Elementary Time Cycles

Essay Contest: *Questioning the foundations*

© Donatello Dolce

Relativity, if rigorously interpreted, only fixes the differential structure of space-time without giving any particular prescription about the boundary of the space-time dimensions. On the other hand the boundary conditions have played a fundamental role since the earliest days of quantum mechanics. Wave-particle duality suggests the introduction in physics of the principle of intrinsic periodicity so that elementary particles turn out to be described as modulated harmonic vibrations of compact space-time dimensions. This enforces the undulatory nature of elementary particles and the local nature of relativistic space-time. For instance, as mathematically proven in recent publications, this conciliates special relativity with quantum mechanics [1], gauge interactions with gravitational interaction [2], and pinpoints a fundamental aspect of the Maldacena conjecture [3]. Here, we present some conceptual elements of this innovative and elegant interpretation of the quantum world giving some historical and philosophical motivations.

I. The 'incomplete revolution' of quantum physics.

Quantum mechanics (QM) has been defined as the "incomplete revolution". In fact, it is an "axiomatic" theory, i.e. based on purely mathematical axioms whose physical meaning, after more than a century, is still debated. From a formal point of view QM works in surprising agreement with our experimental observations. This means that its mathematical formulation must be considered to be correct but over the decades it has left a consistent number of paradoxes and questions unsolved. The elusive nature of the quantum world has been pointed out by the most eminent minds of quantum physics. Einstein addressed the indeterministic nature of QM by saying "God doesn't play dice". Feynman, who is the father of the most powerful and modern formulation of QM, said "I think I can safely say that nobody understands QM".

The most problematic aspects arise when QM is combined with Relativity. In this case the difficulties are also computational and mathematical. In fact, observables in particle physics are calculated through techniques that involve cancellation, a.k.a. renormalization, of infinities (e.g. loop diagrams). To add a single digit in the precision, brute-force calculations are necessary with long simulations in extremely expensive supercomputers. Note that two of the fathers of renormalization theory, the Nobel laureates, Prof. G. 't Hooft and Prof. F. Wilczek, are actively working on foundations of QM –in models that for some aspects are related to the idea below. Indeed, besides the purely conceptual interest, the open questions of QM affect directly most of the ongoing research areas of modern physics. For instance the Higgs mechanism has its origin in superconductivity (Cooper pair), that is to say a quantum phenomenon. The problematics of the Higgs sector are essentially of quantum nature (e.g. vacuum stability, radiative corrections, hierarchy problem, unitarization). The strong evidences of a 125GeV Higgs boson coming from LHC is problematic even for the Standard Model (SM) itself, suggesting for new physics. For instance, such a light Higgs boson implies an unnatural cancellation of quantum corrections of 15 orders of magnitudes. Moreover, this would imply that gauge interactions –and the Higgs mechanism generating masses– are practically decoupled from gravity or that the quantum vacuum of the universe is unstable.

The gravitation interaction in General Relativity (GR) has a well-established geometrical space-time meaning, and concerns about clock synchronization. On the other hand gauge interactions, investigated by LHC and representing the other three fundamental interactions of nature, are based on quantum symmetries. Despite Weyl's [17] and Wheeler's original attempts to interpret gauge symmetries, their geometrical space-time meaning remains obscure and their existence must be postulated mathematically. Furthermore the situation is even worse when QM is combined to GR. One of the most important outcomes of the attempts of a unified description of QM and GR, the so-called gauge/gravity duality or AdS/CFT (Anti de Sitter / Conformal Field Theory) correspondence [16], remains after more than a decade an unproved conjecture, a.k.a. Maldacena's conjecture. All this requires new ideas in particle physics (I would like to mention that even WIMP models, the main candidate for dark-matter, is also very constrained by experiments). The new experimental data are giving rise to a renewed interest in the foundations of physics.

II. Digging into the history of physics

In physics the most groundbreaking ideas are the simple ones. At the beginning of the 19th century experiments suggested that the speed of light was constant. Einstein raised this experimental evidence to a fundamental principle of physics and he derived relativity theory. As noted since the "old" formulation of QM, experiments tell us that a phenomenon with recurrences in time (periodicity) T and space (wavelength) l is to be associated to every elementary particle. This observation is at the base of the famous wave-particle duality, shown in fig.1. In fact, nearly 90 years ago de Broglie noted that the periodicity and wavelength of a relativistic wave can be used to describe the energy momentum, E and p, of the particle through the Planck constant h. That is, $E= h / T$ and $p = h/ l$. The undulatory nature of elementary particles is at the base of the modern description of relativistic QM and it is observed in many experiments [14]. If this intrinsically periodic nature of elementary system is raised to a general principle, an unified description of fundamental quantum and relativistic aspects of physics follows naturally. The rigorous mathematical demonstrations off this unified description can be found in [1-9] or in the citations thereby. In particular the wave-particle duality was introduced by de Broglie in terms of "*we proceed with the assumption of the existence of a certain "periodic phenomenon" of a yet to be determined character, which is to be attributed to each and every isolated energy parcel [elementary particle]*" [19]. We shall propose an elegant realization of this "yet to be determined" "periodic phenomenon" in terms of space-time vibrations with characteristic de Broglie periodicities. We will show how this can unlock in an elegant way important, unsolved (and sometimes forgotten over the decades) problems of physics [1-9]. For the sake of simplicity here we only consider time periodicity T, to which a fundamental energy $E=h/T$ is associated. For instance, in classical-relativistic mechanics an isolated elementary particle has constant energy E, so that the corresponding temporal periodicity T of the de Broglie periodic phenomenon is persistent, i.e. is constant in time. This is similar to a pendulum in the vacuum (we neglect gravitational effect here). If no interaction or friction affects the pendulum, the physics laws say that it will continue to oscillate forever with the same periodicity T fixed by the energy initially given to it. It is important to bear in mind that, according to de Broglie, the energy E and the periodicity T are "two faces of the same coin". This means that the assumption of intrinsic periodicity is fully consistent with relativity as long as the relativistic modulations of periodicity T associated to relativistic variations of kinematical state E are considered. Indeed, we will describe interaction as modulation of periodicity T as for gravitational interaction in GR. Since our world is composed of elementary particles and QM says that elementary particles are intrinsically periodic phenomena, it is natural to assume that reality can be described in terms of elementary cycles i.e angular variables. This is the forgotten lesson of QM that we want to explore [13].

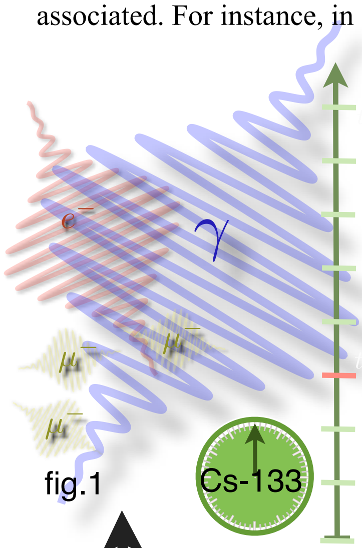

fig.1

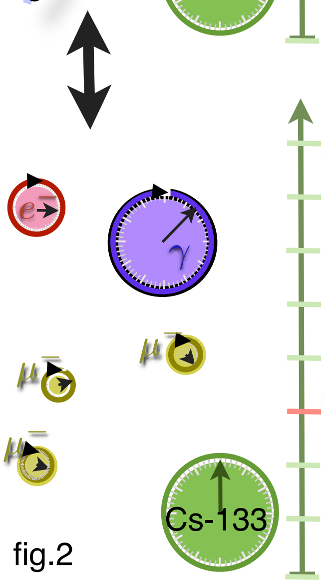

fig.2

The notion of time implicitly contains an assumption of intrinsic periodicity. As Galileo taught us with the pendulum experiment in the Pisa dome, time can only be defined by counting the number of periods of a phenomenon which is supposed to be periodic. The persistence of periodicity guaranties that the unit of time does not vary. Indeed, the modern definition of time is based on the same principle: "A second is the duration of 9,192,631,770 periods of the radiation corresponding to [...] the Cs 133 atom". The importance of the assumption of intrinsic time periodicity is also present in Einstein's definition of relativistic clock: "By a clock we understand anything characterized by a phenomenon passing periodically through identical phases so that we must assume, by the principle of sufficient reason, that all that happens in a given period is identical with all that happens in an arbitrary period" [20]. Under the assumption of intrinsic periodicity, every elementary isolated particle can be therefore regarded as a reference clock, the so-called de Broglie "internal clock" [13,14]. In fact, an isolated particle has persistent time periodicity as an inertial pendulum in the vacuum. In fig.2 we show the same systems of elementary particles of fig.1 (e.g. 1 photon, 1 electron and 3 muons) described in terms of de Broglie "internal clocks".

The assumption that an elementary particle is an intrinsically periodic phenomenon means to enforce the undulatory nature of elementary particles of ordinary QM, but at the same time we also are enforcing the local nature of relativistic time. This turns out to conciliate together SR and QM. Fig.2 also shows the arrow of time defined in terms of the periods of an atomic clock (green). Every instant in time is characterized by a different combination of the phases of the internal clocks constituting the system under observation. Intuitively, this is what we do every day when we fix events in time by combining phases of time periods that we call years, months, days, hours, minutes, seconds, and so on. A system of periodic phenomena is ergodic (i.e. quasi-periodic evolution, such as that of the combination of the rotation and revolution of the Earth and the Moon). If we also consider that the clocks can vary time periodicity through exchange of energy (i.e. interaction), we see that the evolution of a simple system of interacting periodic phenomena is in general very chaotic. Similar to an ordinary calendar or stopwatch the arrow of time in the quantum world can be therefore described in terms of the "ticks" of the internal clocks of the particles, and in principle the external time axis can be dropped. This description of reality in terms of elementary cycles yields a better understanding of problems related to the notion of time in physics [1-3,8,9]. In particular the helicity of the de Broglie internal clock associated to the elementary particles, as our ordinary clocks, can be conventionally chosen to be clockwise or anti-clockwise. Only the reciprocal combinations of the ticks of these clocks are important. These depend on the kinematical state of the observer (relativistic simultaneity) and the periodicities, as the energy, vary with the retarded potential (relativistic causality and time ordering). Every particle is a reference clock and can be used to define an external time axis: the local nature of time is enforced w.r.t. ordinary relativity. This means that the inversion of the helicity of a single clock does not mean to invert the helicity all the other clocks. This simply corresponds to pass from a particle to an antiparticle description.

The time scales of the de Broglie internal clocks are typically extremely fast w.r.t. our experimental resolution in time. In fact the intrinsic time periodicity of a particle is always faster than its Compton time. For instance, the time scale associated to an electron (the lightest particle excepting neutrinos) is 10^{-21} s. This means that for every "tick" of the atomic clock (periodicity 10^{-10} s), an electron does a huge number of "ticks" (10^{10}). The difference between these time scales is comparable with the difference between the age of the universe and a solar year. The modern resolution in time, nearly 10^{-17} s, is still far from these scales. The internal clock of the electron has been indirectly observed in a recent interference experiment [12]. Intrinsic periodicity also yields a semi-classical formulation of the spin and the intrinsic magnetic momentum of the electron as already noticed in 1932 by Schrödinger with his *Zitterbewegung* model.

III. The secret harmony of elementary particles

An elementary system constrained to have intrinsic periodicity can be typically represented as a string vibrating in compact dimensions. Therefore, as shown in fig.3, we shall represent elementary particles as strings vibrating in space-time dimensions. The manifestation of the corresponding harmonic modes is QM with all its peculiar phenomena [1-3,9]. As known since Pythagoras, a vibrating string has discretized (quantized) frequencies. That is a homogeneous string, which in our case corresponds to a free particle, can be represented as a homogeneous string vibrating with fundamental periodicity $T=1/\nu$. Through the Planck constant h, the resulting harmonic frequency spectrum implies a harmonic quantized energy spectrum $E_n = n E = n h \nu$. Intrinsic periodicity represents a quantization condition as in the semi-classical description of a "particle in a box", fig.3.

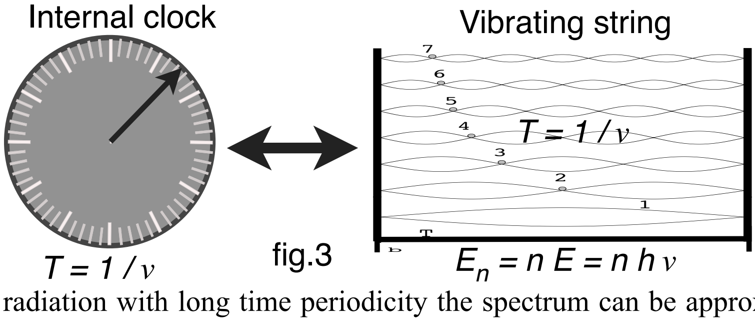

fig.3

We may consider the case of the Black-Body radiation. Photons are massless particles, $M=0$, so that they have infinite Compton time, i.e. the internal clock of the photon is frozen. This means that the photon periodicity can span from zero to infinity. Thus, for those components of the electromagnetic radiation with long time periodicity the spectrum can be approximated to a continuum. On the other hand, for those components with very small periodicities the effect of the periodicity cannot be neglected and the quantization of the energy spectrum $E_n = n E$ becomes evident. This is ordinary Planck's description of the Black-Body radiation, avoiding the so called UV catastrophe.

According to the relativistic Doppler effect, the periodicity of a vibrating string is modulated relativistically. In this way it is straightforward to prove that the harmonic spectrum associated to the intrinsic periodicity, if observed from different reference frames, reproduces exactly the same energy spectrum prescribed in ordinary QFT by the famous "second quantization". This is the first element of a long chain of exact formal correspondences with ordinary QFT. For instance, a vibrating string is the typical classical system which can be described in a Hilbert space, the Schrödinger equation is obtained as the "square root" of the wave equation describing the string dynamics, the evolution is unitary (it can be cut in infinitesimal evolutions), it is possible to define a Hamiltonian (and momentum) operator describing the corresponding harmonic spectrum, and so on. Remarkably, by integration by parts, from this intrinsic cyclic behavior it is also possible to derive exactly the ordinary commutation relations of QM which are implicit in the assumption of periodicity. Furthermore, to establish with good accuracy the frequency $\Delta\nu$, or equivalently the energy $\Delta E = h\,\Delta\nu$ of a periodic phenomenon in which we cannot observe the phase (Hilbert space), it is necessary to count a large number of ticks, according to the Heisenberg relation $\Delta E\,\Delta t \geq h/2$. *Intrinsic periodicity implies an exact formal correspondence with canonical QM.*

Even the most powerful mathematical tool of QFT, i.e. the Feynman Path Integral (FPI), can be ***exactly*** derived in a extremely intuitive way from the assumption of intrinsic periodicity, [1-3,9]. The FPI prescribes that the quantum evolution of a particle is described by the self-interference of all its possible paths starting from A to B. But in classical mechanics it is only possible a single "straight" path between two point, so that in the ordinary formulation the FPI can be only achieved by relaxing one of the pillars of classical mechanics, the least action principle. But this is not the case of a classical cyclic geometry. If we imagine drawing all the possible "straight" lines between two points on a cylindrical surface, fig.4, we immediately see that an infinite set of classical paths with different windings numbers are possible. In a few words, a classical periodic phenomenon can self-interfere and, as proven in [1-3], its classical evolution is naturally described by the ordinary FPI of QM. From this for instance follows a similarly natural interpretation of the double slit experiment as well as of the non-relativistic particle description of the vibrating string (high frequency limit), [1,9].

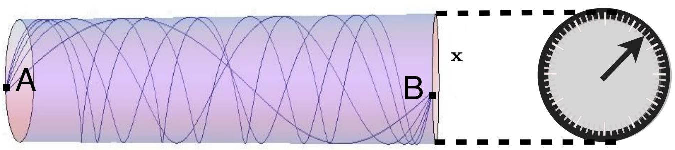

fig.4

In this theory the quantized energy spectrum of a particle is described in terms of vibrations with characteristic periodicities along the time dimension. Similarly, the vibrations along the modulo of spatial dimension and of the angular dimensions describe the quantization of the momentum and of the angular momentum, respectively. The elegance of the theory can be noted from the fact the it represents the full relativistic generalization of the theory of sound, developed by the Nobel laureate Reyleigh at the end of the 19th century and which is actually at the origin of the modern quantum formalism. Matter can be regarded as sound sources vibrating not only in space but also in time. The relativistic waves generated by these vibrations play the role of the mediator of interactions. The idea that quantum mechanics could be related to space-time vibrational modes has recently inspired the Nobel laureate prof. F. Wilczek which has published two papers on the -for some aspects- similar idea of "time-crystal", [11] (a crystal is characterized by a periodic structure, and it can be described by considering a single period, e.g. Brillouin zone).

Indeed the assumption of intrinsic periodicity also yields the Bohr-Sommerfeld quantization which is historically was one of the first quantization prescriptions. Its most famous application is in the quantized orbitals of the Hydrogen atom which can be obtained by considering only closed orbits for the electron, i.e. periodicity conditions in space-time (modulo a twist factor) and in the spherical angles. It can be shown that from this all the possible non-relativistic Schrödinger problems can be consistently solved.

IV. An unexpected unified scenario

As can be extrapolated from recent publications [1-9], the axioms of ordinary QM can be formally derived directly from the simple and natural assumption of intrinsic periodicity of elementary systems. This promotes intrinsic periodicity as the physical principle (the "missing link") for the "incomplete revolution" of QM, see also [13]. In fact intrinsic periodicity is also a way out to Bell's theorem (it states that a theory with local-hidden-variables cannot consistently reproduce ordinary QM). In fact, the quantization condition in the theory is the assumption of intrinsic periodicity, similar to the semi-classical description of a particle in a box. This also means that the theory does not have local-hidden-variables (time is a physical variable that cannot be integrated out and the assumption of periodicity is an element of non-locality in the theory). As noticed in [1-3,9], if we try to reformulate Bell's theorem within our theory we find again a formal correspondence with the inequalities of ordinary QM. Therefore we may actually speak about mathematical determinism and emerging QM. Intuitively, the typical periodic dynamics of a typical quantum system (governed by the quantum electrodynamics) are so intrinsically fast (the intrinsic periodicities associated to electrons are always faster than 10^{-21} s) w.r.t. our resolution in time that at every observation the quantum system appears to be in an aleatoric phase of its cyclic evolution. Therefore, a phenomenon with such a fast periodicity can be imagined as a dice (namely the "de Broglie Dice", see [5]) rolling so fast that its outcomes can only be described statistically. As proven in my papers, and similarly to 't Hooft determinism [21] and stroboscopic quantization [22], the statistic description of a fast periodic phenomenon is formally given by ordinary QM. Thus we can say that "God does not play dice" [Einstein], in the sense that an imaginary observer with infinite time resolution would be able to resolve the underlying deterministic dynamics of a die and in principle predict the outcomes: such an observer would not have fun playing dice. This interpretation of QM not only is interesting for the advances on the conceptual and philosophical knowledge of the quantum world and of the concept of time, it is also interesting to improve computational methods and to address open questions in particle physics. To see this we must introduce interaction in our theory. Since temporal-spatial periodicity and energy-momentum are two faces of the same coin, we can equivalently describe the retarded local variations of energy-momentum occurring during relativistic interactions in terms of corresponding retarded local modulations of temporal-spatial periodicity of the elementary particles. In particular we may note that Einstein derived his description of gravitational interaction of GR by considering the modulations of temporal-spatial periodicity associated to a Newtonian gravitational potential. In general, local modulations of periodicity of reference clocks can be equivalently encoded in corresponding local deformations of the underlying space-time metric [23]. Fig.5, shows the modulation of periodicity in linearize gravity and the corresponding deformation of the metric (i.e. the linearized Schwarzschild metric) [3]. Reference clocks in a gravitational well are known to go slower w.r.t. those outside. Therefore the assumption of intrinsic periodicity is fully consistent with special and general relativity as long as modulations are considered. The intrinsic periodicity of the elementary particle can be formalized by means of periodic boundary conditions. As well-known from string theory, periodic boundary conditions, as well as Dirichlet or Neumann boundary conditions, fulfill the variational principle and therefore preserve all the relativistic symmetries of the theory. In fact relativity sets the differential structure of space-time without giving any particular prescription about the boundary conditions [23] whereas QM was characterized by boundary conditions since its earliest days. It is strange enough, from a mathematical point of view, that ordinary QFT is a differential theory in which only the most general solution of the differential equation is considered whereas the boundary conditions have a very marginal role in explicit computations. The geometrodynamical description of gravity and its relation to clock modulation is well known. The geometrodynamical description of gravitational interaction in GR can be extended to gauge interactions by considering the undulatory nature of elementary particle, as mathematically proven in [3]. The assumption of intrinsic periodicity as a general principle of physics represents a breakthrough towards a unified geometrodynamical description of all the fundamental interactions of nature. Essentially, a gauge field, through a formalism very similar to the one of modulated signals, turns out to describe modulations of periodicity associated to local transformations of flat reference frame. This can be easily understood if we consider, for instance, how an electromagnetic field is generated by the trembling motion of a charged particle in an antenna. Indeed, similarly to GR, such a trembling motion can be described as local transformation of the reference frame of the charged particle. The resulting modulation of periodicity, similarly to the Doppler effect, turns out to be described by the ordinary Maxwell equations. The local modulation of periodicity and the

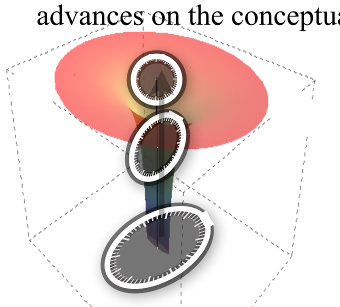
fig.5 Gravity

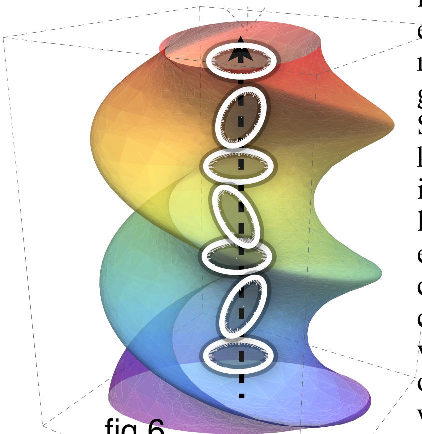
fig.6 Electromagnetism

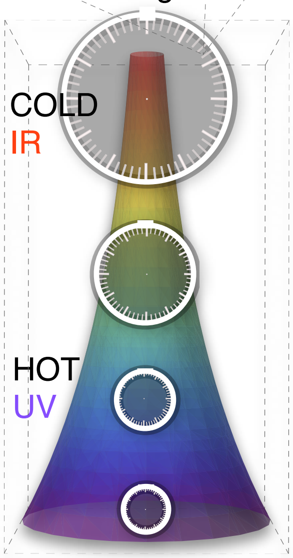

fig.7 (QCD) Quark-Gluon-Plasma

corresponding local "rotation" of reference frame (e.g. *Zitterbewegung*) is shown in fig.6. Remarkably, gauge symmetries, which in ordinary QFT are internal symmetries whose existence must be postulated, turn out to be related to space-time symmetries. That is, through the assumption of intrinsic periodicity, the Equivalence Principle of General Relativity can be extended to gauge interaction, in a realization of Weyl's proposal which originated the idea of gauge invariance. Similar attempts are given by Kaluza's and Wheeler's works. Moreover, when the modulation of periodicity of all the harmonic modes is considered, the classical evolution of such a vibrating string turns out to be exactly described by ordinary quantum electrodynamics (QED). This means that the quantum behavior of gauge interactions can be derived directly from cyclic dynamics, so that, in principle, Feynman diagrams can be expanded in harmonics and calculated semi-classically. Remarkably, the explicitly quantized spectrum associated to intrinsic periodicity could regularize the conceptually problematic infinities of the loop diagrams. This possibility is confirmed by Light-Front-Quantization, twistor theory, holography, AdS/CFT; all these theories have fundamental analogies with our theory. The quantization through the assumption of intrinsic periodicity has important applications in condensed matter where actually semi-classical methods are largely used.

Intrinsic periodicity also implies that the gauge field can only vary by finite amounts in unit *h/e*, so that the magnetic flux is quantized and we obtain the Dirac quantization condition for magnetic monopoles [1,9,14]. In other words, the electric current cannot vary continuously. This reproduces superconductivity in an immediate and material-independent way. Thus, intrinsic periodicity not only provides a geometrodynamical description of gauge interaction, its quantum limit also suggests a gauge symmetry breaking mechanism with fundamental analogies with the Higgs mechanism. This yields the possibility of a geometrodynamical interpretation of the problem of the origin of the masses explored by LHC. The masses of elementary particles are related to the proper time periodicities of vibrating strings [1-9,12,13], similar to the string description of the hadron spectrum of QCD. The question of the origin of the mass spectrum can be therefore reformulated in the following way: *Why is nature playing that particular string chord?*

It is also interesting to note that the extra-dimension in a kaluza-Klein theory acts in perfect mathematical analogy with the Compton periodicity. This reveals an explicit correspondence between QM and extra-dimensional theory [2], similarly to Klein's original proposal, and justifies the good behavior of extra-dimensional theory without introducing any (unobserved) extra-dimension. Considering this dualism, the theory also provides an elegant explanation for the correspondence between quantum behavior and classical geometry at the base the AdS/CFT correspondence, which is one of the greatest open questions of the last decade of theoretical physics [16]. Similarly, fig.7, it is possible to infer the quantum behavior of the quark-gluon-plasma freeze-out (e.g. the asymptotic freedom) from the classical Bjorken hydrodynamical model [2,9,15]. For the general nature of the assumption of intrinsic periodicity, it is not difficult to foresee spectacular applications and predictions (not discussed here), in particle physics, condensed matter, cosmology, quantum gravity, with possible important implications in other branches of knowledge, e.g. biology or economy (cyclic models).

Conclusions

The space-time coordinates in relativity are typically interpreted as emphatically non-compact (except in cyclic cosmology). Nevertheless, if rigorously interpreted, relativity only fixes the differential structure of space-time (e.g. the metric) without given any particular prescription about the boundary conditions [23]. On the other hand, QM with the wave-particle duality implies that every physical system can be described in terms of a set of elementary particles, characterized by an undulatory nature, in which the space-time coordinates enter as angular variables. This suggests a principle of intrinsic periodicity in physics. As proven in my recent publications [1-9], the assumption of intrinsic periodicity, if consistently implemented in relativity w.r.t. the classical variational principle, represents the natural solution of most of the fundamental (and sometimes forgotten) open questions of modern physics. Elementary particles are described as harmonic vibrations of the space-time dimensions. Their characteristic modulated periodicities encode the relativistic kinematics through the Planck constant, similarly to undulatory mechanics. The manifestation of these space-time vibrations is relativistic QM in all its peculiar phenomena. This reveals an elegant new interpretation of the local nature of the relativistic space-time dimensions enforcing the wave-particle duality of QM. As a result, quantum and relativistic dynamics as well as gauge and gravitational interactions can be conciliated in a unified description. This points out a new frontier in physics.

References

[1] **Compact Time and Determinism for Bosons: foundations.** D. Dolce, *Found. Phys. 41:178-203, 2011*, arXiv:0903.3680.
[2] **Classical geometry to quantum behavior correspondence in a Virtual Extra Dimension,** D. Dolce, *Ann. Phys., 327 (9), pp 2354-2387*, arXiv:1110.0316.
[3] **Gauge Interaction as Periodicity Modulation,** D. Dolce, *Ann. Phys., 327 (6), pp. 1562-1592*, arXiv: 1110.0315.
[4] **On the Intrinsically Cyclic Nature of Space-Time in Elementary Particles,** D. Dolce. *13pp. J. Phys.: Conf. Ser. 343 012031, 2012*, arXiv:1206.1140.
[5] **de Broglie Deterministic Dice and emerging Relativistic Quantum Mechanics,** D. Dolce, 10pp. *J. Phys.: Conf. Ser. 306 012049, 2011*, arXiv:1111.3319.
[6] **Deterministic Quantization by Dynamical Boundary Conditions,** D. Dolce, *AIP Conf.Proc. 1246:178-181,2010,* arXiv:1006.5648.
[7] **Quantum Mechanics from Periodic Dynamics: The Bosonic Case,** D. Dolce, *AIP Conf.Proc. 1232:222-227,2010,* arXiv:1001.2718.
[8] **Clockwork Quantum Universe,** D. Dolce, 8pp. *IV Price, FQXi contest 2011 "Is reality Digital or Analog?"*, arXiv:1205.1788.
[9] **Compact Time and Determinism for Bosons,** D. Dolce, *e-Print*: arXiv:0903.3680v1 - arXiv:0903.3680v4.
[10] **How a wave function can collapse without violating Schroedinger's equation,** G. 't Hooft, arXiv: 1112.1811
[11] **Quantum Time Crystals**, F. Wilczek, arXiv:1202.1523
[12] **Search for the de Broglie Particle Internal Clock by Means of Electron Channeling**, P. Catillon, et.al, *Found.Phys.38(2008)659*.
[13] **A missing link: What is behind de Broglie's "periodic phenomenon"?,** R. Ferber, *Found. Phys. Lett., 9, 6, (1996),575–586*.
[14] **Electroweak Symmetry Breaking From Monopole Condensation,** C. Csaki, Y. Shirman, J. Terning, *Phys.Rev.Lett.106:041802,2011*
[15] **Non-equilibrated post freeze out distributions**, V. K. Magas, et.al, *Eur. Phys. J. C30 (2003) 255–261*.
[16] **Anti-de Sitter space, thermal phase transition, and confinement in gauge theories,** E. Witten, *Adv. Theor. Math. Phys.2(1998)505*
[17] **Gravitation and electricity**, H. Weyl, *Sitz.Preuss.Akad.Wiss. Berlin (Math. Phys.) 1918 (1918) 465*.
[18] **Electron in a transverse harmonic potential**, H. Honkanen, *PoS LC 2010 (2010) 004*.
[19] **Researches on the quantum theory,** L. de Broglie, *1924, Phil. Mag. 47 446*
[20] **Principe de relativité,** A. Einstein, 1910, *Arch. Sci. Phys. Natur. 29*
[21] **Determinism in free bosons**, G. 't Hooft, 2003, *Int. J. Theor. Phys., 42, 355,* arXiv hep-th/0104080
[22] **Time without time: a stochastic clock model,** H. T. Elze, O. Schipper, *Phys.Rev. D66 (2002) 044020*, arXiv gr-qc/0205071.
[23] **Boundary Terms in the Action Principles of General Relativity,** J. York 1986 Found. Phys. 16 249