

# Clockwork Quantum Universe

*An essay for the FQXi contest “Is Reality Digital or Analog?”*

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**Abstract.** Besides the purely digital or analog interpretations of reality there is a third possible description which incorporates important aspects of both. This is the cyclic interpretation of reality. In this scenario every elementary system is described by classical fields embedded in cyclic space-time dimensions. We will address these cyclic fields as “de Broglie internal clocks”. They constitute the deterministic gears of a consistent deterministic description of quantum relativistic physics, providing in addition an appealing formulation of the notion of time.

## INTRODUCTION

Among the many starting points to motivate the possible cyclic nature of space-time, published in [1] (see also [2] for more details) and presented to several international conferences [3–5], in this essay we adopt the ‘t Hooft determinism [6–8]. It states that there is a close relationship between the quantum harmonic oscillator with angular frequency  $\bar{\omega} = 2\pi/T_t$ , that is the mode of an ordinary quantum field with energy  $\bar{E} = \hbar\bar{\omega}$ , and a classical particle moving along a circle of periodicity  $T_t$ . By assuming the time period  $T_t$  on a lattice with  $N$  sites, it turns out that if the experimental time accuracy is  $\Delta t \gg T_t$ , at every observation the system appears in an arbitrary phase of its cyclic evolution, *i.e.* on an arbitrary site of the lattice. Since the underlying periodic dynamics are too fast to be observed, the evolution has an apparent aleatoric behavior as if observing a “clock under a stroboscopic light” [7]. The evolution operator  $\mathcal{U}(\Delta t = \varepsilon) = \exp[-\frac{i}{\hbar}\mathcal{H}\varepsilon]$  is given in terms of a  $N \times N$  matrix and the model is analogous to a harmonic system of  $N$  masses and springs on a ring. In the limit of large  $N$ , the frequency eigenvectors  $|\phi_n\rangle$  obey to the relation  $\mathcal{H}|\phi_n\rangle \sim \hbar\bar{\omega}(n+1/2)|\phi_n\rangle$  which actually describes the energy eigenvalues  $E_{n+1/2} = \hbar\bar{\omega}(n+1/2)$  of a quantum harmonic oscillator with periodicity  $T_t$  - apart for an “unimportant” phase in front of the operator  $U(\varepsilon)$  which reproduces the factor 1/2 in the eigenvalues [9–11] and which can be regarded a twist factor on the Periodic Boundary Conditions (PBCs). The idea is that, due to the extremely fast cyclic dynamics, we loose information about the underlying classical theory and we observe a statistical theory that matches QM. For this reason we speak about deterministic or pre-quantum theories. Since the cyclic time interval  $T_t$  is supposed on a lattice, the ‘t Hooft determinism can be classified as purely digital. It recently evolved into the idea of “classical cellular automata” [12], that is a deterministic model which shows interesting correspondences between elementary particles and black holes. However, if we take the continuous limit of the ‘t Hooft deterministic model by assuming an infinite number of lattice sites  $N \rightarrow \infty$ , it is easy to see that the system of springs and masses turns out to be a vibrating string embedded in a cyclic time dimension, that is a bosonic classical field  $\Phi(x, t)$  embedded in a compact time dimension of length  $T_t$  and with PBCs. Formally, through discrete Fourier transform, to a compact variable corresponds a quantized conjugate variable, that is a variable which takes discrete values. Hence, a compact dimension yields to a digital description of the conjugate space. Considering the relation  $\bar{E} = \hbar\bar{\omega}$ , in the specific case of an elementary cyclic system with  $t \in [0, T_t]$  there is associated the quantized energy spectrum  $E_n = n\hbar\bar{\omega} = nh/T_t$ . The energy is the digital conjugate variable of a cyclic time variable. More in general, since we observe an energy-momentum space on a lattice (quantized energy-momentum spectrum) we try to describe QM in terms of cyclic space-time dimensions. In [1] we have shown that, similarly to a particle in a box, relativistic fields can be actually quantized by imposing their characteristic de Broglie space-time periodicities as constraints [1–5].

Our assumption of dynamical periodic fields can be regarded as a combination of the Newton’s law of inertia and de Broglie-Planck hypothesis of periodic matter waves: *elementary isolated systems must be supposed to have persistent periodicities as long as they do not interact*. Such an assumption of intrinsic periodicity is also implicit in the operative definition of time (and for some aspect in the action-reaction law). Time can only be defined by counting the number of cycles of isolated phenomena *supposed* to be periodic. For a consistent formalization of time, in physics there must be an assumption of periodicity for free elementary systems! In modern physics, a second is defined as the duration

of 9,192,631,770 characteristic cycles of the Cs atom ( $T_{Cs} \sim 10^{-10}s$ ). For the central role of time in physics, the assumption of isochronism of the pendulum made by Galileo in the cathedral of Pisa can be regarded as one of the foundational acts of physics. Such an assumption of persistent periodic phenomena allowed a sufficiently accurate definition of time to study the motion of bodies and in turn the formulation of theories of dynamics. The definition of relativistic clock given by A. Einstein [13] is: “by a clock we understand anything characterized by a phenomenon passing periodically through identical phases so that we must assume, by the principle of sufficient reason, that all that happens in a given period is identical with all that happens in an arbitrary period”. The whole information of such a relativistic clock is contained in a single period. Thus we say, using the language of extra dimensional theories, that the period is a analog compact dimension with PBCs. In this way a non interacting cyclic field can be regarded of as analog “de Broglie internal clocks” [14, 15], that is to say as relativistic fields with intrinsic de Broglie time periodicities  $T_t = h/\bar{E}$ .

Since the measure of time is a counting process it also has a digital nature. This intrinsically contains the Heisenberg uncertain principle. In fact, in a “de Broglie clock”, to determine the energy  $\bar{E} = \hbar\bar{\omega}$  with good accuracy  $\Delta\bar{E}$  we must count a large number of cycles, that is to say we must observe the system for a long time  $\Delta t$ , according to the relation  $\Delta\bar{E}\Delta t \gtrsim \hbar$ . Moreover, since periodicity conditions mean that the only possible energy eigenmodes are those with an integer number of cycles, we obtain the Bohr-Sommerfeld quantization condition (for instance, it can be shown that the periodicity condition  $E_n T_t = nh$  can be more in general written as  $\oint E_n dt = nh$  for interacting systems). This allows to solve many non-relativistic quantum problems [1, 2].

For the covariant formulation of the theory we must consider that the de Broglie time periodicity induces spacial de Broglie periodicities  $\lambda^i$ , and that these space-time periodicities, as well as the energy-momentum quantized spectrum, transforms under Lorentz. In other words, since  $T_t = h/\bar{E}$ , the de Broglie time periodicity must be regarded as dynamical. As every time interval, the time periodicity  $T_t$  transforms in a relativistic way. The proper-time intrinsic periodicity  $T_\tau$  fixes the upper bond of the time periodicity  $T_t$  because the mass is the lower bond of the energy. For instance, denoting the reference system by the spatial momentum  $\bar{\mathbf{p}}$ , where  $p_i = h/\lambda^i$ , we have  $T_\tau \geq T_t(\bar{\mathbf{p}})$  and  $\bar{M}c^2 \leq \bar{E}(\bar{\mathbf{p}})$ . The heavier the mass the faster the proper-time periodicity. Hence, even a light particle such as the electron has (in a generic reference frame) intrinsic time periodicity equal or faster than  $\sim 10^{-20}s$ , *i.e.* the time periodicity in a generic reference frame is always faster than its proper-time periodicity. It should be noted that the periodicity is many orders of magnitude away from the characteristic time periodicity of the cesium atomic clock, which by definition is of the order of  $10^{-10}s$ , and that it is extremely fast even if compared with the present experimental resolution in time ( $\sim 10^{-17}s$ ). Thus, for every known matter particle (except the neutrino) we are in the case of too fast periodic dynamics as in the 't Hooft determinism. The de Broglie intrinsic clock of elementary particles can also be imagined as a “de Broglie deterministic dice” [3], that is a dice rolling with time periodicity  $T_t$ . We inevitably have a too low revolution in time, so that at every observation the system appears in an aleatoric phase of its evolution. As for a clock observed under a stroboscopic light or a dice rolling too fast, we can only predict the outcomes statistically. For the results presented in [1] and summarized here we see that the statistical description associated to intrinsically periodic phenomena actually matches ordinary QM [1]. We may also note that, on a cyclic geometry such as a cylinder, there exist many possible classical paths, characterized by different winding (digital) numbers, between every initial and final point. Thus a field with PBCs can self-interfere. Its evolution is described by a sum over classical cyclic paths (characterized by digital numbers) which actually matches the ordinary Feynman Path Integral of QM.

In this essay we will only describe some published results or announce some others that will be published soon. The reader interested in more technical details or to the mathematic proofs may refers to [1].

## RELATIVISTIC GEARS

The relativistic generalization of the Newton’s law of inertia can be formulated in the following way: every isolated elementary system has persistent four-momentum  $\bar{p}_\mu = \{\bar{E}/c, \bar{\mathbf{p}}\}$ . On the other hand, the de Broglie-Planck formulation of QM prescribes that a four-momentum must be associated to the four-angular-frequency of a corresponding field, according to the relation  $\bar{\omega}_\mu = \bar{p}_\mu c/\hbar$ . Here we will assume that every elementary system is described in terms of intrinsically periodic fields whose periodicities are the usual de Broglie-Planck periodicities  $T^\mu = \{T_t, \tilde{\lambda}_x/c\} = 2\pi/\omega_\mu$ . As the Newton’s law of inertia doesn’t imply that every point particle moves on a straight line, our assumption of intrinsic periodicities does not mean that the physical world should appear to be periodic. In fact, the four-periodicity

$T^\mu$  is fixed dynamically by the four-momentum through the de Broglie-Planck relation

$$T^\mu = \frac{2\pi}{\bar{\omega}_\mu} = \frac{h}{\bar{p}_\mu c}. \quad (1)$$

The variation of four-momentum occurring during interactions implies a variation of the intrinsic periodicities of the fields. This guarantees time ordering and relativistic causality.

Similarly to the 't Hooft deterministic model, the free cyclic field  $\Phi(\mathbf{x}, t)$  is a tower of frequency eigenmodes  $\phi_n(x)$  with energies  $E_n(\bar{\mathbf{p}}) = n\hbar\bar{\omega}(\bar{\mathbf{p}})$ ,

$$\Phi(\mathbf{x}, t) = \sum_n A_n \phi_n(\mathbf{x}) u_n(t), \quad \text{where} \quad u_n(t) = e^{-i\omega_n(\bar{\mathbf{p}})t}. \quad (2)$$

By bearing in mind the relation  $\bar{E}(\bar{\mathbf{p}}) = \hbar\bar{\omega}(\bar{\mathbf{p}})$ , the quantized energy spectrum  $E_n(\bar{\mathbf{p}})$  is nothing else than the harmonic frequency spectrum  $\omega_n(\bar{\mathbf{p}}) = n\bar{\omega}(\bar{\mathbf{p}})$  of a vibrating string with time periodicity  $T_t(\bar{\mathbf{p}})$ <sup>1</sup>. This quantization is the field theory analogous of the semiclassical quantization of a “particle” in a box, it also shares deep analogies with the Matsubara and the Kaluza-Klein (KK) theory [16]. Since in this case the whole physical information of the system is contained in a single four-period  $T^\mu$ , our intrinsically four-periodic free field can be described by a bosonic action in compact four dimensions with PBCs

$$\mathcal{S}_{\lambda_s} = \int_0^{T^\mu} d^4x \mathcal{L}_{\lambda_s}(\partial_\mu \Phi, \Phi). \quad (3)$$

It is important to note that PBCs minimize the action at the boundaries, in particular the ones of the compact time dimension. Therefore PBCs have the same formal validity of the usual (Synchronous) BCs assumed in ordinary field theory. This is an essential feature because it guarantees that all the symmetries of the relativistic theory are preserved as in usual field theory. In particular it guarantees that the theory is Lorentz invariant. For instance we can consider a generic global Lorentz transformation

$$dx^\mu \rightarrow dx'^\mu = \Lambda_V^\mu dx^\nu, \quad \bar{p}_\mu \rightarrow \bar{p}'_\mu = \Lambda_V^\nu \bar{p}_\nu. \quad (4)$$

By definition,  $T^\mu$  is such that the phase of the field is invariant under four-periodic translations  $\exp[-ix^\mu \bar{p}_\mu] = \exp[-i(x'^\mu + cT'^\mu) \bar{p}'_\mu]$ . In this way we see that the four-periodicity is actually a contravariant four-vector. It transforms under global Lorentz transformations as every generic space-time interval

$$T^\mu \rightarrow T'^\mu = \Lambda_V^\mu T^\nu \quad (5)$$

and the phase of the field is a scalar quantity under Lorentz transformations - de Broglie phase harmony. The space time periodicity  $T^\mu$  can be thought of as describing a reciprocal energy-momentum lattice  $p_{n\mu} = n\bar{p}_\mu$ . This can be also inferred by noticing that after the transformation of variables eq.(4), the integration region of the free action eq.(3) turns out to be

$$\mathcal{S}_{\lambda_s} = \int_0^{T'^\mu} d^4x' \mathcal{L}_{\lambda_s}(\partial'_\mu \Phi, \Phi). \quad (6)$$

Therefore, in the new reference system, the new four-periodicity  $T'^\mu$  of the field is actually given by eq.(5), that is eq.(6) describes a system with four-momentum  $\bar{p}'_\mu$  eq.(4).

The underlying Minkowski metric induces the following constraint on the dynamical periodicities

$$\frac{1}{T_\tau^2} \equiv \frac{1}{T_\mu} \frac{1}{T^\mu}$$

which, considering the above de Broglie-Planck relation, is nothing else than the relativistic constraint  $\bar{M}^2 c^2 = \bar{p}^\mu \bar{p}_\mu$ .

The resulting compact 4D formulation reproduces, after normal ordering, exactly the same quantized energy spectrum of ordinary second quantized fields. In particular, for a massive field with mass  $\bar{M}$  we will find the energy spectrum

$$\bar{E}(\bar{\mathbf{p}}) = n\sqrt{\bar{\mathbf{p}}^2 c^2 + \bar{M}^2 c^4}$$

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<sup>1</sup> As we will see, the theory can be even regarded as a particular kind of string theory where there is a compact world-line parameter instead of a compact world-sheet parameter.

of ordinary quantum field theory. Furthermore, it is easy to see that in the rest frame ( $\bar{\mathbf{p}} \equiv 0$ ) this quantized energy spectrum is *dual* to the KK mass tower  $M_n = E_n(0)/c^2 = n\bar{M}$ . Indeed, for such a massive field, the assumption of periodicity along the time dimension means that in the rest frame the proper-time  $\tau$  there has intrinsic periodicity

$$T_\tau = T_t(0) = \frac{h}{\bar{M}c^2}$$

The invariant mass  $\bar{M}$  is not a parameter of the action by it is fixed geometrically by the reciprocal of the proper-time intrinsic periodicity  $T_\tau$  of the elementary field. In other words, by imposing intrinsic time periodicity, the world-line parameter  $s = c\tau$  turns out to be compact with PBCs. It behaves similarly to the XD of a KK field with zero 5D mass and with fundamental mass  $\bar{M}$ . As a consequence the world-line compactification length  $\lambda_s = cT_\tau$  is the Compton wave length of the field. In order to bear in mind these analogies with an XD field theory we say that, the world-line parameter play the role of a *Virtual* XD (VXD) with compactification length  $\lambda_s$ . It is interesting to note that, originally, T. Kaluza introduced the XD formalism as a “mathematical trick” and not as a *real* XD [17].

## QUANTUM GEARS

Now we briefly show that our cyclic description of reality provides a remarkable matching with the canonical formulation of QM as well as with the Feynman Path Integral (FPI) formulation. The evolution along the compact time dimension is described by the so called bulk equation of motions  $(\partial_t^2 + \omega_n^2)\phi_n(x, t) = 0$  - for the sake of simplicity in this section we assume a single spatial dimension  $x$ . Thus the time evolution of the energy eigenmodes can be written as first order differential equations  $i\hbar\partial_t\phi_n(x, t) = E_n\phi_n(x, t)$ . The periodic field eq.(2) is a sum of on-shell standing waves. Actually this is the typical case where a Hilbert space can be defined. In fact, the energy eigenmodes form a complete set with respect to the inner product

$$\langle\phi|\chi\rangle \equiv \int_0^{\lambda_x} \frac{dx}{\lambda_x} \phi^*(x)\chi(x). \quad (7)$$

Therefore the energy eigenmodes can be defined as Hilbert eigenstates  $\langle x|\phi_n\rangle \equiv \phi_n(x)/\sqrt{\lambda_x}$ . On this base we can formally build a Hamiltonian operator  $\mathcal{H}|\phi_n\rangle \equiv \hbar\omega_n|\phi_n\rangle$  and a momentum operator  $\mathcal{P}|\phi_n\rangle \equiv -\hbar k_n|\phi_n\rangle$ , where  $k_n = n\bar{k} = nh/\lambda_x$ . Thus the time evolution of a generic state  $|\phi(0)\rangle \equiv \sum_n a_n|\phi_n\rangle$  is actually described by the familiar Schrödinger equation

$$i\hbar\partial_t|\phi(t)\rangle = \mathcal{H}|\phi(t)\rangle. \quad (8)$$

Moreover the time evolution is given by the usual time evolution operator  $\mathcal{U}(t'; t) = \exp[-\frac{i}{\hbar}\mathcal{H}(t - t')]$  which turns out to be a Marcovian operator:  $\mathcal{U}(t''; t') = \prod_{m=0}^{N-1} \mathcal{U}(t' + t_{m+1}; t' + t_m - \varepsilon)$  where  $N\varepsilon = t'' - t'$ .

From the fact that the spatial coordinate is in this theory a cyclic variable; by using the definition of the expectation value of an observable  $\hbar\partial_x F(x)$  between to generic initial and final states  $|\phi_i\rangle$  and  $|\phi_f\rangle$  of this Hilbert space; and integrating by parts eq.(7), we find

$$\langle\phi_f|\hbar\partial_x F(x)|\phi_i\rangle = \langle\phi_f|\mathcal{P}F(x) - F(x)\mathcal{P}|\phi_i\rangle. \quad (9)$$

Assuming now that the observable is such that  $F(x) = x$  [18] we obtain the usual commutation relation of ordinary QM:  $[x, \mathcal{P}] = i\hbar$ . With this result we have checked the correspondence with canonical QM. Furthermore, it is possible to prove the correspondence with the FPI formulation. In fact, it is sufficient to plug the completeness relation of the energy eigenmodes in between the elementary time evolutions of the Marcovian operator. With this elements at hand and proceeding in a complete standard way we find that the evolution of the cyclic fields turns out to be described by the usual FPI which, in phase space, can be written in this way

$$\mathcal{Z} = \lim_{N \rightarrow \infty} \int_0^{\lambda_x} \left( \prod_{m=1}^{N-1} dx_m \right) \prod_{m=0}^{N-1} \langle\phi| e^{-\frac{i}{\hbar}(\mathcal{H}\Delta\varepsilon_m - \mathcal{P}\Delta x_m)} |\phi\rangle. \quad (10)$$

This important result has been obtained without any further assumption than PBCs and has a simple classical interpretation. In a cyclic geometry there is an infinite set of possible classical paths with different winding numbers that link every given initial and final points. If we imagine to open this cyclic geometry we obtain a lattice with period

$T^\mu$  of initial and final points linked by classical paths. Thus there are many possible classical evolutions of a field from an initial configuration to a final configuration, which can self-interfere similarly to the non-classical paths of the FPI. However there is a fundamental conceptual difference with respect to the usual Feynman formulation: all these possible paths are classical paths, that is they are classical paths with different winding numbers. This means that in this path integral formulation it is not necessary to relax the classical variational principle in order to have self-interference.

The non-quantum limit of a massive field, that is the non-relativistic single particle description, is obtained by putting the mass to infinity so that, as shown in [1, 2], in an effective classical limit, only the first level of the energy spectrum must be considered. This leads to a consistent interpretation of the wave/particle duality and of the double slit experiment. The quantities describing only the first energy level are addressed by the bar sign. For instance, the Lagrangian of the fundamental mode  $\bar{\Phi}(x)$  is  $\bar{\mathcal{L}}_{\lambda_s}(\partial_\mu \bar{\Phi}(x), \bar{\Phi}(x))$ . Note that the fundamental mode  $\bar{\Phi}(x)$  coincides with the mode of Klein-Gordon field with energy  $\bar{E}$  and mass  $\bar{M}$ . Therefore it can be always quantized through second quantization. It can be shown that the analysis of the geometrodynamics of the de Broglie periodicities that we will perform below can be extended to ordinary field theory. On the other hand a massless field has infinite Compton wavelength and thus an infinite proper-time periodicity. Its quantum limit is at high frequency where, in fact, the PBCs are important. In this limit we have discretized energy spectrum, in agreement with the ordinary description of the black-body radiation (no UV catastrophe). The opposite limit described by a continuous energy spectrum is when time periodicity tends to infinity.

In the original 't Hooft toy model the period  $T_t$  was assumed to be of the order of the Planck time and its cyclic dynamics associated to some sort of hidden variables [19]. Furthermore the Hamiltonian operator was not positive defined. In our case, the assumption of intrinsic periodicities comes from PBCs. Therefore we have the remarkable property that QM emerges without involving any hidden-variable. The theory can in principle violates the Bell's inequality and we can actually speak about determinism. Moreover, similarly to the KK theory where there are no tachyons, a cyclic field can have positive or negative frequency modes but the energy spectrum describes always positive energies and the Hamiltonian operator is positive defined.

## GEOMETRODYNAMICS

To introduce interactions we must bear in mind that the four-periodicity  $T^\mu$  is fixed by the inverse of the four-momentum  $\bar{p}_\mu$  according to the de Broglie-Planck relation eq.(1)<sup>2</sup>. As already said, an isolated elementary system (*i.e.* free field) has persistent four-momentum. On the other hand, an elementary system under a generic interaction scheme can be described in terms of corresponding variations of four-momentum along its evolution with respect to the free case

$$\bar{p}_\mu \rightarrow \bar{p}'_\mu(x) = e_\mu^a(x) \bar{p}_a. \quad (11)$$

In other words we describe interactions in terms of the so called tetrad (or virebein)  $e_\mu^a(x)$ . Thus the interaction scheme eq.(11) turns out to be encoded in the corresponding variation of the space-time periodicities

$$T^\mu \rightarrow T'^\mu(x) \sim e_a^\mu(x) T^a, \quad (12)$$

that is in the corresponding deformation of the compactification lengths of a periodic field. Roughly speaking, interactions can be thought of as stretching of the compact dimensions of the theory. Equivalently, the interaction eq.(11) turns out to be encoded in the corresponding curved space-time background, which in the limit of weak interaction can be approximated as

$$\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x) \sim e_\mu^a(x) e_\nu^b(x) \eta_{ab}. \quad (13)$$

This result can be double checked by considering the transformation of space-time variables

$$dx_\mu \rightarrow dx'_\mu(x) \sim e_\mu^a(x) dx_a. \quad (14)$$

Under the approximation of weak interaction we are assuming that the  $T^\mu$  transforms as an infinitesimal interval  $dx^\mu$ . After this transformation of variables (diffeomorphism) with determinant of the Jacobian  $\sqrt{-g(x)}$ , the free action

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<sup>2</sup> For the sake of simplicity, we work in the approximation of weak interactions so that  $T^\mu$  transforms as an infinitesimal interval  $dx^\mu$ , but the results can be extended to the general case.

eq.(3) turns out to be

$$\mathcal{S}_{\lambda_s} \sim \int_{e_a^\mu(x) T^a} d^4x \sqrt{-g(x)} \mathcal{L}_{\lambda_s}(e_a^\mu(x) \partial_a \Phi(x), \Phi(x)). \quad (15)$$

Therefore, the periodic field which minimizes this action has four-periodicity  $T'^\mu$ , eq.(12), or equivalently has four-momentum  $\bar{p}_\mu$ , eq.(11). We conclude that a field under the interaction scheme eq.(11) is described by the solutions of the bulk equations of motion on the deformed compact background eq.(13) and compactification lengths eq.(12).

This geometrodynamical approach to interactions is interesting because it actually mimics very closely the usual geometrodynamical approach of GR. In fact, if we suppose a weak Newton potential  $V(\mathbf{x}) = -GM_\odot/|\mathbf{x}| \ll 1$ , we find that the energy on a gravitational well varies (with respect to the free case) as  $\bar{E} \rightarrow \bar{E}' \sim (1 + GM_\odot/|\mathbf{x}|) \bar{E}$ . According to eq.(12) or eq.(1), this means that the de Broglie clocks in a gravitational well are slower with respect to the free clocks  $T_i \rightarrow T_i' \sim (1 - GM_\odot/|\mathbf{x}|) T_i$ . Thus we have a gravitational redshift  $\bar{\omega} \rightarrow \bar{\omega}' \sim (1 + GM_\odot/|\mathbf{x}|) \bar{\omega}$ . With this schematization of interactions we have retrieved two important predictions of GR.

Besides this we must also consider the analogous variation of spatial momentum and the corresponding variation of spatial periodicities [20]. According to the relation eq.(13) the weak newtonian interaction turns out to be encoded in the usual Schwarzschild metric

$$ds^2 \sim \left(1 - \frac{GM_\odot}{|\mathbf{x}|}\right) dt^2 - \left(1 + \frac{GM_\odot}{|\mathbf{x}|}\right) d\mathbf{x}^2. \quad (16)$$

We have found that the geometrodynamical approach to interactions actually can be used to describes linearized gravity and that the geometrodynamics of the compact space-time dimensions correspond to the usual relativistic ones.

As well known, see for instance [20], it is possible to retrieve ordinary GR from a linear formulation by including self-interactions. More naively, as we will show in detail in a forthcoming paper, we can add “by hand” a kinetic term (with appropriate coupling) to the Lagrangian in curve space-time eq.(17) in order to describe the dynamics of the metric  $g_{\mu\nu}$  which is the new *d.o.f.* of the theory. Thus, in order to neglecting quantum corrections we replace the Lagrangian  $\sqrt{-g} \mathcal{L}_{\lambda_s}$  of eq.(17) with its non-quantum limit  $\sqrt{-g} \bar{\mathcal{L}}_{\lambda_s}$  and we add the kinetic term (with appropriate coupling  $16\pi G_N$ ) for the metric tensor obtaining the Hilbert-Einstein Lagrangian

$$\bar{\mathcal{L}}_{HE} = \sqrt{-g} \left[ -\frac{g^{\mu\nu} \mathcal{R}_{\mu\nu}}{16\pi G_N} + \bar{\mathcal{L}}_{\lambda_s}(e_a^\mu \partial_a \bar{\Phi}(x), \bar{\Phi}(x)) \right]. \quad (17)$$

This naive procedure is similar to what we usually do in electromagnetism when we add the term  $F_{\mu\nu} F^{\mu\nu} / -4e^2$  to describe the kinematics of the gauge field. Intuitively, because of its geometrical meaning, the Ricci tensor is the correct mathematical object to describes the variations of the space-time compactification lengths at different space-time points. Bearing in mind eq.(1), we note that actually such a kinetic term must encode the content of four-momentum in different space-time points. By varying the metric eq.(17) yields to the usual Einstein equation  $\mathcal{R}^{\mu\nu} = -8\pi G_N \mathcal{T}^{\mu\nu}$ . Here we do not discuss issues related to the variation of boundary terms of the Hilbert-Einstein action and related BCs [21]. As well known the Einstein equation can be obtained from different action formulations which different by boundary terms. With these simple and heuristic arguments we have shown that field theory in compact space-time is in agreement not only with special relativity but also with GR.

In forthcoming papers we will show that, by writing eq.(11) as a minimal substitution, such a geometrodynamical approach to interactions can be also used to describe ordinary gauge interactions. Gauge fields will turn out to “tune” the variation of periodicities, allowing a semi-classical interpretation of superconductivity [2]

A cyclic field turns out to be dual XD theory where the world-line parameter play the role of a VXD. On the other hand we have shown that it also matches ordinary quantum field theory [1]. From the dualism to XD theories and from the geometrodynamical approach to interactions described above, we aspect to find that the classical evolution of the periodic fields along a deformed VXD background corresponds to the quantum behaviors of the corresponding interaction scheme. Since, by using Witten’s words, in AdS/CFT “quantum phenomena [...] are encoded in classical geometry” we find that a relativistic field theory in compact 4D provides the possibility of an intuitive interpretation of the Maldacena conjecture. We will apply this idea to a simple Bjorken Hydrodynamical Model for Quark-Gluon-Plasma (QGP) logarithmic freeze-out [22]. In first approximation the energy momentum of the QGP can be supposed to decay exponentially (similarly to the Newton’s law of cooling for a thermodynamic system [23]). This interaction scheme is described by the conformal warped tetrad  $e_\mu^a = \delta_\mu^a e^{-ks}$ , where  $s$  is the proper time, that is by a *virtual* AdS metric. Actually, we find the classical configurations of cyclic fields in such a deformed background reproduces basic aspects of AdS/QCD.

## CONCLUSIONS

The formalism of cyclic space-time dimensions provides a consistent description of both the digital aspects arising from QM and the analog aspects typical of relativity. Must be noted that (general and special) relativity sets the differential structure of space-time without giving particular prescriptions for BCs. On the other hand, BCs have played an important role since the earliest days of QM (for instance as in the Bohr atom or as in the particle in a box). We have seen that relativity is compatible with cyclic analog space-time dimensions, since the periodicities transform in a covariant way. In the limit of infinite time periodicity, that is in the case of low energy massless fields such as the IR electromagnetic fields of a Black-Body radiation, we have a purely analog limit where the elementary system is described by fields with approximatively a continuous energy spectrum. In the case of small time periodicity, for instance as in a UV electromagnetic field in the black body radiation with respect to the thermal noise, the analog field description of the theory is gradually replaced by digital corpuscular aspects: the energy spectrum turns out to be quantized and the initial and final configurations of the fields form a periodic lattice, so that the evolution is described by a sum over classical paths with different winding numbers.

These intrinsic time periodic fields can be identified with the so call “de Broglie internal clocks”. Similarly to an analog or digital stopwatch, every moment in time is determined by the combination of the phases or the “ticks” of periodic cycles (typically: years, months, days, hours, minutes and seconds). Every value of our external temporal axes (defined with reference to the digital “ticks” of the Cs-133 atomic clock) is characterized by a unique combination of the “ticks” of all the “de Broglie internal clocks” constituting the system under investigation. In this scenario the long time scales are provided by massless fields with low frequencies (long time periodicities). This, however, is an oversimplified picture since, as we have seen, the clocks can vary periodicity through interaction (exchange of energy) and that periods depends dynamically on reference systems according to the relativistic laws. Moreover the combination of two or more clocks, that is to say a non elementary system, with irrational ratio of periodicities gives ergodic, or even more chaotic, evolutions. Independently of the assumption that the de Broglie internal clocks are clockwise or anticlockwise, the flow of time is uniquely determined by the combinations their “ticks” and the variations of their periodicities. Hence, the flow of time can be effectively described in terms of the “ticks” of these de Broglie internal clocks. This formulation is particularly interesting for the problem of the time arrow in physics.

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