

Topological solitons of ellipsoid field - particle menagerie correspondence

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Abstract: *Universal formalism of QFT operates on abstract entities like point particles, not asking about their spatial structures, dynamics behind processes like photon production. Topological solitons, vacuum analogues of Abrikosov vortices, bring hope to understand field configurations behind Feynman diagrams, by the way explaining many fundamental assumptions, like spin, charge or strangeness quantization by constructing them as topological charges. Skyrme has made popular this kind of approach to mesons and baryons, while perfect situation would be a single field which family of topological solitons corresponds to our whole particle menagerie and their behavior. Here is discussed such unifying all interactions simple attempt - practically by just expanding field of unit vectors of Faber's electron model, which can be seen as reformulation of electromagnetic field to enforce charge quantization, by a single degree of freedom corresponding to quantum phase: of rotations around this direction (vacuum dynamics changes from S^2 to $SO(3)$). This simple model also suggests intuitive answers to many fundamental questions of physics.*

1) Introduction: why topological solitons for varying number particle systems?

Abrikosov vortices (called fluxons) are quanta of magnetic field prisoned in superconductor. They are topological solitons - the reason of quantization is that the quantum phase (order parameter) has to enclose within any loop, making phase difference an integer multiplicity of 2π , by the way defining magnetic flux going through this loop ($\Delta\varphi = \frac{q}{\hbar} \oint_{\partial S} A \cdot dl = \frac{q}{\hbar} \int_S B \cdot dS$). Spin can be defined in analogous way: quantum rotation operator says that rotating φ angle a spin s particle corresponds to multiplying wavefunction by $e^{is\varphi}$. Fluxons can be seen as spin ± 1 particles this way. Such two-dimensional hedgehog-like configuration has spin 1, but we can also imagine its 3D

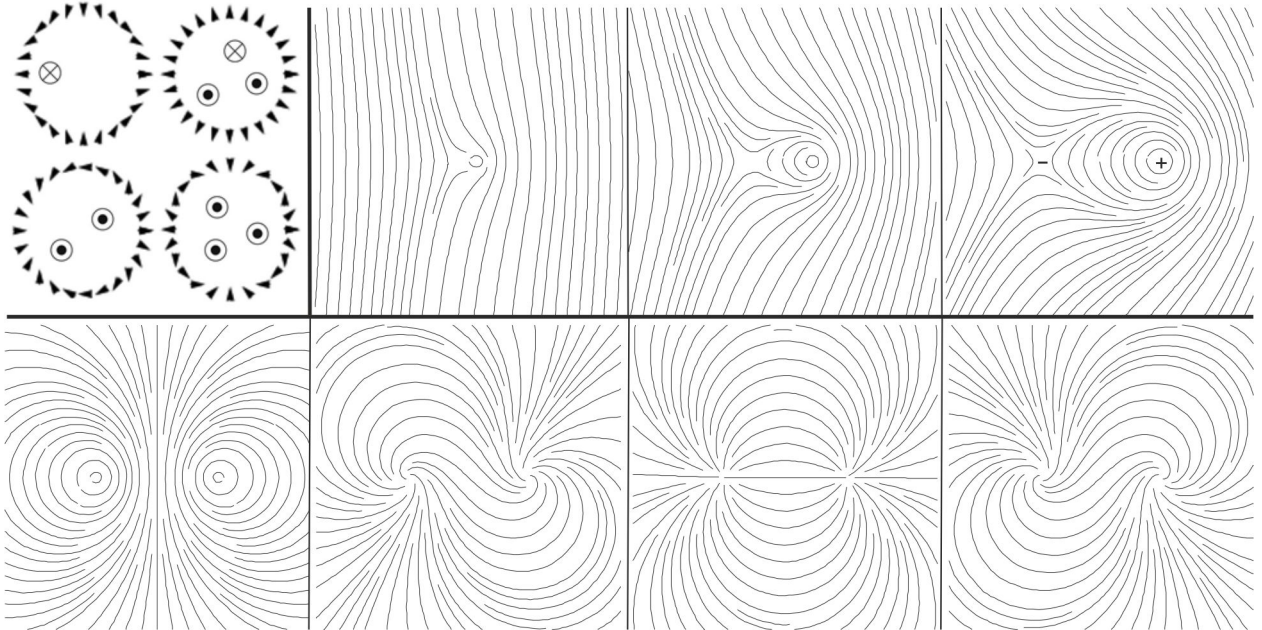


Figure 1: Top left: quantization of magnetic flux going through superconducting ring (arrowheads represent quantum phase). Right: Creation of spin -1/+1 soliton-antisoliton pair from vacuum - the further they are, the larger tension of the field: opposite charges attract (the same repels). Vacuum fluctuations of "just start such process and then soon return" correspond to virtual pair creation in perturbative QFT picture. Bottom: gauge invariance for two spin 1 solitons: phase evolution is constant rotation as for de Broglie's clock (drawn by [3]).

analogue, getting charge-like spatial configurations. We see that simple field of directions (unit vectors) already leads to quantized spin-like and charge-like configurations. We can represent this field as there is fixed shape prolate ellipsoid in each point (with equal two shorter radii). The essence of this paper is to distinguish these two shorter radii to add single degree of freedom of rotations around the main axis - use field of tri-axial ellipsoids instead. Thanks of it, there are three basic families of solitons, simplest charge will topologically need to have spin, and generally family of solitons seems to qualitatively correspond to basic particle menagerie and their behavior. While dynamics of rotations of the original axis will correspond to electromagnetic interaction like in Faber's electron model ([1], [2]), the additional perpendicular rotation corresponds to quantum phase, enforced by the soliton to constantly rotate as for de Broglie's clock of particle.

The natural question is: what about quantum mechanics in such classically looking picture? If someone is not satisfied by a different answer, there are known and used quantization methods for solitons([4]). There are also suggestions that such procedure is not necessary - that the picture of classical field theory and its quantum description are equivalent, like looking at coupled pendulums/crystal structure classically or through normal modes/phonons. We can ask about trajectories of the centers of solitons, but classical mechanics only approximates their behavior - additionally there is some extremely complex evolution of the field involved. For example possibility of video recording trajectories of Abrikosov vortices does not prevent from observing interference [5], tunneling [6] and dual Josephson effect [7] for them. Great intuition about quantum effects for localized classical field configurations provides recent experiments of Couder et al.: using oil droplets bouncing on vibrating liquid surface and so conjugated with waves they periodically create. This wave-particle duality allowed to observe: interference in double-slit experiment [8] (particle goes a single path, but it interacts with waves it created - going through all paths), tunneling [9] (the field, depending in complex way on the whole history, controls behavior in practically unpredictable way), orbit quantization [10] (to find resonance with the field, the clock has to perform integer number of periods in closed orbits) and Zeeman effect analogue [11] (splitting discrete family of orbits by analogue of magnetic field). The clock in physics is rather not external like here, but should be a periodic process hidden in particles - such interpretation of quantum mechanics was started by de Broglie [12]: that with particle's energy ($E = mc^2$) comes some internal periodic motion ($E = \omega\hbar$). Recently this clock/Zitterbewegung was directly observed for electrons [13]. Such intrinsic oscillations, and so constantly created waves around, are natural for many different solitons, generally called breathers [14].

Conjugated with particles \hbar -related waves are one of reasons that classical mechanics is only an approximation - further corrections can be expressed by power series in \hbar using WKB semiclassical approximation [15] of quantum mechanics. Like in Couder's tunneling, extreme complexity of the history dependent field suggests that thermodynamical picture would be more appropriate for larger times. Standard diffusion-like approaches disagree here with thermodynamical predictions of quantum mechanics, but it turns out they only approximate required maximization of uncertainty - correcting it recent models based on Maximal Entropy Random Walk obtain expected agreement, for example predicting thermalization to the quantum ground state probability density ([16], [17]).

To practically operate on varying number of solitons, it would be convenient to consider ensembles of possible scenarios - Feynman diagrams of perturbative QFT picture. Continuous soliton processes can be performed partially, what corresponds to virtual particles in QFT nomenclature. Starting such process corresponds to soliton's vibrational degree of freedom, which absorbs some of energy soliton gains - corresponding to self-energy diagrams for mass renormalization.

2) From Faber's electron to "breathing" topological solitons of ellipsoid field

Skyrme [18] has made popular using topological solitons to model mesons and baryons. Trying to use them for leptons was probably started a few decades later by Faber. He uses field of unimodular quaternions, what can be imagined as a direction from $S^3 \in R^4$ in each point. The potential term

makes the equator of this sphere ($S^2 \in R^3$) energetically preferred, so the field can be imagined as mainly field of unit vectors. The necessity of going out of the equator appears for example in the center of hedgehog-like ($v(x) = x/\|x\|$) configuration: no direction can be chosen in a continuous way there, so the field has to glue in one of 2 poles of the larger S^3 - choosing the sign of electron's spin in this model (does not change while rotating electron!). This unavoidable leaving the equator costs energy due to assumed Higgs-like potential: with topologically nontrivial minimum around zero. The minimal energy required for given topological constrains is just the rest energy-mass of this particle/soliton. Lorentz invariance makes it also the inertial mass. Assigning a hedgehog configuration to charge, curvature of the field drops with distance from the center - defining electric field this way leads to standard electromagnetic interaction [1]. So finally the model can be seen as that symmetry of the original field S^3 was spontaneously broken to some minimum of Higgs-like potential (S^2) and the massless Goldstone bosons of dynamics within this minimum correspond to standard electromagnetism.

This model does not have a place for the wave nature of particles: de Broglie's clock. There is required at least one additional phase-like (S^1) degree of freedom for that - we will choose it as just rotations around the direction: use SO(3) vacuum dynamics instead of Faber's S^2 . Physical representation could be rotating some asymmetric object - mathematically simplest is tri-axial ellipsoid (with three different radii). Its identifying vectors with opposite ones makes the spin is as in physics: multiplicity not of 1, but of 1/2, like in Fig. 2. This kind of spin 1/2 solitons of 2D field of light polarization ellipses (from circular to some linear) are already observed in started by Berry topological optics ([19],[20]). Another advantage of using ellipsoid field is its natural representation: just real symmetric 3x3 (4x4 with gravity) tensor field, like stress-energy or energy-momentum tensor - its 3 eigenvectors are axes of ellipsoid and eigenvalues correspond to radii. Skyrme models uses similar tensor field, but more abstract: complex. The potential term used there has usually minimum in zero matrix or in unit matrix (corresponding to sphere) - the most essential change of presented model is using potential with minimum in fixed shape ellipsoid, like:

$$V(M) = \sum_i (\lambda_i - \lambda_i^0)^2, \text{ where } \lambda_i \text{ are eigenvalues of } M, \lambda_i^0 \text{ are constants of the model.}$$

So energetically there is preferred some ellipsoid - the minimum of potential, corresponding to the equator in Faber's model. Vacuum dynamics is their rotations, which correspond to electromagnetism (Faber) plus dynamics of rotations around the axis: "quantum phase" (plus gravity while

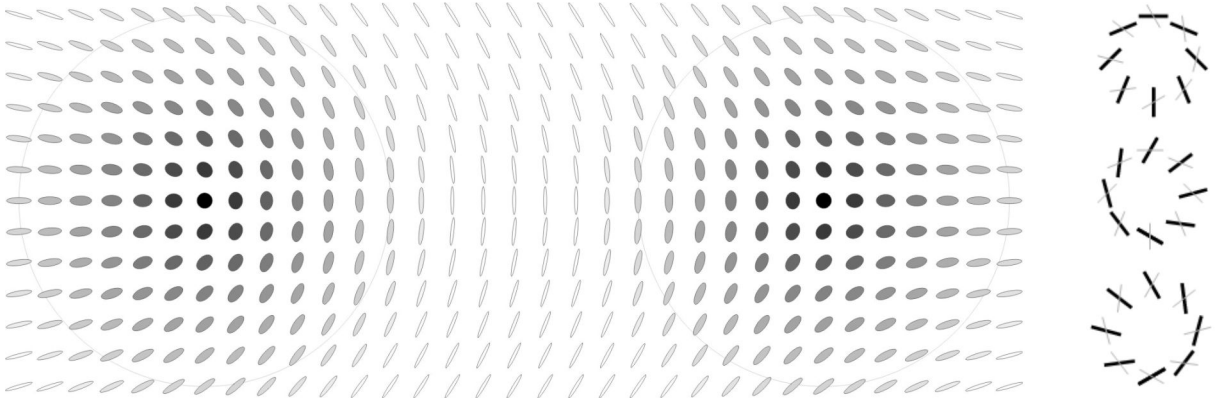


Figure 2: Configuration of spin +1/2, -1/2 pair of ellipse field - the phase makes exactly half rotation in circles around singularities. No phase can be chosen in the centers - the field is enforced to equalize radii there, what costs energy (the darker ellipse, the larger potential energy). All phases should rotate with time evolution (de Broglie's clock).

using 4D ellipsoids). Topological constraints may enforce them to get out of this Higgs minimum to equalize some radii, giving solitons some minimal energy: mass.

The rotations of quantum phase should cost less energy than corresponding to EM, so the ellipsoid should be approximately prolate: there is one longer axis (called the main axis) which rotations correspond to EM and two shorter similar axes (auxiliary axes). Evolution should be governed by some wave equation: generally saying that sum of tensions in all four directions is zero - structural tension of being topological soliton may require tension in time direction: necessary internal periodic process - de Broglie's clock.

3) Topological solitons - particle menagerie correspondence

The most important income of adding perpendicular rotation degree of freedom to Faber's model, is growth of soliton family: from only spin $\pm 1/2$ electrons to looking like our whole basic particle menagerie.

First of all, observe that as there are three different axes, both spin-like configuration from Fig. 2 and charge-like configuration from Fig. 3 can be made in three essentially different ways: by choosing the axis orthogonal to the plane or the axis creating the hedgehog - we basically get three families of spins/charges.

Let us now look at lepton-like configurations: the simplest charges, like hedgehog configurations of one of three axes. Looking at a sphere around, the main axis is perpendicular to it and some auxiliary axis has to form a direction field on the sphere - topological "hairy ball theorem" says it cannot be done in a continuous way: there is required some singularity. Intuitively, the energetically cheapest way to do it is by two opposite spin-like singularities going in opposite directions, like in Fig. 3.

While charge is point-like here, spin not only corresponds to a direction, but to a curve-like structure - let us call them spin curves. These vacuum analogues of Abrikosov vortices appear for example when $\text{Re}(\psi) = 0$ and $\text{Im}(\psi) = 0$ submanifolds intersect transversally for a wavefunction ψ - quantum phase makes 2π rotation around such so called vortex lines ([21]) in Schrödinger picture. Phase change is related to electromagnetic field: $\Delta\varphi = \frac{q}{\hbar} \int A \cdot dx$, so if there is a charge

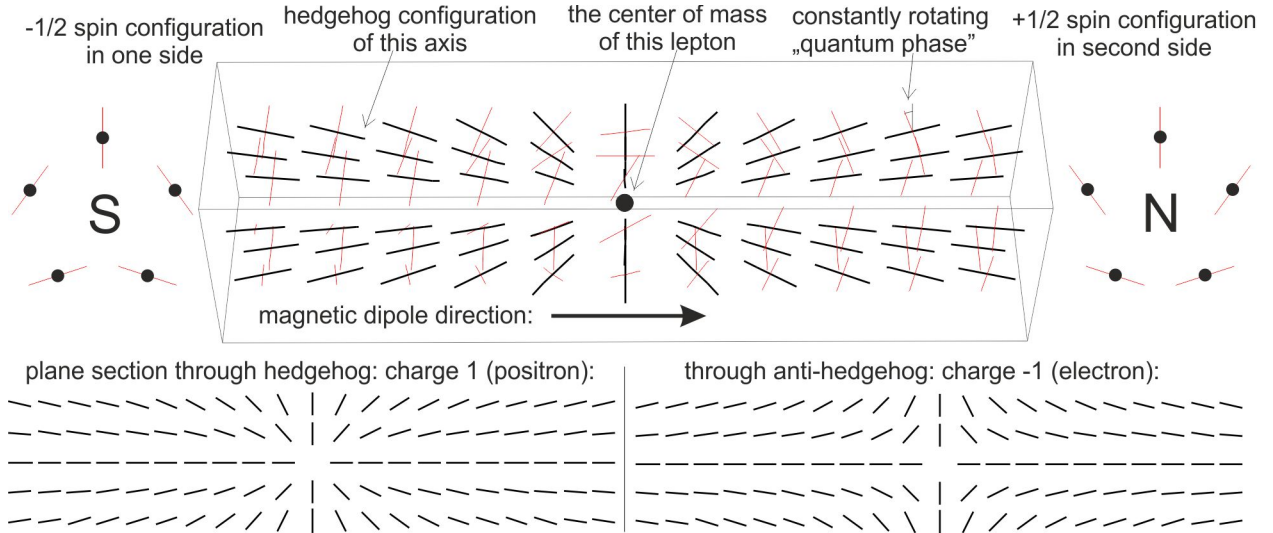


Figure 3: Top: hedgehog configuration of one of 3 axes representing one of 3 leptons. Decreasing curvature corresponds to electric field. The impossibility of arranging the second axis on a sphere around (hairy ball theorem) enforces it e.g. to have two opposite spin-like configurations: opposite magnetic poles. Bottom: configuration of this axis for hedgehog and anti-hedgehog.

(q) involved like for superconductor, such vortex is quant of magnetic field. Charge is not directly involved with the spin curves, which are kind of more fundamental constructs here (neutrino is simpler than electron). However, minimizing energy of the field means that like opposite/identical charges attracts/repels, the same is expected for spin curves, like in Fig. 1.

So let us consider basic particle system: electron-positron couple (positronium). They naturally attract, however, if their spins are aligned in parallel way, time expected to annihilate is nearly thousand times larger than for anti-parallel alignment. This huge difference can be seen through magnetic dipole moments: that these fermions are tiny magnets and magnets repel if aligned parallelly, attract when anti-parallel (drops with $1/r^4$). We have the same in ellipsoid field, like in Fig. 4 - the opposite spin curves going out of leptons make them behave like tiny magnets. So as in Faber's model, leptons here also need to have spin, but this time not as some fixed internal $\pm 1/2$ value, but which projection corresponds to magnetic dipole direction and changes while rotation. There are recent experiments surprisingly showing that while scattering, positronium behaves like electron alone([22]), suggesting large distance between electron and positron and so required additional stabilization in parallel alignment. Besides Cooper pairs, another example of relatively large distance fermion bondings are halo nuclei [23], in which neutrons can stay bonded for milliseconds in distance a few times larger than range of strong interaction.

Another lesson from para and ortho positronium is while the first one produces two photons, the second decays into three - like one of them corresponds to rotating one spin 180° to get from repelling to attracting spin alignment. It brings us to another fundamental question: what optical photon is? While in QED one is satisfied with abstract result of creation operator, here we need to find a concrete field configuration and continuous dynamics for this rather non-instant creation process ([24]). What we definitely know is that it carries energy, momentum and angular momentum, so because of conservation laws, these properties was involved while photon production. The key here is the angular momentum, which does not only mean some abstract spin, but is very physical tiny torque: can be directly used to rotate macroscopic objects [25]. From the other side, photon does not have magnetic dipole moment: in contrast to fermions, its spin 1 corresponds to something different: just changing spin of involved electron by 1: e.g. from $-1/2$ to $+1/2$. Like

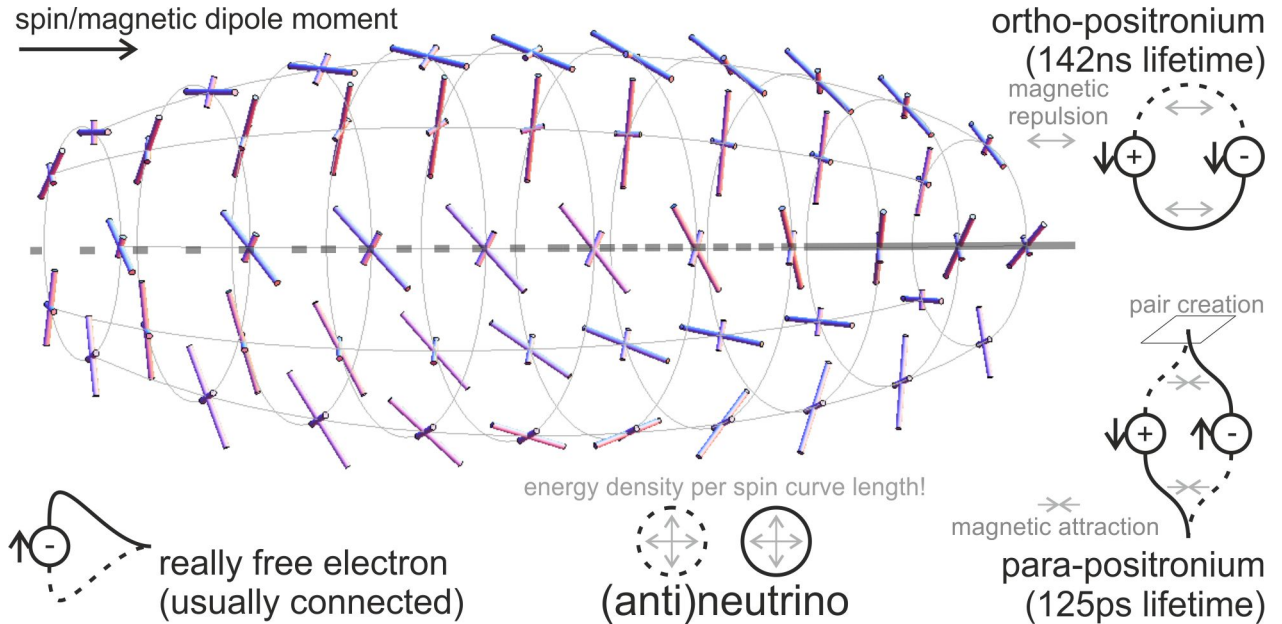


Figure 4: Configuration of electron from the point of view of auxiliary axes and some basic constructions.

for positronium, such spin change can be made by simple 180° rotation, corresponding to photon's angular momentum (Fig. 5). It is angular acceleration first, then deceleration to stop the rotation, what through Noether theorem requires creation of twist-like wave: optical photon. Thanks to lack of viscosity of EM field, instead of dispersing, such twist-like wave can travel undeformed through light years.

What seems most controversial about these spin curves/vacuum Abrikosov vortices/vortex lines here, is that they rather require some small rest mass/energy density per length: to equalize two axes in the center. The ones going out of electron should be the lightest of three possibilities: radii required to equalize should be nearly equal. Essentially different is the largest radius - equalized in muon and taon configurations and so giving them much larger masses. In opposite to spin curves, charge-like configurations require equalization of not two, but all three radii - have much larger masses. In contrast, light donut-like spin curve constructions are good candidates for neutrinos: they should be very hard to interact with: can be essentially involved only in very close interactions and have relativistic speeds because of huge energy/mass proportion. Additionally we will see that they should be produced while beta decay. Another required property is neutrino oscillations, allowed in this model: one type of spin curve can transform into another by rotating all axes 90° correspondingly, e.g. compensating energy difference by length change. By analogous rotation, muon and taon tend to get to the lowest energy charge: electron. The mass of such spin loop depends on its length - it might be various here, what could be one of reasons for difficulty of neutrino mass measurement.

Another disturbing consequence is the mass of spin curves going out of some particles - energy minimization should make them extremely short and of neutrino mass scale, probably built into tiny corrections of e.g. parallel/anti-parallel alignment. To make these spin curves macroscopic, e.g. as hypothesized cosmic strings [26], there would be required extreme magnetic fields. Their topological stability and additional mass/energy density might be the missing factor in magneto-hydrodynamics considerations, which still have essential difficulties to understand plasma instabilities or the coronal heating problem [27]: that against thermodynamical intuition, while the surface of Sun has thousands of Kelvins (where heating comes from), its corona has millions. Modern approaches to this problem usually base on magnetic reconnections of so called magnetic flux ropes or tubes - observed line-like structures, surprisingly holding ions in stable way and releasing energy while reconnecting or shortening - what we should expect from vacuum analogues of Abrikosov vortices.

4) Complexity grows: mesons, baryons, nuclei ...

Curve-like structures also allow for further possibilities: of additional internal rotation like in Möbius strip and of knotted structures. The first possibility creates nasty topological conflict in the center - all three axes have to equalize in more complex structure than in the center of charge, giving it larger mass. Decay modes and lack of spin suggest to assign pion to such simplest

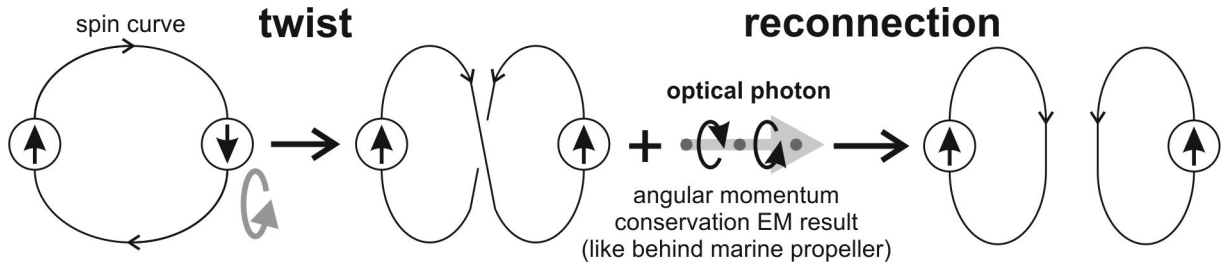


Figure 5: Topological mechanism of fermion coupling and of breaking it by changing spin projection by 1: e.g. from $-1/2$ to $+1/2$. It requires production of shape preserving twist-like wave: photon.

construction and further mesons for more internal rotations. Analogous construction can be also charged - like free electron in Fig. 4, but with additional internal rotation. Pion usually decays into muon, suggesting that due to the central singularity, such "Möbius strip"-like construction has minimal energy if the loop is muon-like. Internal tension can be reduced by twisting into eight-like shape and reconnecting like in Fig. 6. Pion decay to muon and neutrino could be explained e.g. by reconnecting with some external spin curve. Strangeness is integer here - opposite strangeness could lead to different decay times, explaining short/long living kaons. It is also counted a bit differently: ± 1 for pions, ± 2 for kaons.

While considering knotted constructs, we need to have in mind the possibility of reconnections, which could easily destroy the structure. There are ways to prevent it - for example when spin curves are of different types, what can be stabilized by their interconnectedness, like in Fig. 6. It also shows the necessity of using different axes for these curves - the lowest density and beta decay suggests the longer curve going through is electron-like. Besides energy density, suggestions that the loop around is usually muonic are: pion production while decay of strange baryons or while baryon collisions - it could elongate such loop, which then reconnects/fragmentates into smaller pieces (mainly mesons), like in Lund string phenomenological model of hadronization [28].

The structure of muon spin loop around enforces some rotation of the main axis of electron spin curve - we know this kind of rotation from hedgehog configuration: such 180° rotation corresponds

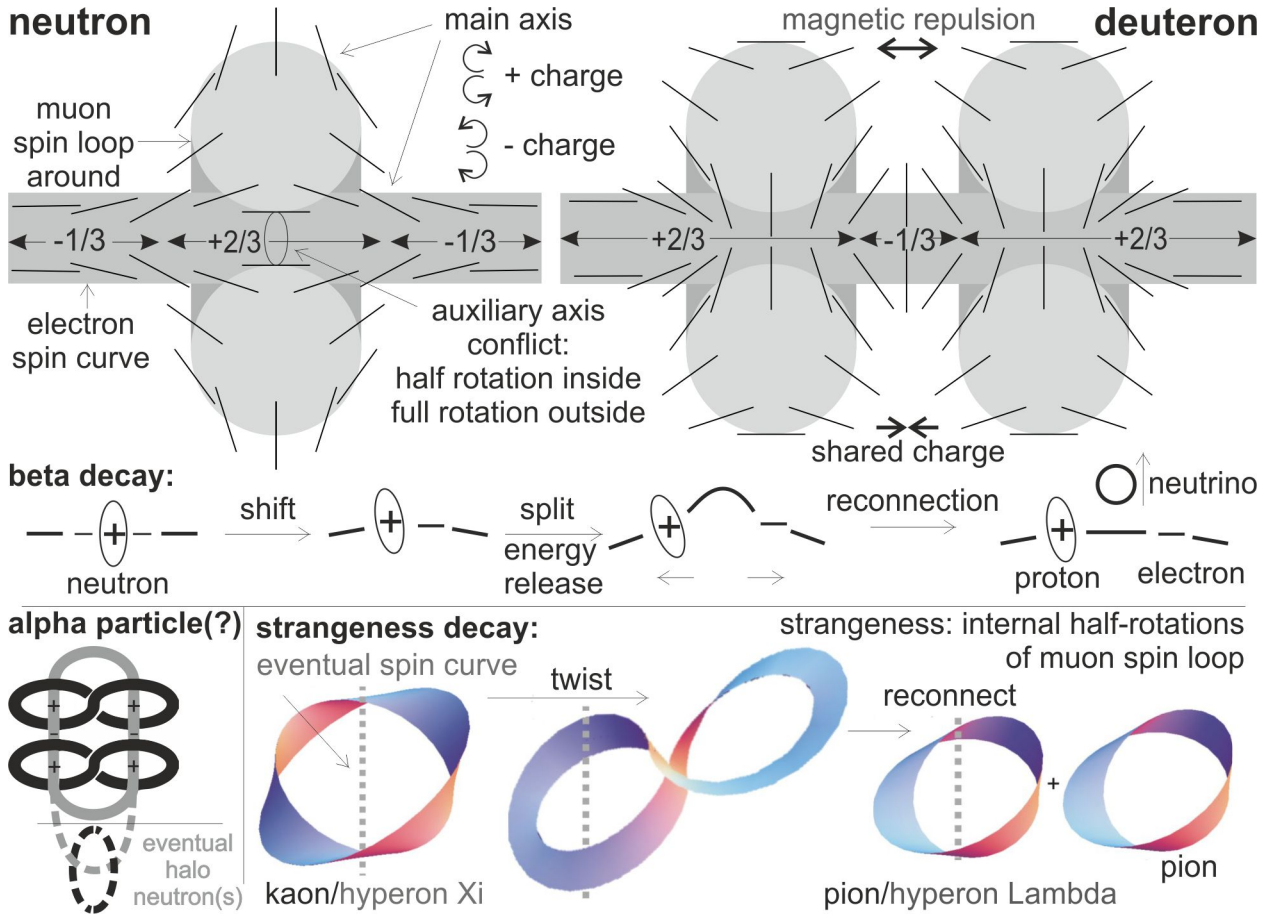


Figure 6: Top: Schematic plane section of the main axis for neutron and deuteron-like solitons - electron spin curve with muon spin loops around. Middle: schematic picture of beta decay: released energy difference goes partly to the spin curve and leaves the system by neutrino creation. Bottom: some possibility for alpha particle soliton and process of reducing strangeness.

to +1 elemental charge. Locally there are possible fractions of such complete rotation. The central vortex costs more energy if the main axis is not parallel, what makes that the total charge has to be quantized and gives tendency to reduce the length of eventual series of opposite fractional charges. At least partial charge is required while interconnecting with muon spin loop around, so neutron has to recompense it for example with two opposite charges, e.g. like for the quark model. To relate further to quark model, from perspective of deep inelastic scattering, loop-like constructs can be interpreted as being made of two compartments while looking from a side, baryon constructs of three.

We have now two main contributions to neutron's mass: the length of muon spin loop and lengthening the charge. There is also a conflict of (not drawn in Fig. 6) auxiliary axes - in opposite to proton, neutron's main axis configuration cannot harmonize auxiliary axes of both spin curves, requiring additional radii equalization - additional mass. The last two mass contributions nearly vanish in proton - the charge allows for better agreement, explaining its lower mass. Elementary charge can be shared here between two baryons, like for deuteron in Fig. 6 - muon loops repel each other, but shared charge gives them attraction as kind of additional "strong" force. Imagining hydrogen and helium isotopes suggests that for three baryons such sharing single charge becomes less energetically optimal and completely unstable for four or more. There is required a different mechanism to explain that alpha particle is lighter than two deuterons by almost 24MeV - one of possibilities is additional reconnection between such two charge sharing couples to shorten muon loops a bit. More quantitative considerations are required to understand larger nuclei in presented model - they might behave like a liquid of such charge sharing clusters, generally aligned parallel to the spin of nucleus. The stability of even-even nuclei might be explained by preferred pairing of such charge sharing clusters of the same number of nucleons (2 or 3). There are known much more particles (or rather resonant states) than mentioned here, but they are very unstable - less stable configurations are also available to explore here: local energy minimums, e.g. choosing different axes permutation.

Electric dipole moments are usually zero here as measured. Electric quadrupole moments are usually nonzero - for deuteron seem to be in agreement with measurement (predicting about 0.4fm distance between nucleons), while nonzero required for neutron is very difficult to test experimentally. Another unusual possibility of this model is disentangling the knot, what could reduce the baryon number - required stability is crucial test of this model. Decaying to a lower energy configuration requires a mechanism, production of a carrier for this released energy: photon, neutrino or larger particle - being in the global energy minimum is not necessary for stability. However, like in supersymmetric or other unifying theories, in opposite to spin and charge, baryon number cannot be ultimately conserved here - in really extreme conditions like while hypothetical baryogenesis, this conservation needs to be violated e.g. through proton decay. Candidates for such extreme conditions may be the centers of neutron stars - instead of destroying baryons through hypothetical Hawking radiation after black hole creation, start "burning" them just before collapsing to this singular state of matter (exceeding all finite limits!). Huge released energy could prevent collapse and make the star a source of currently difficult to explain gamma sources.

5) Electromagnetism, gravity and conclusions

Although this extremely simple to formulate model seems to qualitatively reconstruct basic particle menagerie and behavior, there are required quantitative considerations for full agreement and predictive capabilities, for example to derive constants of the standard model using just a few more fundamental ones.

There are many ways to make potential term preferring some set of eigenvalues (radii) λ_i^0 , like

$$V(M) = \sum_i (\lambda_i - \lambda_i^0)^2 \quad \text{or} \quad V(M) = \sum_i (\text{Tr}(M^i) - s_i)^2 \quad \text{where} \quad s_i = \sum_k (\lambda_k^0)^i$$

The choice of the proper one will be definitely required while comparing particle masses. Before considering this near-singularity behavior, we should well understand vacuum dynamics - when the potential term vanishes: $M = ODO^T$ for constant $D = \text{diag}(\lambda_i^0)$ and some orthogonal matrix O . So vacuum dynamics is described by $SO(3)$ field. A simple choice of the kinetic term is just to use tension of the field ([29]): $\sum_{i=0}^3 \sum_{jk} (\partial_i M_{jk})^2$. After (arbitrarily!) choosing time direction, Legendre transform leads to standard Lagrangian density:

$$\mathcal{L} = \sum_{jk} (\partial_0 M_{jk})^2 - \sum_{i=1}^3 \sum_{jk} (\partial_i M_{jk})^2 - V(M)$$

Looking at local rotations, shifting ϵ in i direction corresponds to $O \rightarrow (1 + \epsilon(\partial_i O)O^T)O$ rotation - we can define affine connection as $\Gamma_i = (\partial_i O)O^T = -O(\partial_i O)^T$. Denoting

$$(\Gamma_0)_{ij} = \frac{\epsilon_{ijk} E_k}{\lambda_j - \lambda_i}, \quad (\Gamma_d)_{ij} = \frac{\epsilon_{ijk} B_k^d}{\lambda_j - \lambda_i} \quad \text{for } d = 1..3,$$

the Lagrangian density becomes ([29]): $\mathcal{L} = E \cdot E - \sum_d B^d \cdot B^d - V(M)$. So basically vacuum behavior can be described using Maxwell equations. The additional degree of freedom of B field corresponds to quantum phase Faber's model was missing.

If instead of 3x3, we would use 4x4 tensor field - 4D ellipsoid field, the fourth axis can be seen as local time axis: central axis of light cone. Above Lagrangian density would says that its dynamics is governed by additional Maxwell's equations: for gravity. This expansion of Newton's gravity is called gravitomagnetism and is currently used as approximation of GRT, e.g. for theoretical predictions for recent Gravity Probe B experiment ([30]). Gravitational mass probably requires some additional assumption here, like incompressibility ($\det(M) = \text{const}$) causing that topological solitons additionally deform gravitational field. Finally vacuum dynamics for presented Lagrangian is generally:

$$\text{electrodynamics} + \text{dynamics of quantum phase} + \text{gravitomagnetism}$$

Situation changes near the centers of solitons: deformational degrees of freedom have to be activated, what can be seen as weak and strong interactions: weak corresponding to deformation of single spin curve and strong as interaction between them like through shared charge.

Unfortunately there is essential problem with the above Lagrangian - around hedgehog configuration energy density drops like $1/r^2$ instead of required $1/r^4$. Faber model corrects it by defining E , B not from the connection, but from the curvature: straightforward as expected for the hedgehog configuration. He gets standard electromagnetic interaction between charges this way ([1]). The same could be done for ellipsoid field. However, for differences e.g. between magnetic dipole moments of electron and muon, it seems essential that not all rotations are equally energetic. It was expressed by using not only rotation (O), but the whole tensor (M) in the kinetic term, or equivalently by using eigenvalues while defining E , B from connection.

To sum up, besides choosing preferred eigenvalues, it seems there remains large freedom of choice for both potential and kinetic terms. Obtaining electromagnetism (+gravitomagnetism) is relatively simple, while fitting them to known properties (also de Broglie's clock) seems difficult, but could lead to reduction of required constants of standard model - effective in presented approach.

Most importantly, this approach brings hope to finally find intuitive understanding of physics - complementing abstract QFT description with very concrete structures and mechanisms behind. Particles from leading to ultraviolet divergence point entities, become concrete spatial field configurations here. This simple approach is all or nothing - a single discrepancy would make it go to trash; instead it does not stop bringing succeeding simple answers to difficult questions.

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