.

$$V(\phi) = \frac{m^2}{2}\phi^2 \tag{1}$$

$$V(\phi) = \frac{\lambda}{4}\phi^4 . \tag{2}$$

During inflation,[‡] the field rolls a distance $\delta\phi_{\mathrm{roll}}$ in the time the universe takes to roughly double in size,

$$\delta\phi_{\mathrm{roll}} = -\frac{1}{V}\frac{dV}{d\phi} . \tag{3}$$

The field rolls faster when the derivative of the potential ($dV/d\phi$) is larger, just as a ball rolling down a steep hill accelerates more quickly. During the same interval, the typical quantum fluctuation is

$$\delta\phi_{\mathrm{jump}} = \pm\sqrt{V(\phi)} . \tag{4}$$

[‡]This discussion reflects the "textbook" treatment of inflation; see [5]. It relies on general relativity – which controls the expansion of the universe, and quantum mechanics, which governs the hopping of the field.

The equations here use "natural units", and the energy density of the universe (roughly $V(\phi)$) must be less than one if the evolution of the universe is governed by general relativity, rather than quantum gravity. Since these equations were derived from general relativity, we need $V(\phi) < 1$ for our discussion to be self-consistent: this fixes the maximum value of ϕ that we can consider. Conversely, inflation ends when the field can roll to the minimum of the potential in less than the time it takes for the universe to double in size; $\phi \sim 1$ for the models here. Finally, the free parameters in the potentials (m^2 or λ) determine the depth of the primordial fluctuations. Observations of the microwave background – the "baby photo of the universe" – allow us to measure these fluctuations a little less than 400,000 years after the big bang. It can be shown that this fixes $\lambda \sim 10^{-14}$ and $m^2 \sim 10^{-12}$

So far, so good – and, so far, no mention of the multiverse. But can the quantum hopping of the field become bigger than the classical rolling? Looking at the $\lambda\phi^4$ potential, the quantum hopping and the classical rolling will become equal when $|\phi| \gtrsim 430$. However, $|\phi|$ can grow as large as ~ 4500 before $V(\phi) \approx 1$, invalidating the assumptions used to derive these equations. An entirely analogous argument applies to the $m^2\phi^2$ model.

We have every reason to believe that the universe "starts" with $V(\phi) \sim 1$. Very early in the inflationary era – long before the fluctuations now inside our visible universe are laid down – the quantum hopping will *dominate* the classical rolling. In this case, ϕ is just as likely to increase as it is to decrease, and large regions of the universe remain at very high densities. Inflation ends only in isolated patches, or pockets. These regions are vastly larger than the volume of our visible universe, and are separated by regions of exponentially expanding spacetime with an enormous energy density. This is of course the "multiverse" I described in the introduction.

Let's pause for a moment. The quantum fluctuations with which inflationary models ensure the formation of stars and galaxies appear to be equally capable of generating a multiverse. Not only that, with apparently natural choices of initial conditions, these inflationary models *will* generate a multiverse, so far as we can tell. In this sense, the multiverse is not a theory on its own; it is the *predicted* outcome of well-posed cosmological models, constructed from the familiar ingredients of gravity and quantum field theory. This may seem like a small distinction, but it is a vital qualification: simply *postulating* a multiverse is scientifically sterile, whereas specific inflationary models that predict multiverses are eminently testable.[§] Finally, we

[§]It has been suggested that in this sort of stochastic inflation scenario, inflation may end with a single, very large jump and bypass the "rolling" phase entirely. This is related to both the "youngness" [11] and the "Boltzmann brain" paradoxes [12]. Significant progress has been made in this area, but the argument here implicitly assumes that the conventional predictions for inflationary models are reliable, even in models that have a stochastic phase at high energies.

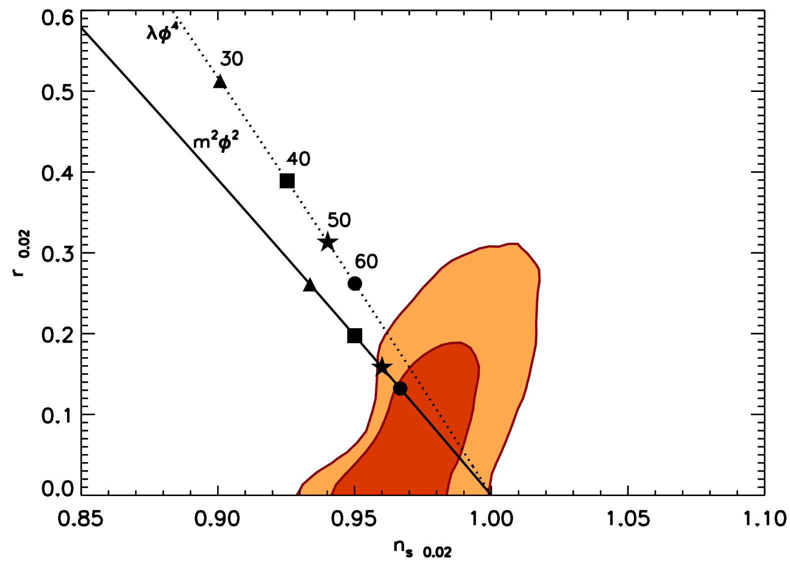

Figure 1: Joint 68% (inner) and 95% (outer) bounds on two variables which characterize the primordial perturbations, derived from a combination of WMAP5 and SuperNova Legacy Survey data. Predictions for our two inflationary models are superimposed. The numbers refer to the logarithm of the size of universe during the inflationary era. Cosmological perturbations are generated when this quantity is around 60, so $\lambda\phi^4$ inflation is significantly inconsistent with the data. [Adapted from [6]] .

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have identified two different *flavors* of multiverse – the $m^2\phi^2$ multiverse, the $\lambda\phi^4$ multiverse. There is an almost limitless class of possible multiverses, but for now we will work with these two examples.

Testing the Multiverse

The precise pattern of density perturbations produced by inflation depends on the detailed shape of the model's potential, $V(\phi)$. Observationally, we deduce this pattern from the distribution of galaxies in the sky, and the mix of the hot and cold spots in the cosmic microwave background. Models with different potentials make different predictions, and a major milestone of the current "golden age" of cosmology is that observations are now of such high quality that $\lambda\phi^4$ inflation is significantly inconsistent with the data, as seen in Figure 1.

To be formal, given the *prior* that the primordial perturbations in our universe were generated by $\lambda\phi^4$ inflation, the *odds* of discovering the observed perturbations in the sky are less than 0.01, whereas given the prior of $m^2\phi^2$ inflation, the odds are substantially better. Future experiments – both planned and actually under way – will dramatically shrink the contours in Figure 1 and, unless the central values move significantly, we will soon be able to reject $\lambda\phi^4$ inflation with far higher confidence. Given this, it is tempting to infer that our visible universe is not embedded in a $\lambda\phi^4$ multiverse. By inferring this, we apparently not only falsify a model of inflation, but a model of the *multiverse*.

In reality, the situation is more complex. Even a "perfect" dataset cannot reduce the odds to zero, thanks to *cosmic variance*. Consider an experiment that returns a map of the microwave sky,[¶] from which we extract the temperature as a function of angular direction, $T = T(\theta, \phi)$. To make contact with theoretical predictions we turn this map into a sum over the spherical harmonics, $Y_{lm}(\theta, \phi)$ (analogously to the decomposition of the tone of a musical instrument into its fundamental note and overtones), such that

$$T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} a_{lm} Y_{lm}(\theta, \phi)\,. \tag{5}$$

The l parameter fixes the level of detail in the harmonic – broad structures correspond to low l, whereas fine structure corresponds to high l. Since any map of the sky has a finite resolution, there is a cutoff above which the a_{lm} cannot be measured. The *monopole*, or C_0, is set by the average temperature of the microwave background; the *dipole* C_1 is dominated by the peculiar motion of the observer, relative to the average motion of all the mass-energy in the visible universe.

For $l \geq 2$, however, the a_{lm} and C_ℓ are functions of the density perturbations in the universe, whose detailed properties are predicted by our choice of inflationary model. Since most cosmological theories predict that the universe is, on average, the same in each direction, we are more interested in the variation with respect to l than to m, so we average over the m parameter to find the *angular power spectrum*,

$$C_\ell = \sum_{m=-l}^{l} |a_{lm}|^2\,. \tag{6}$$

The observed C_ℓ for our universe are shown in Figure 2. Unfortunately, an inflationary model does not predict the specific values of the a_{lm}, but gives the statistical distribution from which they are drawn. The C_ℓ are all averages of $2\ell + 1$ random numbers. We can predict the expected value

[¶]The microwave background is famously a blackbody, with a temperature of roughly 2.72K. However, the blackbody temperature varies as a function of position, with δT measured in microKelvin.

for the C_ℓ, but the actual value we see in the sky corresponds to a single throw of the dice.[‖] To "roll the dice" a second time, we would have to look at a different sky from our own, which is not possible without traveling a substantial distance across the visible universe, or waiting for a very long time (although see [7, 8]). This randomness is known as "cosmic variance" and is, at root, due to our inability to view the entire universe – which is itself a result of the finite speed of light, not to mention the technological challenges posed by interstellar travel.[**]

To understand the consequences of cosmic variance, let us turn to another gambling analogy. Imagine you sit down at a poker table with three strangers, and at the end of the first round, your tablemates are holding flushes, whereas you have three aces, a losing hand. Assuming honest players and a fair deck, the odds of this happening in the first hand are minuscule, but they *will* happen if you play long enough. You might well worry that you are participating in a rigged game, but you could test this hypothesis by continuing to play, assuming your patience, trust, and available funds were not already exhausted. Eventually you would have enough data to establish whether you were in an honest game. However, if the other players walk out into the night after the first hand you can never be *certain* (in the statistical sense) whether you were swindled, or merely the unfortunate witness to an honest but exceptionally rare event. As cosmologists we can only play the hand we are given. We can "repeat an experiment", but we are still looking at the same sky, just as our hypothetical card player can take a second look at the cards on the table, but cannot draw a new hand.

Thanks to cosmic variance, an inflationary model that predicts the existence of the multiverse will contain individual pockets within which the C_ℓ have all made large fluctuations from their expected values. Moreover, in some of these pockets the fluctuations will ensure that the microwave background mimics that which would be predicted by any other model of inflation. The odds of a given pocket in a $\lambda\phi^4$ multiverse overlapping with our expectations for a representative pocket in an $m^2\phi^2$ multiverse are minute[††] but calculable in principle. Crucially, they are not *identically* zero. Moreover, because the odds are not exactly zero, in an infinite number of $\lambda\phi^4$ pockets, some observers will erroneously conclude that they are embedded in an $m^2\phi^2$ multiverse.

As a consequence of *extreme cosmic variance* [10] we will never be able to

[‖] If you roll two dice, the most likely outcome is that they sum to 7, but they can add to any integer between 2 and 12.

[**] There are a number of suggestions for reducing the impact of cosmic variance (e.g. [9]), but none of them would reduce it to zero.

[††] Certainly less than 1 in 10^{1000}. In practice, we would look beyond the cosmic microwave background to whether our universe was consistent with a given model of inflation, but the same argument would still apply.

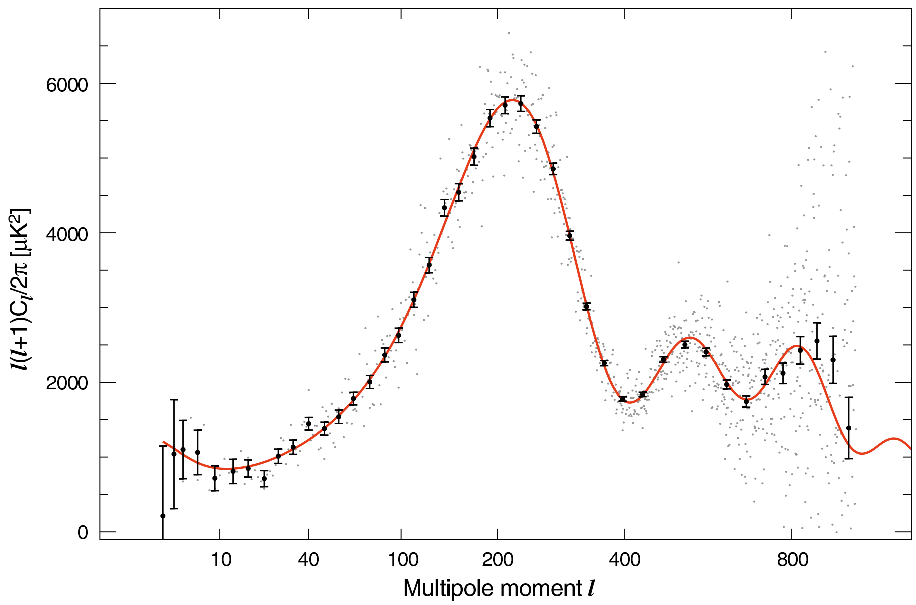

Figure 2: The observed C_ℓ for the cosmic microwave background. Grey points denote individual C_ℓ, solid bars are binned data, and the red line is the best fit to the "standard" cosmological model. The error budget is *dominated* by cosmic variance at low ℓ; at high ℓ the spread in the measured C_ℓ increases due to the finite resolution of the WMAP telescope. [Figure: WMAP5 Science Team].

definitively rule out the claim that our universe is embedded in either a $\lambda\phi^4$ *or* an $m^2\phi^2$ multiverse. To extend our poker analogy, if a player lays down five aces we *know* he is cheating. But a player who consistently gets dealt four aces may just be very, very lucky – and if you play for long enough, sooner or later an honest player will indeed be that lucky. Likewise, we can never know with certainty whether or not our sky is actually a remarkably rare "lucky" pocket in a $\lambda\phi^4$ multiverse.

The Good News or the Bad News?

A pessimistic interpretation of this argument is that we have indeed reached a limit of what is ultimately possible in physics, and in the strictly formal sense this is true. In practice, though, the news is not so dire. Theoretical models in physics are frequently tested statistically, rather than at the five aces level. The threshold for the definitive discovery of a new particle at an accelerator is widely said to be "5σ". This implicitly allows that there is a small but still finite chance that the purported signal is due to random events. Assuming Gaussian statistics, the odds of an $N\sigma$ deviation from an expected background are $P(N\sigma) = \mathrm{efrc}(N\sigma)$; erfc is the complementary error function, and $P(5\sigma) \sim 10^{-12}$, far *less* than the odds that cosmic variance would cause the occupants of a $\lambda\phi^4$ multiverse to believe they are living in an $m^2\phi^2$ pocket.

We can – at least in principle – repeat this experiment an arbitrary number of times. After 100 successful repetitions, the odds that all of these (assumed independent) datasets would yield the same, spurious signal fall to $(10^{-12})^{100} = 10^{-1200}$. However, since it is difficult enough to build a single Large Hadron Collider, economic realities alone will deter us from driving the likelihood of a spurious signal down to the 10^{-1200} level. Moreover, this estimate only accounts for the possibility of the experiment being fooled by a large statistical fluctuation – for large enough values of $N\sigma$, any erroneous claims of discovery are more likely to be due to unmodeled processes in the detector, or undetected mistakes, than to massively unlikely statistical fluctuations.

Extreme cosmic variance alone ensures that we cannot raise our confidence in any multiverse theory to an arbitrarily high level. However, we can still reject the hypothesis that we live in a *specific* multiverse with as much (and in practice, vastly more) confidence than we use to establish other claims in particle physics. While the multiverse forces us to confront a possible upper limit on the degree of rigor with which we can test fundamental ideas, these limits are still far beyond the level of certainty we can obtain in practice. What we have lost is not certainty itself – something we never possessed – but the illusion of the possibility of certainty. Philosophically (and perhaps psychologically) this is a crucial distinction, but happily it is not one that need change the way we practice fundamental science.

Coda: What About the Landscape?

Of course, the previous discussion has largely ignored the thorniest part of the problem, the "string landscape" [3] which is far more baroque than the two simple multiverse models discussed above. Within the string landscape, the analog of the potential depends on many fields rather than one, and these fields can come to rest in up to 10^{500} different minima, so the apparent laws of particle physics will depend on which of these 10^{500} minima is chosen by our pocket.

The argument above indicates that the string landscape (and string theory itself) should not be ruled out of bounds *solely* because it predicts the existence of a multiverse. However, recall that our ability to confidently – albeit not definitively – conclude that the visible universe is *not* embedded in a $\lambda\phi^4$ multiverse depends on having a clear description of a representative pocket within this scenario. The rich structure of the string landscape ensures even if string theory really is the "theory of everything", it may lack a characteristic and distinctive set of predictions, both for particle physics and for the cosmological properties of the observable universe. Consequently, it is unclear whether we can test the string landscape in the ways that we might test the $\lambda\phi^4$ or $m^2\phi^2$ multiverse.

The problem here is one of *mathematical* technology: we lack the tools to properly map the string landscape, just as Ayer's contemporaries lacked the tools to map the far side of the moon. Moreover, there are persuasive arguments that a brute force search of the landscape is an inherently intractable proposition [13], so any progress in this area is likely to require breakthroughs in understanding the deep structure of string theory. However, if we can compute correlations between observables within the pockets of the string landscape, it is conceivable that the landscape might then be subjected to similar tests to those described above. Like Ayer, we cannot know whether this mapping will remain forever beyond our reach. But perhaps the fear that we cannot explore – and test – the string landscape may one day be as dated as claims that we could never behold a mountain on the far side of the moon.

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