

How the Totality of Mathematics Shapes Physics

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Abstract

I explore the idea that the Cosmos and the Physics that rules it are a product of the totality of Mathematics. Mathematical objects like E8 and the Mandelbrot Set are important landmarks of Math's internal structure – and we are fortunate to have mapped them – but there is much more to discover. There may be other constructions of pure Math, yet to be revealed, which like these examples exist as timeless ideals or archetypes of form that shape natural law by their very existence. If nature is ruled by Mathematics in some measure, it has been shaped by *all* of the applicable Maths since the beginning of time – and nature therefore already honors mathematical rules we have yet to learn. This gives Mathematics, and its study, profound relevance to progress in Physics. Some of the Maths we already know about may play a greater role in Physics than we realize, but the subject of pure Mathematics is also worthy to develop further – because it is likely to reveal many elements of structure and form that are utilized by nature in its laws.

Introduction

It is obvious that there is a similarity between mathematical and physical relations, a shared order and sensibility that makes the two alike in many ways. But as one delves deeper into the physical sciences, it is astounding how mathematical the universe is seen to be, and it makes one wonder if perhaps the universe is based on Mathematics, or if instead Math is the image of natural law – a product of what we know as Physics. The Math of what makes sense physically and the Physics of what makes sense mathematically are both valid outlooks and worthy topics to consider. In this paper; I mainly discuss ideas about how the totality of Math inspires Physics, but this does not exclude or deny the validity of views that Math is primarily based on relations arising in physical systems, or change the fact that we could not be here to elucidate Math apart from a physical reality for us to inhabit. The fact remains; there are some structures that can be described as mathematical invariants, which arise as unchanging patterns within the core of Mathematics, or enduring features of the mathematical

landscape, that are timeless and exist apart from any specific construction of that form. Sometimes these forms are characterized by their ease of definition, or by their preservation of some well-defined mathematical property, but sometimes we get lucky and a form satisfies several important criteria at once. Such is the case with the humble circle, which is the simplest form that contains space by virtue of its continuous boundary, and has the minimal perimeter of any such figure. But if we write out the equation for the unit circle, $r=1$, we find that we have also defined a whole family of unit spheres S^n – where the circle is designated S^1 and the common sphere is S^2 – merely by sequentially increasing the dimensional range of the operator. So, there is a definite benefit to writing things in the language of Mathematics; we gain insight into a hidden order.

That Mathematics is a precursor to Physics is more difficult to prove, than the utility of Math as a descriptive tool for Physics. But, as shown above, using Math to describe things sometimes leads to generalizations that are not apparent – until we cast things in mathematical language. Thus the utility of Math in Physics goes way beyond playing the part of a tool, or a descriptive framework, as it often reveals hidden properties of a system and new dimensions of a problem and its solution. So using Math to model physical systems is a very effective way to do Physics. Eugene Wigner suggested Mathematics' unreasonable effectiveness in Physics [1], meant that not only should we model physical systems *with* Math, but we should also conclude that physical systems are modeling the order found *in* Math itself. Of course; the idea that forms found in reality spring from a mathematical ideal is not new, and certainly did not start with Wigner, because it is well known that the early Greek scholars including Plato spoke on archetypes of ideal form [2], existing in the abstract, which have limited expressions or projections in the physical realm. Simple figures, like the circle, are examples illustrating that there is mathematical perfection which can be faithfully represented in the physical world. However; there are limitations to how nearly any ideal of form can be approximated by its physical representation – no matter how carefully we craft it. Even so; nature creates, in various ways, precise renderings of ideal forms all around us, such as the Sun and full Moon which depict a circle or sphere rather well.

The question of whether ideal forms can predate their physical representation, and to what extent all physical forms are a representation of their mathematical ideal, remains open. If we give credence to the Mathematical Universe Hypothesis (or MUH) of Max Tegmark [3], Mathematics is a structure existing apart from human minds, which has a life of its own because the concepts are self-consistent or make sense mathematically – and thus it is the basis for physical existence. In my view; this is a quite reasonable outlook, but I first began considering it almost 30 years ago, when I started examining the connections between the Mandelbrot Set and Cosmology [4]. The idea that the Mandelbrot Set can inspire or shape cosmological evolution requires it to somehow be there at the universe's inception, or to exist apart from the universe – which is to say that it is external to the universe we inhabit. So I entertained a proposition similar to Tegmark's MUH long before his framing of it. A related

metaphor I particularly like is the Theory of Theories [5] of Phil Gibbs, which summarized states that real-world Physics is the product of all the mathematical formulations, co-existing in theoretical space, which are averaged in a path integral weighted by each model's utility or applicability to physical relations, and their congruent representation. In Phil's words "the laws of physics are a universal behaviour to be found in the class of all possible mathematical systems." So things are shaped by the totality of Math, instead of just one perfected model. Therefore; while some people look for a singular final theory, I exalt the fact that different theories can all point to a similar result, and in this diversity I find a hidden order.

I offer a rather different view of how the universe springs from pure Math which will give the reader much to ponder. I'll leave important questions unanswered, and issues unaddressed, to clearly elucidate my main point. I will be as rigorous as space allows, though, and I'll likely answer some questions the reader might not ask. I've been pondering these things for some time already, you see. In 1986 I discovered a variation of the algorithm for the Mandelbrot Set, the Mandelbrot Butterfly figure (in Fig. 1 below). What I saw strongly suggested that the Mandelbrot Set – which is a pleasant diversion for many – is also a map of cosmological evolution. In that same timeframe, I had a few phone conversations with Benoit Mandelbrot, and engaged in a period of research, but the idea languished for years before I took it up about 15 years ago, when I resumed my study and began writing for presentation – what the Mandelbrot Set tells us about Cosmology. So I've been exploring the idea of, and context for, a universe springing from Mathematics, for almost 30 years now. The question I've pondered most is "Now that I know it is so, why should this be true?", but it is not so easy to explain why Math of itself should give rise to Physics. It has taken a lot of learning and study, to become qualified to give an adequate description of what I have found in this area – because my work dives deep into the fundamentals of theoretical Physics and the depths of higher Mathematics, and it spans the entire life of the Cosmos, but it is an exciting journey for those who are bold enough to take it.

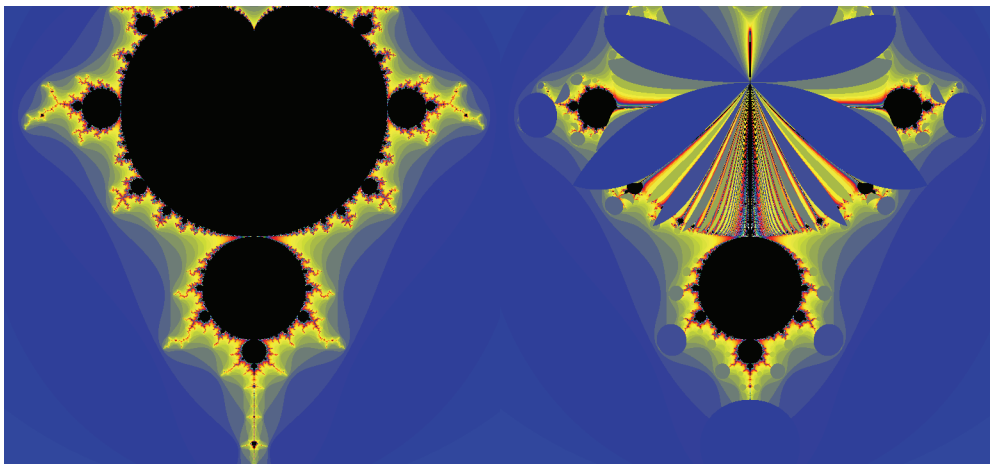


Fig. 1 – The Mandelbrot Set and Mandelbrot Butterfly

Why should pure Mathematics shape Physics?

What makes Mathematics so relevant to Physics, that it seems to be the one indispensable tool, and do those factors also assure that Math has a defining role, by helping to shape natural laws? Many before and since Wigner have asked those questions, and there are a host of different answers. Since I first saw the strong connections to Physics of an object born of pure Math almost 30 years ago; I assume here that Math *must* play a defining role, and I then ask why this is true, knowing that pursuing this knowledge also gives insight into how Math finds expression in Physics. So what determines which mathematical fundamentals find expression in Physics? Is it what most closely mimics physical reality? Intriguingly; I have found that the concepts and entities most central or fundamental to Math also have the greatest relevance to Physics. It is seen that there exist mathematical invariants, objects and patterns that embody fundamental truths about how Math operates, and rules that reflect the internal consistency of various concepts. These universal or invariant patterns find expression across disciplines – and those patterns seem to hold special importance to nature. It seems odd, for example, that there are exactly four normed division algebras, instead of more or fewer. The Real, Complex, Quaternion, and Octonion, number types – and their associated algebras – give us the full range of numerations. Those are all of the well-behaved natural algebras, in terms of repeatability and reversibility. But this array also shows us the limit of sums of squares, the parallelizable spheres, and other related patterns of Math. In fact; one could equally well assert that sums of squares and properties of spheres are what defines the limit on the number of valid algebras.

Of course; the imaginary unit i is itself one of the archetypal entities of Math, and it brings us from Real to Complex numbers, which have both Real and imaginary components. If we instead add 3 or 7 imaginary components; we get the Quaternion and Octonion numbers, respectively. Where the Real numbers and Real components define a fixed extent, the imaginary numbers and components represent a specific range of variability instead – which makes them useful for describing regular variations of all kinds. When we seek laws of nature, this is largely concerned with finding regularities in the changes seen in the natural world, according to Wigner – as with Galileo's demonstration that massive objects all fall at the same rate. But are the regularities seen in nature a reflection of the consistent patterns or internal consistency found in Math, and its ability to provide a context for form to arise? I think there is, but nature works from a larger palette of mathematical structures than those we have discovered or devised. With the help of computers, we have mapped an impressive array of objects heretofore unseen, including the largest exceptional group E8 and my personal favorite the Mandelbrot Set. Given that the Standard Model is based in symmetry groups, I can see why Garrett Lisi's exceptionally simple theory employs E8, as it is the very epitome of preserved symmetries. By contrast; the Mandelbrot Set is a catalog of examples where symmetry preserved at one level of scale is broken for larger structures, as self-similar structures are also conformal at the edges to an asymmetric boundary.

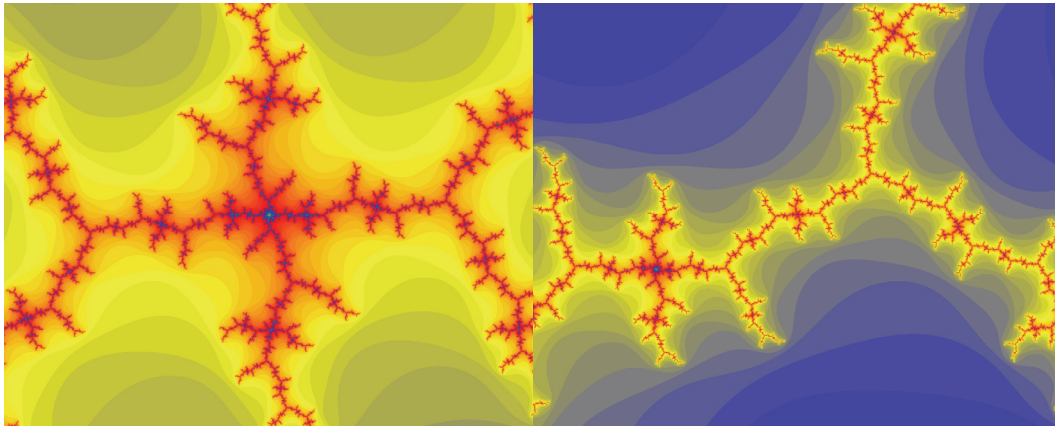


Fig. 2 – A highly symmetrical structure is part of a larger asymmetrical pattern

Final Thoughts

Since I have left myself with no more time for elaboration, I will share a few thoughts on how I think Math in its entirety has shaped Physics, and relate a conversation I had at FFP10. The search for laws of nature continues, and I think the idea there should be a theory explaining everything is sound, but I would prefer to believe that the right answer is actually a fabric of theories that work well together. I think the fact our two prevailing pillars, Relativity and Quantum Mechanics, yield incompatible answers tells us that there are missing pieces of the fabric – which have to be filled in to give us the whole picture. If we need not only the study of symmetry, but the side by side analysis of how symmetries are preserved and broken – then there is a clear road to further progress. But the full answer may not be so clear, as fundamental questions are often unasked. However; I had the opportunity between lecture periods at FFP10, to ask of Nobel laureate Gerard 't Hooft “What does the calculating? Do we need Planck-sized atoms of space?” And he said “We don’t need atoms of space or whatever, because the laws of nature do the calculating for us.” Lacking a better explanation; I am assuming that he means the laws of nature are inherently mathematical. In a nutshell; the laws of Mathematics and the laws of nature are one and the same. The best summary of how Math engenders Physics would be that we will inevitably find, as we continue our quest to discover the depths of higher Math, more and more pieces of an underlying pattern that nature has already put to use.

References

1. Wigner, Eugene – The Unreasonable Effectiveness of Mathematics; *Communications in Pure and Applied Mathematics*, Vol. 13, No. I. Feb 1960, pp. 1-14
2. Early Greeks – Platonic solids from Wikipedia; http://en.wikipedia.org/wiki/Platonic_solid
3. Tegmark, Max – The Mathematical Universe [arXiv:0704.0646](https://arxiv.org/abs/0704.0646) and “Our Mathematical Universe: My Quest for the Ultimate Nature of Reality” [book entry at Amazon](#)

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4. Dickau, Jonathan – Mandelbrot Cosmos, Amygdala Issue 7, Nov 1987
 5. Gibbs, Philip – The Theory of Theories, Cyclotron Notes and ‘Event-Symmetric Space-Time’