Theoretical proof of biased will of nature as the origin of quantum mechanical results

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Abstract

Although quantum mechanics can accurately predict the probability distribution of outcomes in a large number of identical systems, it cannot predict the result of individual experiment. So, Schrodinger and others had hypothesized the existence of free will in every particle which causes randomness in individual results. This free will theory however *failed to quantitatively* explain the quantum mechanical results. In this article, by assuming the will of particles to be biased to satisfy the goals of the collective system or universe, we mathematically derive the correct spin probability distribution without using quantum mechanical formalism (operators and Born's rule). Similarly, by using biased will, we exactly reproduce quantum mechanical spin correlation in entangled pairs of particles. Finally and most importantly, using the biased will approach, we develop a scientific justification for the form of quantum mechanical wave function of free particle (which is conventionally and till date a postulate of quantum mechanics). Thus, we prove that the will of the object biased by universe is the origin of quantum mechanical results. So, we can say that mindless mathematical laws of quantum mechanics give rise to aims and intention in complex systems because the microscopic entities from which the larger system is made up of, are biased to contribute in achievement of the collective goal of the system. Finally, in this paper we show that by using our biased will theory, we can find answers to many important philosophical questions such as how life and intelligence could have been created in nature.
Keywords: Biased will, Goal oriented behavior, Quantum biology, Quantum entanglement

1. Introduction:

It is well known that classical or Newtonian mechanics, although deterministic, is an approximate theory that fails at the microscopic level. After discovery of first quantum mechanical formalism by Schrodinger in 1926, quantum physics has established itself as a robust and accurate theory of nature explaining nearly all the physical phenomena observed in the universe both at the micro and macroscopic level. But most important limitation of quantum mechanics is that it is a stochastic theory providing only a probabilistic prediction of experimental results. The accuracy of its prediction can be confirmed only if an infinitely large number of experiments are carried out in identical systems. In a single experiment on an individual system, the quantum mechanics cannot predict the result with 100% accuracy (i.e. with probability one). That's why in their famous paper, Einstein, Podolskey and Rosen [1] expressed the view that present day quantum mechanics is incomplete and hoped that it may be completed in future by including some local hidden *realistic* (*pre-existing*) variables in the system being measured. By assuming these local hidden variables to be the cause of outcomes, in 1964, Bell [2] derived an inequality expression for the spin correlation among entangled pairs of particles which contradicted the quantum mechanical spin correlation. Consequently, Aspect et al and others [3-7] have carried out experiments which decisively violated the Bell's inequality and thus ruled out the future possibility of local hidden variables to complete the quantum theory. Recently, non-local realistic theories [8-10] have also been proved to be incompatible with quantum mechanics and relativity. While reporting the experimental results demonstrating the inconsistencies of non-local realistic theories, Groblacher [10] wrote, "Our result suggests that giving up the concept of locality is not sufficient to be consistent with quantum experiments, unless certain intuitive features of realism are abandoned". To escape from this no-go situation, originally Schrodinger [11] and then Coway and Kochen [12] proposed that every elementary particle in the universe has some amount of free will (a kind creativity) which causes the uncertainty or randomness in the experimental result. However, this free will theory could neither quantitatively prove the quantum mechanical results nor provide any basis for postulates of quantum mechanics.

In this article, instead of taking the will of nature as completely free, we have taken it to be biased to satisfy some laws and intentions (or motivations) of the universe. By this, we could form a foundation on which the postulates of quantum mechanics can stand and for some cases we could directly derive established quantum mechanical results without using quantum mechanical formalism (such as operators and Born's rule). In our case, because there is a will in each particle or system, of course there will be some amount of randomness in the outcomes of individual experiments. However, the biasing of the will by nature generates a specific pattern or distribution in the outcomes of repeated experiments carried out on identical systems. Although an individual entity may look like wondering randomly, the collection of such entities show a pattern in results trying to achieve a certain goal of the universe or collection. In order to develop a scientific proof of the above phenomena, in the next section-2, we will first see how by considering the biased will of the system and without using quantum mechanical formalism such as operators and Born's rule, we can correctly predict the probability distribution of spin along any arbitrary direction that agrees with the quantum mechanical predictions. In section-3, using the biased will approach, we will derive the expression for expectation value of spin correlation between two entangled particles that again exactly matches with the conventional quantum mechanical relation. We will also discuss how opposite spin may be understood in a single pair of entangled particles separated by huge (space-like) distances without need of superluminal information transfer. In section-4, we theoretically justify the form of generalized quantum mechanical wave function of a free particle using biased free will so that interference of matter waves and expressions for quantum mechanical operators can be explained. Thus, by means of three different cases, we prove that the origin of quantum mechanical results is the *will* of the objects biased by the universe. Since the biasing of will is to achieve certain overall goal for the ensemble, all macroscopic systems have amount of goal oriented behavior depending upon extent of coherence in its constituent parts. Thus, we understand that mindless mathematical laws gives rise to aims and intention in complex systems because the microscopic entities from which the larger system is made up of are biased to help in achieving the collective goal. As an experimental evidence of it, we can cite the recently reported adaptive mutation in genetic material DNA [13]. Although mutation was earlier thought to be purely random, it has been experimentally observed that when E. Coli bacteria is subjected to low oxygen atmosphere, mutation in them occurs in a direction that makes them more virulent. Finally in section-5, using the above theory of biased will, we try to find answers to some important philosophical questions and discuss the consequences of it on emerging branches of science like quantum biology where we can model the processes and behaviors exhibited by living organisms [14-15].

2. Derivation of spin probability distribution from biased will:

Consider a fundamental particle such as an electron whose spin angular momentum '***s***' is quantized that can be of either $+\hbar/2$ or $-\hbar/2$ along the direction of measurement. Let the electron initially passes through a Stern-Gerlach magnet so that its spin is aligned along Z-axis and let it be $+\hbar/2$. Of course, in this case spin along other two perpendicular directions are undefined. Now we want to find out the probability that its spin is found to be $+\hbar/2$ when measured along any arbitrary direction making an angle θ with Z-axis as shown in Fig.1. Because of presence of will of the particle, there will be a uncertainty in result in the individual experiment which can be $\pm\hbar/2$. But whatever be the result, angular momentum of the single particle is not conserved (it is initially $+\hbar/2$ along Z-axis and finally $\pm\hbar/2$ along OM as shown in Fig.1). This happens because of superiority of law of quantization of spin over the law of conservation of angular momentum.

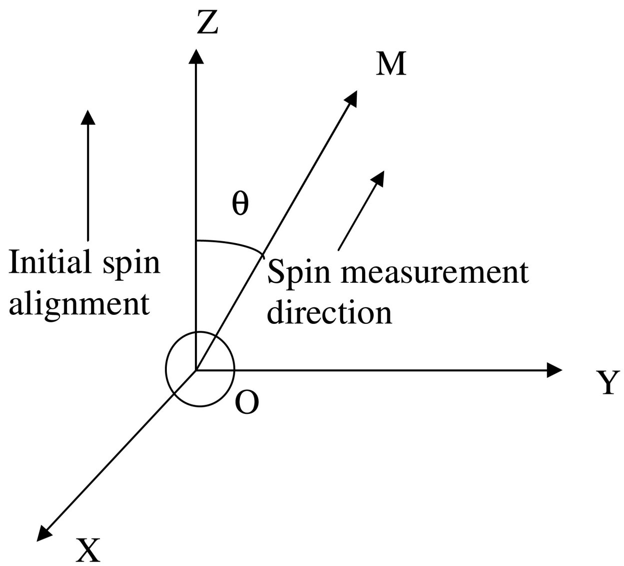

Fig.1 Direction of spin measurement (Initially particle spin is aligned along OZ and then spin is measured along OM)

However, if we carry out the experiment on a collection of large number of identical particles, *the probability **p** will be so biased by nature that total angular momentum of the collection is conserved as far as possible along OM in which direction the particles have freedom to have any one of the two possible spin values*. If ***p*** is the probability of getting spin $+\hbar/2$ along OM, then (1-***p***) is probability of getting spin $-\hbar/2$. For N number of identical particles on which experiment is carried out, to satisfy the law of conservation of angular momentum along OM,

Initial total angular momentum along OM= Final total angular momentum along OM

Or $$N\frac{\hbar}{2}\cos\theta = pN\left(+\frac{\hbar}{2}\right) + (1-p)N\left(-\frac{\hbar}{2}\right)$$

Or $$p = \frac{1+\cos\theta}{2} = \cos^2\left(\theta/2\right) \tag{1}$$

Thus, Eq. (1) exactly reproduces the conventional quantum mechanical probability distribution to get spin $+\hbar/2$ along any arbitrary direction. We have derived it by use of the concept of biased will of nature without applying quantum mechanical operators and Born's rule.

3. Derivation of spin correlation in quantum entangled particles using theory of biased will:

Spin correlation in a pair of entangled particles A and B is given by the product of their measured spins along pre-decided directions (spins are taken to be +1 or -1 excluding the constant part $\hbar/2$ or $\hbar$). If we select to measure the spin of particle A along unit vector $\vec{a}$ and spin of particle B along $\vec{b}$, quantum mechanics predicts that expectation (or average) value of spin correlation is given by,

$$\left\langle P(\vec{a},\vec{b}\right\rangle = -\vec{a}\cdot\vec{b} = -\cos\theta \tag{2}$$

Where θ is the angle between unit vectors $\vec{a}$ and $\vec{b}$.

Above quantum mechanical spin correlation has been experimentally validated by numerous authors. Now we will derive the same average spin correlation given by Eq. (2) using the concept of biased will of nature without using quantum mechanical formalism.

Let us consider N number of entangled pairs (or twins) of particles A_1B_1, A_2B_2, A_3B_3,….. A_NB_N emerging from a common source of spin zero. Particles in each pair A_iB_i move in opposite directions before being exposed to experimental setups for measurement of spin along certain directions $\vec{a}$ and $\vec{b}$ as shown in Fig.2.

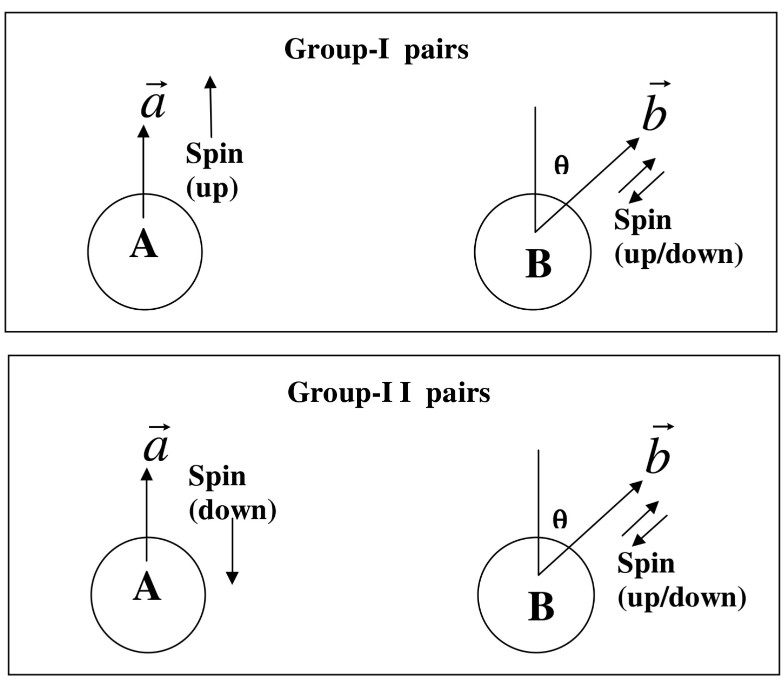

Fig.2 Two different groups of members in ensemble of entangled pairs
(For Group-I pairs, first measurement result is spin-up and for Group-II it is spin-down)

Although spin is a property of individual particle, the correlation (or product) of spins in a twin is only a *property of twin* which can be +1 (if A_i and B_i both are aligned along or opposite to $\vec{a}$ and $\vec{b}$ respectively) or -1 (if either of A_i and B_i is aligned along and other is opposite to pre-decided direction). So, for the measurement of correlation, each twin or pair A_iB_i represents a *single member* (or coherent pair) in statistical ensemble of N number of twins. However, measurement of spin is a two step process in which at first we have to measure the spin of (say) A_i and then go for B_i. Only after knowing the spin state of B_i, we get to know the spin correlation. Just after the spin measurement of A_i (and before measuring B_i), all the members in the ensemble cannot be considered identical since some of A_i's have spin along unit vector $\vec{a}$ and others have spin opposite to it. If p_A is the probability of getting spin of A_i along direction $\vec{a}$, $p_A N$ members are exactly identical in the sense that all of them have A_i along $\vec{a}$ before expressing their spin correlation. Let us call this group of members which are identical before experiment on B_i as Group-I. Similarly, $(1-p_A)N$ numbers of pairs constitute identical members of Group-II all of which have spin oppositely aligned to direction of $\vec{a}$ as shown in Fig.2.

Now for Group-I, if p_B is the probability of getting spin along the direction of $\vec{b}$, as per our axiom of biased will of nature, p_B will be such that total angular momentum in group-I along the measurement direction $\vec{b}$ is conserved (As only in this direction, particle B_i has freedom to have any spin). Since, initial angular momentum of all twins before birth is zero, final angular momentum along the direction of $\vec{b}$ in Group-I which has $p_A N$ members is also zero. So mathematically from Fig.2, we get,

$$p_A N \cos\theta + p_B (p_A N)(+1) + (1-p_B)(p_A N)(-1) = 0$$

Or
$$p_B = \frac{1-\cos\theta}{2} \tag{3}$$

Now, all of $p_A N$ members in group-I have one partner spin along $\vec{a}$ and $p_B p_A N$ members have other partner spin along the direction $\vec{b}$. So, correlation or product of spin for each of $p_B p_A N$ members is +1. Hence, $(1-p_B)p_A N$ members have correlation equal to -1.

Expectation (or average) value of spin correlation in Group-I is then given by,

$$\left\langle P(\vec{a},\vec{b}) \right\rangle_I = \frac{\textit{sum of correlations of all members}}{\textit{Number of members}}$$

Or
$$\left\langle P(\vec{a},\vec{b}) \right\rangle_I = \frac{p_B p_A N(+1) + (1-p_B) p_A N(-1)}{p_A N}$$

Or
$$\left\langle P(\vec{a},\vec{b}) \right\rangle_I = 2p_B - 1$$

Putting the value of p_B from Eq. (3) in above,

$$\left\langle P(\vec{a},\vec{b}) \right\rangle_I = -\cos\theta \tag{4}$$

Similarly, we can proceed to calculate the average correlation for Group-II which has $(1-p_A)N$ number of identical members all of which have spin oppositely aligned to direction of $\vec{a}$ as shown in Fig.2. If p'_B is the probability of getting spin along the direction of $\vec{b}$ in group-II, as per our theory of biased will of nature, it will be such that total angular momentum in group-II along the measurement direction $\vec{b}$ is conserved. Since, initial angular momentum of all twins before birth is zero, final angular momentum along the direction of $\vec{b}$ is also zero. From Fig.2, mathematically we get,

$$-(1-p_A)N\cos\theta + p'_B(1-p_A)N(+1) + (1-p'_B)(1-p_A)N(-1) = 0$$

$$p'_B = \frac{1+\cos\theta}{2} \tag{5}$$

Product of spin for each of $p'_B(1-p_A)N$ members is -1. Hence, $(1-p'_B)(1-p_A)N$ members have correlation equal to +1. Average value of spin correlation in Group-II is then given by,

Or $$\left\langle P(\vec{a},\vec{b})\right\rangle_{II} = \frac{p'_B(1-p_A)N(-1) + (1-p'_B)(1-p_A)N(+1)}{(1-p_A)N}$$

Or $$\left\langle P(\vec{a},\vec{b})\right\rangle_{II} = -2p'_B + 1$$

Putting the value of p'_B from Eq. (5) in above,

$$\left\langle P(\vec{a},\vec{b})\right\rangle_{II} = -\cos\theta \tag{6}$$

Thus from Eq. (4) and (6), we find that irrespective of whether the twin is in Group-I or II, the average value of correlation is $(-\cos\theta)$. So, we can generalize the result and write the expectation value of correlation as,

$$\left\langle P(\vec{a},\vec{b})\right\rangle = -\cos\theta \tag{7}$$

Thus, we could derive the expectation value of spin correlation in entangled particles which exactly matches with the relation derived by conventional quantum mechanical formalism. We can prove that Eq. (7) is relativistically invariant i.e. it holds independent of state of motion of observer. The proof is as follows. If measurements of spin of A_i and B_i are carried out in space like separated regions, certainly for some observers, event at B_i will happen before A_i. In that case, those observers will classify the members to Group-I and Group-II as per the spin result at B_i and apply the law of conservation of angular momentum along direction $\vec{a}$ since they can know about the correlation only after spin measurement of A_i. Thus, adopting a similar procedure as before, they will also derive the same spin correlation given by Eq. (7).

It is interesting to analyze the case of a single pair of entangled particles when spin is measured along same direction for both of them ($\theta = 0$). It has been a surprise to everyone how one particle in the pair gets knowledge about the spin of other partner so that it can be aligned in opposite direction relative to other especially if both are separated by space-like region and superluminal speed of information is not allowed. We can understand this from Eq. (7) which dictates that for $\theta = 0$, average spin correlation is perfect i.e. $\left\langle P(\vec{a},\vec{a})\right\rangle = -1$. So, if there are millions of pairs of particles with which experiments are carried out, each of them must contribute a spin correlation of -1 (none +1) to the sum of correlations so that average remains -1 otherwise it will shift towards +1. *This means, each of the millions of pairs must have opposite spin which we can state in terms of probability that "probability for getting opposite spin must be one"*. So, even if we carry out the experiment on a single entangled pair, it must show opposite spins in its partners. Thus we conclude that opposite spin observed in a single pair of entangled particles is not due to superluminal information transfer from one to other, rather it is due to the fact that *spin correlation is a property of pair as a whole* (which can be called coherence) and it becomes -1 due to Eq. (7). Avoiding the superluminal information transfer is important as it violates principle of causality since ordering of two events occurring in space-like separated regions can be changed by state of motion of observer.

4. Quantum mechanical wave function of a free particle from biased will of nature:

It is well known that interference of matter waves can be explained only if generalized quantum mechanical wave function of a free particle of momentum p and energy E is taken as,

$$\psi(r,t) = Ae^{\frac{i}{\hbar}(\vec{p}\cdot\vec{r}-Et)} \tag{8}$$

Where r is space coordinate, t is time, A is a constant and $\hbar$ is reduced plank's constant. Fundamental justification for the above form of wave function is also very important as it the only basic equation from which the two conventional quantum mechanical operators (momentum and energy operators) in formalism are derived. In this section, by using our biased will approach, we will theoretically derive the generalized form wave function given by Eq. (8).

Let us suppose that the complex function related to the extent of presence of a single free particle at any point of space-time is given by,

$$\psi(r,t) = f(r,t)e^{ig(r,t)} \tag{9}$$

Where, magnitude $f(r,t)$ and phase $g(r,t)$ are two arbitrary real functions of space-time. If we consider the extent of presence of particle at each point of space 'r' as an independent variable, then each of these values of $\psi(r,t)$ can be represented as an orthogonal vector $\psi(r,t)\left|r\right\rangle$ (where $\left|r\right\rangle$ is unit eigen vector) in a multidimensional mathematical space (called Hilbert space) and the total $\left|\psi\right\rangle$ is represented as a vector sum of these components. But due to quantization of the presence of the particle, it can only be detected at a single point in space at any time. So, each point of space will have a probability for appearance of the particle. To have a probabilistic interpretation of $\psi(r,t)$, ***total presence must be equal to one (i.e.*** $\left|\psi\right\rangle$ ***must be a unit vector) and it must be written as a sum of its scalar components***. In any multidimensional vector space, the resultant can be written as a scalar sum of projections of its vector components along itself (i.e. $\left|\psi\right\rangle$) as all of them are collinear.

Since projection of $\left|\psi\right\rangle$ on $\left|r\right\rangle$ is $\left|r\right\rangle\left\langle r|\psi\right\rangle$,

projection of $\left|r\right\rangle\left\langle r|\psi\right\rangle$ along $\left|\psi\right\rangle$ is $\left|\psi\right\rangle\left\langle\psi|r\right\rangle\left\langle r|\psi\right\rangle = \left|\psi\right\rangle\left|\psi(r,t)\right|^2$.

Thus each projection of the total system along position eigen vectors contributes a collinear component to constitute the total system. So, all these magnitudes can be added to get,

$$\left|\psi(r_1,t)\right|^2 + \left|\psi(r_2,t)\right|^2 + \left|\psi(r_3,t)\right|^2 + \ldots\ldots\ldots\ldots = 1 \tag{10}$$

Now since the above Eq. (10) is a scalar equation, we can have probabilistic interpretation and conclude that $\left|\psi(r,t)\right|^2$ is the probability density of physically finding the particle at r at time t (Here, we have actually proved Born's rule).

Since for a free particle, space must be physically *symmetric a*nd particle must *continue to exist* with time (*these are the biasing by the universe*), probability $\left|\psi(r,t)\right|^2 = \psi^*(r,t)\psi(r,t)$ must be independent of r and t for the case of *free* particle. This indicates, using Eq. (9), $\psi^*(r,t)\psi(r,t) = \left(f(r,t)\right)^2$ must be independent of space-time i.e. $f(r,t) = A$, where A is a constant. Thus Eq. (9) reduces to,

$$\psi(r,t) = Ae^{ig(r,t)} \tag{11}$$

To know the mathematical form of function $g(r,t)$ (whether it is a linear or nonlinear function of space time), for simplicity, let us consider only one coordinate say, *x*.

So,
$$\psi(x) = Ae^{ig(x)}$$

Generally the phase is given with respect to a reference angle which can be arbitrarily chosen by different observers. But, difference of phase between any two space points must be same for all observers stationary with respect to each other irrespective of their location (*required for symmetry of space*). If $g(x_1)$ and $g(x_2)$ are phases at points x_1 and x_2 as recorded by observer located at 'O' and $g(x_1')$ and $g(x_2')$ are phases recorded at same points by another observer having a shifted origin,

$$g(x_2) - g(x_1) = g(x_2') - g(x_1') \qquad (12) \qquad \text{and} \qquad x_2 - x_1 = x_2' - x_1' \qquad (13)$$

Dividing Eq. (12) by (13),

$$\frac{g(x_2) - g(x_1)}{x_2 - x_1} = \frac{g(x_2') - g(x_1')}{x_2' - x_1'}$$

In differential form,

$$\frac{dg(x)}{dx} = \frac{dg(x')}{dx'}$$

Since both sides of above equation are functions of different variables, the ratio must be a constant, say '*k*'. So, $\dfrac{dg(x)}{dx} = k$

Integrating above differential equation and taking $g(x) = 0$ at x=0,

$$\int_0^{g(x)} dg(x) = \int_0^x k dx \qquad \Rightarrow \qquad g(x) = kx$$

Thus we see that phase must be a linear function of x. Since, all four coordinates of space time must be considered at par, phase $g(r,t)$ must also be linear function of space-time. Taking into account the opposite sign of time with respect to space in relativistic space-time metric (+---),

$$g(r,t) = kr - \omega t$$

The constant *k* before *r* happens to be same as $p/\hbar$ and constant ω before *t* happens to be same as $E/\hbar$ where *p* is momentum, *E* is total energy and $\hbar$ is universal constant equal to reduced Plank constant. So we get, $g(r,t) = \dfrac{1}{\hbar}(\vec{p} \cdot \vec{r} - Et)$

Putting the above in Eq. (11), we get,

$$\psi(r,t) = Ae^{\frac{i}{\hbar}(\vec{p} \cdot \vec{r} - Et)} \qquad (12)$$

Above equation is same as the desired Eq. (8) for free particle wave function with fixed momentum and energy. Using this equation, as mentioned in conventional text books [16], we can now prove the quantum mechanical momentum operator to be ($-i\hbar\nabla$) and energy operator to be ($i\hbar \partial/\partial t$). Using Eq. (12), we can also generate interference pattern and form wave packets for localized particles. Thus, we have found that the generalized form of free particle wave function on which the whole of quantum mechanics stands can be derived from our biased will of nature.

5. Conclusion and Scientific implications

In this paper, by assuming that the inanimate particles have a will and that is biased to satisfy the laws of universe such as conservation laws and symmetry of space, we have derived the correct spin probability distribution with angle without using quantum mechanical formalism such as operators and Born's rule. Similarly, by using the biased will, we have exactly reproduced the quantum mechanical spin correlation in entangled pairs of particles. Finally, we have developed a theoretical justification for the form of generalized quantum mechanical wave of a free particle using biased will of nature so that interference and quantum mechanical operators can be derived on which the whole of quantum mechanics stands. Thus, we have proved that origin of quantum mechanical results lies in will of the objects biased by the whole. Scientific implications of above analysis can be significant since we can extrapolate our findings to infer that motivations of the universe not only in form of conservation laws and spatial symmetry but also in form of other intentions such as minimizing potential energy, collective goal of complex system etc. affect the quantum mechanical results. This type of behavior in complex systems has been experimentally observed (adaptive mutation in DNA [13, 17]). Using the above described theory of biased will, we can now get answers to many philosophical questions such as, (a) How do goal oriented systems arise? (b) Is goal oriented behavior a cosmic trend or accidental? (c) What decides the event, causality or teleology? (d) Why do living systems pursue goal of growth and reproduction despite being made up of inanimate objects (like atoms) of the universe? (e) How are intelligent systems different from unintelligent systems?

The answer to first question (a) is that since *all* processes occurring in the universe are governed by quantum physical laws which incorporate the biased will of the nature (as demonstrated in this paper), every system is inherently goal oriented whether we are able to perceive it or not. Answer to second question (b) is that some amount of goal oriented behavior is there in every system since the time of creation. It is purely a physical/cosmic trend, not accidental. That's why repeated generations during evolution of living organisms have made them more and more fit to be adaptable to the changing environment. In response to third question (c), biased will theory of this paper indicates that in addition to causality, result is also affected by teleology i.e. a purpose in future. To answer the question (d), we can consider the recently reported results that quantum coherence (or entanglement) has been observed even in macroscopic many-body systems [18]. So, I think, living bodies have life in them because of quantum coherence among their constituent parts. From the observation of laser where a photon hitting an excited atom causes stimulated emission of another photon in coherence with the incoming photon, we can conclude that nature has a tendency to increase the coherence. Since, living beings are coherent assembly of inanimate particles, they try to increase this coherence by pursuing the goal of growth and reproduction. To answer the final question (e), let us first give a functional definition of intelligence i.e. those systems that show goal oriented behavior for self preservation are called intelligent. Thus, all living organisms are intelligent since they pursue self preservation because of quantum coherence. So, necessary part for being intelligent is not the brain, but quantum coherence. However, during evolution, some species (like insects and animals) developed brain which also works on the principle of quantum coherence to make them capable of making more number of goal oriented behaviors which are very much required for survival in a fiercely competitive environment. Of course, present understanding of the quantum coherence in living organisms and brain is at a very preliminary stage, but I am hopeful, human civilization one day will be able to understand and apply this most interesting phenomenon of nature for its benefit.

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