Essay written for the FQXi 2009 contest: What is Ultimately Possible in Physics?

Ordinary Analogues for Quantum Mechanics

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Abstract: Matching quantum behaviour with our ordinary macroscopic experience is commonly regarded to be impossible. The difficulty to relate the quantum world to common sense experience stems partly from the fact that classical physics was not sufficiently advanced to deal with macroscopic particle-wave systems at the birth of quantum mechanics. Physicists therefore lacked references to compare quantum with analogous macroscopic behaviour. After consideration of some recent experiments with droplets steered by waves, we examine possibilities to give some intuitive meaning to the rules governing the quantum world.

Introducing the essay question

Upon pondering over the question "*What is ultimately possible in physics?*", various interrogations emerge. How could one interpret *ultimately*? Is there an *ultimatum*, a final statement in physics, after which one could say "*Physics is finished*"? Are there issues, for instance fundamental principles, beyond which we could not go past? How can we describe the boundary between the possible and the impossible in physics? Anyway, does such a boundary exist? And if so what is at the edge?

If history has a lesson, it is certainly that prophesied terminations of physics end up entirely wrong. 19th century physicists strongly believed that their understanding of nature was near to complete. According to Laplace, an all knowing intellect (also referred to as Laplace's demon) *"would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom"* [1]. In 1871, Maxwell testifies the opinion that "*seems to have got abroad that in a few years all the great physical constants will have been approximately estimated, and that the only occupation which will then be left to men of science will be to carry on these measurements to another place of decimals*" [2, p. 244]. In the 20th century, physicists have become more prudent. For instance, Dirac advanced in 1979 : "*It seems clear that the present quantum mechanics is not in its final form*" [3]. So predicting what is ultimately possible in physics is a risky business. Today's consensus is that there is no end to physics. Kristine Larsen related it to Popper's falsifiability principle: "*There is no end to scientific endeavor. A true scientific theory is always open to be disproved*" [4]. Moreover, the craft of physics implies creative processes. So even if some final-looking form of a theory of everything might emerge, that theory would allow to create and practice new physics that nobody has ever dreamed of. I will therefore focus on a slightly reformulated version of the essay question "*What is possible in physics, that has been declared impossible?*" I will investigate a famous impossibility declaration in quantum physics and illuminate it with some recent experiments that seem to contradict it. This reveals some room for possibilities.

Impossibility to explain quantum interference in any classical way

Since its inception, various impossibility and no-go statements have emerged in quantum physics. Probably, the first impossibility statement one comes across when being introduced to quantum physics, is that it is impossible to fully understand quantum physics, due to a lack of intuitive representation of the on-going processes.

Richard Feynman's quantum lectures introduction may serve as an example: "*We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics*" [5, p. 1-1]. He first reminds us how classical entities behave when they are detected after crossing a screen with two holes. For bullets, "*the probabilities just add together. The effect with both holes open is the sum of the effects with each hole open alone... So much for bullets. They come in lumps, and their probability of arrival shows no interference*" [5, p. 1-3]. For water waves, "*the intensity can have any value, and it shows interference*" [5, p. 1-4]. Electrons, that serve as general example for quantum particles, behave differently. Although they arrive in lumps, their detection pattern is similar to that of waves. There is interference between the contributions of both holes. It is possible to detect electrons one by one, with long intervals between successive electrons and still an interference pattern builds up. Moreover it is impossible to track an electron's path without destroying the detection pattern. If one determines through which hole the electron goes, the interference pattern vanishes. So neither bullet-like analogies, nor wave-like analogies are complete. Feynman further warns "*One might still like to ask: 'How does it work? What is the machinery behind the law?' No one has found any machinery behind the law. No one can 'explain' any more than we have just 'explained'. No one will give you a deeper representation of the situation. We have no ideas about a more basic mechanism from which these results can be deduced*" [5, p. 1-10].

Explaining the double-slit interference for quantum particles in a "clear and ordinary way"

John Bell, who like Feynman dug deep into the foundations of quantum physics, had another opinion about possible machineries behind quantum interference patterns. Let me quote at length an instructive text from one of his papers dealing with the different interpretations of quantum physics [6]: "*While the founding fathers agonized over the question 'particle' or 'wave', de Broglie in 1925 proposed the obvious answer 'particle' and 'wave'. Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.*"

Such peppered opposite viewpoints, Feynman's impossibility to give a deeper representation on one side and Bell's opinion of de Broglie's clear and ordinary explanation at the other side, are common in the field of foundational quantum physics. They feed famous controversies. The least one can say is that there seems to be room for unexplored possibilities, as confirmed by the numerous attempts for alternative interpretations and speculations.

Even before classical "hidden-variable" theories have been ruled out by modern violations of Bell-type inequalities, there were at least two important obstacles that hindered the development of pilot wave theories[1]. Firstly, pilot wave theorists like de Broglie and Bohm did not come up with handy tools. Although they developed its dynamics, their competing arguments generally remained on a

1 The pilot wave model is an example of a hidden-variable theory: a theory aiming at supplementing quantum mechanics with a classically deterministic framework. John Bell showed that such a theory could be tested experimentally against conventional quantum mechanics, with the help of an inequality that emerges from correlated measurements of entangled (= quantum-correlated) photons. Since the 1980s, numerous experiments have showed that the locally causal hidden-variable theories are ruled out.

metaphysical plane. Theorists like Dirac or Feynman, on the contrary, developed concise ket algebra for the former, path-integral diagrams for the latter. For the professional physicist, tools[2] are more important than deeper interpretations of quantum mechanics. So it is little wonder that Feynman's voice resonates stronger than de Broglie's.

The second main obstacle for pilot wave models is the fact that, at the birth of quantum theory, classical physics was not sufficiently advanced to handle ordinary pilot waves. Pilot wave based quantum theory could not benefit from any historical classical research on pilot wave systems. Therefore one could hardly see any advantage to this model, certainly not the young theorists, like Pauli, Dirac, Heisenberg. Why use pilot waves, if there is not any analogous classical pilot wave theory which one could compare to quantum physical phenomena? De Broglie and Bohm focused on quantum world applications of pilot wave theories, but never tried to put them to the test with ordinary macroscopic physics.

Ordinary macroscopic pilot wave experiments

Louis de Broglie first invoked the idea of a pilot wave in 1924 [8]. Incidentally, the first experiments on ordinary pilot waves with embedded particles were performed some 80 years later [9]. Yves Couder and his group managed to let droplets bounce on liquid oscillating surfaces for very long times. The wave generated by the droplet steers the path of that same droplet at consecutive bounces, causing it to become a so-called walker droplet [10][3]. The wave steering the trajectory of the droplet is therefore a classical instance for a pilot wave. These experiments are an interesting example of what Maxwell called "*illustrative experiments*": "*their aim is to present some phenomenon to the senses of the student in such a way that he may associate with it the appropriate scientific idea*" [2, p. 243]. They are a nice pedagogical alternative to the dominant paradoxical bullet and wave analogues. The bouncing droplets arrive in lumps, they are sustained by waves and are themselves sources of waves. They show characteristic interference and diffraction patterns for dual wave-particle systems [11]. If some external influence disturbs the bouncing droplet, the wave and the droplet decohere and the interference pattern fades away. If one considers the interference effect, this system behaves just like the electrons in Feynman's lecture. Furthermore, these experiments have an adjusting value for pilot wave theories, because their experimental behaviour shows unexpected features. This might be taken into account by pilot wave theoreticians. In reality, pilot waves do not exactly behave the way de Broglie and Bohm theorized their behaviour in quantum physics. An interesting feature is that visited locations continue to act as wave-sources, providing non-local influences. Clustering occurs when nearby droplets get caught by other droplets' wave-fields, yielding a whole set of quantized orbitals [12]. Unpredictable tunnelling takes places in the macroscopic domain when a bouncing droplet happens to approach close to the barrier [13]. This research field is wide open, as concluding remarks of Ref. [13] testify : "*More work is needed to understand in depth the relation between the trajectories of the droplet and its waves. Of particular interest is the link between the memory effect due to the superposition of past waves and the observed uncertainty. Although our experiment is foreign to the quantum world, the similarity of the observed behaviours is intriguing.*"

The characteristic feature of these ordinary pilot wave experiments is the periodical motion of a particle (= the droplet) that serves as a source for the wave, which in turn feeds back to the motion of the particle. Therefore, a wave embedding a bouncing droplet exhibits unexplored wave-particle

2 See David Kaiser's *Drawing theories apart* [7] for an exemplar study of the influence and dispersion of tools in physics.

3 Bouncing and walker droplets in pilot-waves may be viewed in the video conferences available on internet: *Une dualité onde-particule à échelle macroscopique ?* Yves Couder, 19 octobre 2006, and *La goutte, un exemple de dualité onde matière*, Yves Couder, 9 mai 2007

duality.

In principle, ordinary pilot wave systems are not limited to droplets and liquid surfaces. We could imagine other wave environments embedding other little travelling objects, whose periodical motions have same phase. Little oscillating springs, vibrating strings or rotating needles in a radio-wave environment might do. David Bohm and Basil Hiley suggested a pilot wave with active information [14]. With the technological advancement in radio frequency signals, steering the phase of a particle seems to be attainable in physics. It is worthwhile to perform research on such systems, not only as pedagogical illustrative experiments, but also in order to test to which extent they simulate quantum behaviour.

So, contrarily to the situation at the birth of quantum mechanics, ordinary pilot waves are now in the experimental domain. The de Broglie - Bohm interpretation, which was speculative in the 20^{th} century, has therefore become testable for macroscopic systems. It is of interest to reconsider quantum pilot waves with this return of experience.

Tools for ordinary quantum analogues

The other obstacle, the lack of computational tools for pilot wave theories, remains. In order to fully apprehend the features of real ordinary pilot waves, more experimental research is needed. New theoretical tools will probably emerge for their description. This undoubtedly will take several years.

Upon closer examination of the several possible ordinary pilot waves, there may however be cases for which existing quantum tools are suited. For example, a system where rigid straight line-shaped particles, say needles, are embedded in a cloud of identical needles seems particularly suited to be described by conventional quantum computational tools. Each needle may be represented by a vector of same length superposed on the needle. For instance, a needle whose centre is located at (x,y,z), may be represented by a vector $\boldsymbol{u}(x,y,z,\varphi)$ with centre at (x,y,z) directed along the spatial orientation φ of the needle. Classical models have already been developed in this sense [15][16]. However, determining the dynamics in such systems is no simple matter due to the rotational motion and the line-shape of the needles. Mukoyama and Yoshimura report that "*the scattering process is very sensitive to the rotational phase*" [17].

Alternatively, we could apply the whole quantum mechanical machinery to this system[4]. We rewrite vector $\boldsymbol{u}(x,y,z,\varphi)$ as a quantum-mechanical ket-vector $|x,y,z,\varphi>$. We define a measurement process as the determination of impact positions on a screen. So, observing the trajectory of a needle by sight is not allowed as a measurement process. The measurement result is then determined by a point localizing the impact. Because the needle has some length, the result will always show an indetermination equal to the length of the needle. There is an unbeatable uncertainty in measurement processes for needles, that emerges from the fact that the observable "impact position" never gives a full picture of the needle. The possible measurement results are spread out over the volume swept by the needle. Furthermore, as long as we have not detected a needle with another needle, its state is unknown. So the state may be represented mathematically by a weighted sum of all possible locations and orientations. Formally, it is in a state of superposition of all possible states:

4 The vector is the mathematical abstraction for an arrow. In quantum mechanics, vectors are named kets and written as |label>, where "label" specifies the arrow, for example with physical properties or with a greek letter. An illustration of the description of a needle with ket-vectors may be found in the video "*Quantum probabilities with ordinary objects*", Arjen Dijksman, January 1st 2009, available on internet.

$$|\psi> = \text{sum}\ (a_i\ |x_i,y_i,z_i,\varphi_i>) \qquad (1)$$

where a_i denotes the normalizing weight of each possible location and orientation. So $|\psi>$ is a statical snapshot description of our knowledge of the system.

The advantage of using quantum-mechanical state vectors is that the dynamical parameters of the needles may be included in the ket notation. If the needle is travelling with a velocity $\boldsymbol{v}$ and rotating at angular velocity ω about a given axis, we abstract it into the vector $|x,y,z,\varphi,\boldsymbol{v},\omega>$[5]. The generic snapshot 3-dimensional vector $|x,y,z,\varphi>$ therefore is in fact a short hand notation for the set of dynamical states $|x,y,z,\varphi,\boldsymbol{v},\omega_1>$, $|x,y,z,\varphi,\boldsymbol{v},\omega_2>$, $|x,y,z,\varphi,\boldsymbol{v},\omega_3>$, etc.

So again, if we do not know the exact dynamical state of the needle, formally, it might be seen as a weighted sum over all possible states :

$$|\psi> = \text{sum}\ (a_i\ |x_i,y_i,z_i,\varphi_i,\boldsymbol{v}_i,\omega_i>). \qquad (2)$$

In this way, a single needle is described in an infinite dimensional vector-space, taking account of all its physical properties. Moreover, the whole cloud of needles may also be represented by a vector (i.e. a vector of vectors).

Freely rotating needles rotate about their centre. In the frame of reference of a needle, the velocity of the tips of the needle is directed perpendicularly to the needle itself. Therefore, using the head to tail rule, the vector difference $d|\psi> = |\psi_2>-|\psi_1>$ between two subsequent snapshots of $|\psi>$ will make a 90° angle with $|\psi>$, in the limit that the snapshots are taken infinitesimally proximate. Mathematically, with $d\varphi$ the small angle over which the needle has rotated, this may be written with complex notation[6]:

$$\exp(j\pi/2).d|\psi> = |\psi>.d\varphi \qquad (3)$$

This trivial differential equation may be written in different ways. The first factor may be written as the imaginary unit j. Depending on the varying quantities, we could express the infinitesimal angle $d\varphi$ as a product $\omega.dt$ of instantaneous angular velocity and infinitesimal time (provided we have defined time) or as a product $k_x.dx$ of instantaneous wave number and infinitesimal coordinate difference directed along the x-axis, etc. We could also multiply both sides by a constant number. A suggestive way to write the differential equation of evolution of needle-like objects is :

$$j\ \hbar\ d|\psi>/dt = \hbar\ \omega\ |\psi> \qquad (4)$$

where the inserted constant may have any appropriate value that transforms angular velocities into another physical quantity. In this form, physicists will immediately recognize the similarity with the time-dependent Schrödinger equation which describes the behaviour of quantum particles and which has been discovered along a completely different path in 1926 [18].

If the needle is placed in a dense field of other needles, it will constantly be disturbed by collisions.

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5 Freely rotating needles have in fact 2 degrees of independent rotational freedom. It would be more precise to write : $|x,y,z,\varphi,\omega_f,\omega_s>$. The same is valid for the orientation in 3D, which should be written with respect to two axes (declination and right ascension).

6 This equation means that if we rotate the little arrow joining the tips of two subsequent snapshots of the needle by a an angle of $\pi/2$ radian (= 90°), we get an arrow that represents the needle times half the measure of the little angle over which the needle has been rotated. The letter j denotes the imaginary unity (i is already used as index for the possible states).

The instantaneous angular velocity ω must therefore be decomposed into several components, each component describing the influence of each external perturbation. The dynamics become tricky but the general form remains.

$$j \hbar\, d|\psi>/dt = (\hbar\, \omega_0 + \hbar\, \omega_1 + \hbar\, \omega_2 + \hbar\, \omega_3 + ...)\, |\psi> \quad (5)$$

The different ω_i-components denote the constituent internal motions of the needle, due to interaction potentials in the cloud of pervading needles, so we could write it equivalently as some sort of energy or potential terms.

$$j \hbar\, d|\psi>/dt = (E_0 + E_1 + E_2 + E_3 + ...)\, |\psi> \quad (6)$$

Written under this form, it is a truly unifying equation describing the way objects of two different domains evolve: quantum particles in the microscopic domain and rod-like objects in the macroscopic domain. However different macroscopic needles might be from quantum systems, the analogy is manifest. The evolution of macroscopic needles follows the same general mathematical rule as the evolution of quantum systems. Unlike its quantum counterpart, the macroscopic formulation has a straightforward interpretation, which may be expressed in ordinary words[7]. We could say for example that the velocity of the needle tips due to self-rotation is always perpendicular to the arrow itself, and that this velocity is proportional to the product of 3 numbers: the angular velocity, the infinitesimal time over which this difference has been taken and the length of the needle.

Two needles may simply scatter with gain or loss of rotational motion. Two needles may also scatter with multiple successive contacts, which are known as chattering collisions [17] and so form temporary composite systems. Vortices may occur [19]. Three needles may aggregate strongly. And aggregates of aggregates may form even bigger structures. There are various similarities with the quantum world which may be specified more precisely through experimental and theoretical research on interactions in dense clouds of line-shaped particles.

The pilot wave approach for clouds of needles is of interest when the orientation of the needle varies in phase with collective motions of the surrounding needles. The evolution equation (6) can then be seen as a wave equation describing as well the dynamics of the single needle as the collective motion of the surrounding needles. The needle orientation and the phase of the wave evolve in coherence. The orientation φ may be seen as a wave property and abstracted as a complex exponential $\exp(j\varphi)$ outside the ket. Only those needle orientations that are compatible with the wave are physically relevant for pilot-wave systems. The normalization coefficient in equation (2) therefore becomes complex :

$$|\psi> = \text{sum}\,(a_i \exp(j\varphi_i)\, |x_i,y_i,z_i,\boldsymbol{v_i},\omega_i>) \quad (7)$$

The projection of a needle on a given axis will evolve sinusoidally. This will affect scattering probabilities. Two needles will therefore collide with a probability proportional to the projection of the first needle times the projection of the second needle on the axis perpendicular to the line joining the centres of both needles. As all needles evolve in phase with the same pilot wave, the probability to detect a needle with another needle is therefore equal to the square of the normalized

7 There are popular quotes attributed to Rutherford or Einstein that say: "*If you can't explain something to a six-year-old, you really don't understand it yourself*", "*You do not really understand something unless you can explain it to your grandmother*", "*If you can't explain your physics to a barmaid it is probably not very good physics*"... With ordinary analogues, quantum mechanical principles may be explained to grandmothers, to children and to barmaids.

wave-function, a result which is analogous to Born's interpretation of quantum probabilities.

A pilot wave also reduces considerably the number of possible composite needle systems mentioned earlier. The spinning degrees are then no longer independent, but related to the fundamental frequencies of the pilot wave and therefore induce discrete spinning ratios. This is a feature that is also present in quantum particle physics, but not in classical bullet or billiard ball physics. So intuitively, the physics of dense clouds of rotating needles is nearer to quantum physics than to classical physics. Although their behaviour may be handled as well with classical as with quantum computational tools, my preference goes to the latter because they are operational in both microscopic and macroscopic domains and help to grasp intuitively some weird-declared behaviours of quantum systems. It is a nice manner to "*provide a sensible language for the theory*" and "*a new way of reading the equations*"[8][20].

A complete description of the behaviour of needle-shaped objects goes behind the scope of this essay. All in all, it is an example among others that reveals room for possibilities, even beyond declared impossibilities.

Conclusion

We started with an impossibility statement that forbids giving deeper representations of quantum processes. We then noticed that there are actually classical experiments which give results analogous to the quantum world and ended up with an ordinary system with everyday objects that may be described with the help of quantum mechanical computational tools. The declared impossibility to give a deeper representation of single particle interference may therefore be seen, not as a scientifically proven impossibility, but rather as a cultural one, emerging probably from our physics education that prescribes that macroscopic objects be described with classical physics tools. Our ordinary macroscopic world is however neither classical nor quantum-mechanical. It simply is. For most practical purposes, classical tools are best suited to describe macroscopic phenomena, but there are some macroscopic wave-particle systems that may as well be investigated using quantum tools. It is therefore doubtful that ordinary representations of quantum mechanics belong to the realm of impossibilities. Faraday noted in his laboratory book: "*still try, for who knows what's possible*" [21]. This requires that one doubts certainties, constantly. Ultimately in physics, it will always be possible to question acquired knowledge.

"*Thus the questions (and the quest) go on.*" ~ George Sudarshan [22]

8 Lee Smolin identified five problems in contemporary physics. The second problem deals with the foundations of quantum mechanics and might be resolved "*by making sense of the theory as it stands*".

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